Hydrodynamique de films liquides en présence de transferts de chaleur et/ou d'évaporation: prise en compte des effets Marangoni et couplage transferts-hydrodynamique

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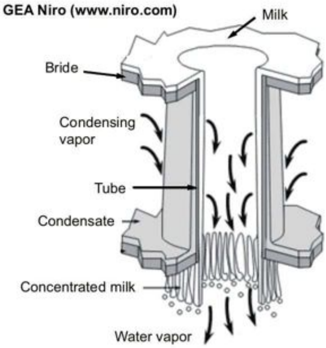
Contents

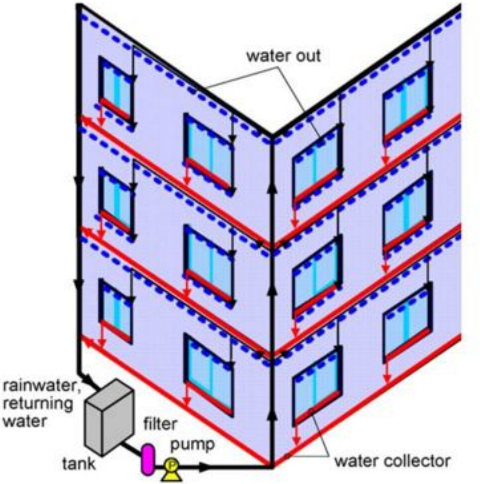
Examples of processes involving falling films

Food industry



- concentration of milk by falling film evaporator

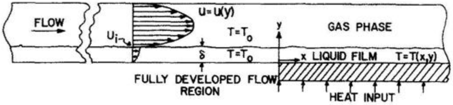
Building industry[1]



- cooling of building surfaces (latent heat)

[1] He and Hoyano Energy and Building (2008)

Waves and heat transfer enhancement[2]



Nusselt number

$$\mathrm{Nu} = \frac{q_w h_N}{k(T_w - T_b)}$$

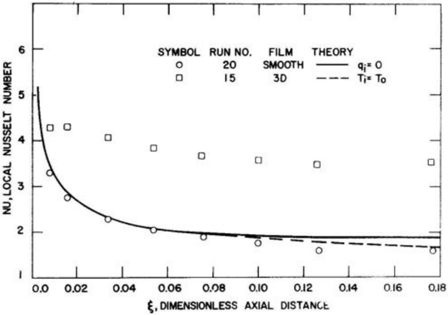
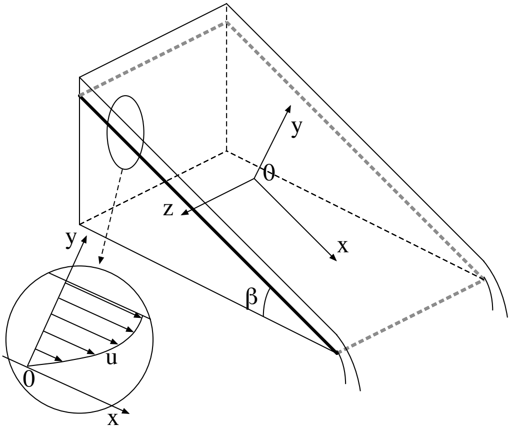


FIG. 9. A comparison between heat transfer results for smooth film flow and three-dimensional wave flow.

[2]Frisk and Davis IJHMT (1972)

Falling film hydrodynamics : Phenomenology

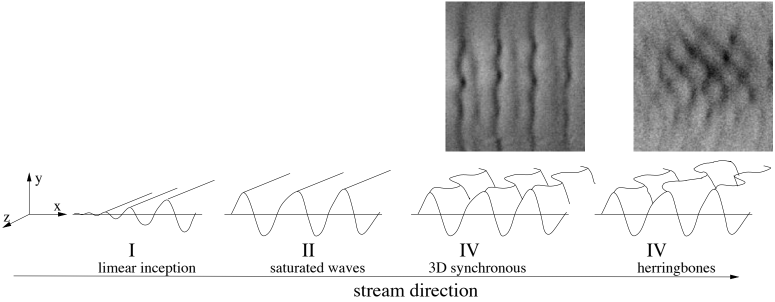
Position



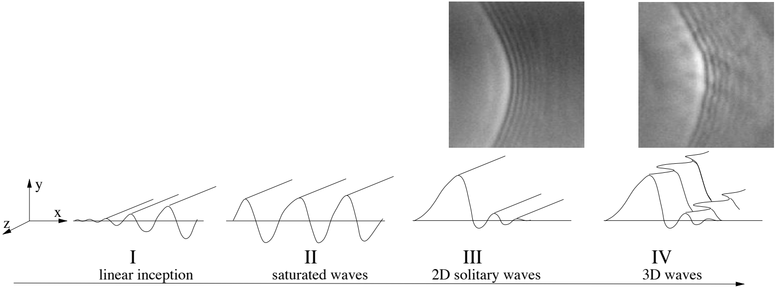
surface tension σ, viscosity μ, density ρ, gravity g
inclination angle β, inlet flow rate per wetted perimeter $\bar{q}_N$
streamwise x, spanwise z, cross-stream y directions

A series of symmetry breakings

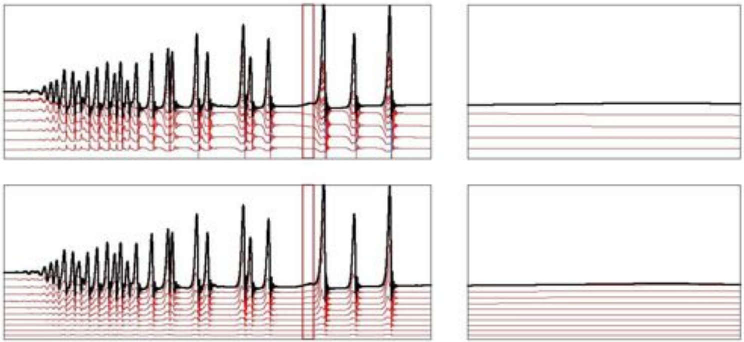
high frequency



low frequency



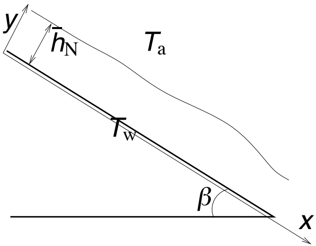
Noise-driven dynamics



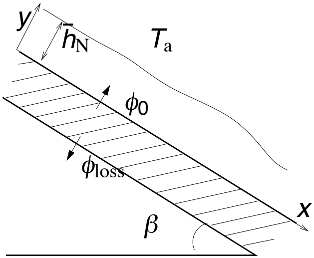
Kapitza experiments[3] alcohol film on vertical wall $Re = 6.07$, $\Gamma = 529$, length $L = 80$ cm, decelerated 8 times

[3]Kapitza & Kapitza Zh. Ekper.Teor. Fiz. 19, 105-120 (1949)

Heated flim flows[4]



specified temperature : ST case



specified flux : HF case

[4]Kalliadasis et al. JFM (2003) ; Ruyer-Quil et al. JFM (2005) ; Scheid et al. JFM (2005) ; Trevelyan et al. JFM (2007) ; Scheid et al. Europhys. Lett. (2008); Scheid et al. PRE (2008)

Evaporation ? : a very crude modelling

- evaporation is modelled through a constant heat transfer coefficient α and Newton's law of cooling $-\lambda \nabla T \cdot \mathbf{n} = \alpha (T - T_0)$
- this assumption works well is the atmosphere can be assumed to be passive (contant pressure, no shear) and the vapor is dilute (only diffusion)
- a better description is a wevenumber dependency of α[5]

[5] H. Machrafi, A. Rednikov, P. Colinet and P.C. Dauby, PRE **91** (2015)

Reduced parameters

Shkadov notations (Shkadov, 1977)

length scale h_N in the y direction
stretched length scale κh_N in the x and z directions
κ tuned such that $g \sin\beta$ and $\sigma \partial_{xxx} h$ are of same order
$\leadsto \kappa = We^{1/3} = (l_c/\bar{h}_\mathrm{N})^{2/3}$

- reduced Reynolds number $\delta = h_\mathrm{N}^3/\kappa = 3Re/\kappa$ with $h_\mathrm{N} = \bar{h}_\mathrm{N}/l_\nu$ and $l_\nu = \nu^{4/3}/(g\sin\beta)^{1/3}$ [6] which measures inertia
- viscous dispersion parameter $\eta = 1/\kappa^2 \ll 1 = (\bar{h}_\mathrm{N}/l_c)^{4/3}$ compares elongational viscosity and capillary damping
- reduced inverse slope $\zeta = \cot\beta/\kappa$
- modified Marangoni number $M = Ma/\kappa = \frac{\gamma \Delta T}{\rho g \bar{h}_\mathrm{N}^2 \sin\beta}\frac{1}{\kappa}$
- Biot number (Newton's law of cooling) $B = Bi\, h_\mathrm{N} = \frac{\alpha \bar{h}_\mathrm{N}}{\lambda} = \frac{\alpha l_\nu}{\lambda} h_\mathrm{N}$

[6] In fact this definition is 45 times Shkadov's original definition

Low dimensional modelling

evolution equations for thickness h, flow rate $\mathbf{q}$ and temperature at free surface $\theta \equiv T(y = h)$

$$
\begin{aligned}
\delta\, \partial_t \mathbf{q} \quad = \quad & \frac{5}{6} h\, \mathbf{i} - \frac{5}{2} \frac{\mathbf{q}}{h^2} \boxed{- \frac{5}{4} M \nabla \theta} + \frac{5}{6} h \nabla (\nabla^2 h) \\
& + \delta \left[\frac{9}{7} \left(\frac{\mathbf{q} \cdot \nabla h}{h^2} - \frac{\mathbf{q}}{h} \cdot \nabla \right) \mathbf{q} - \frac{8}{7} \frac{\nabla \cdot \mathbf{q}}{h} \mathbf{q} \right] \\
& + \eta \left[\frac{13}{4} \frac{\mathbf{q} \cdot \nabla h}{h^2} \nabla h + \frac{13}{16} \left(\frac{\nabla h}{h} \cdot \nabla \mathbf{q} - \frac{\nabla \cdot \mathbf{q}}{h} \nabla h \right) \right. \\
& + \frac{3}{4} \frac{\nabla h \cdot \nabla h}{h^2} \mathbf{q} - \frac{23}{16} \frac{\nabla^2 h}{h} \mathbf{q} - \frac{73}{16} \left(\frac{\mathbf{q}}{h} \cdot \nabla \right) \nabla h \\
& \left. + \frac{7}{2} h \nabla \cdot \left(\frac{\nabla \mathbf{q}^T}{h} \right) + h \nabla \cdot \left(\frac{\nabla \mathbf{q}}{h} \right) \right],
\end{aligned}
$$

where $\nabla = (\partial_x, \partial_z)$, $\mathbf{q} = (q, p)$ and $\mathbf{i}$ is the streamwise unit vector.

averaged energy balance :

$$
\begin{aligned}
Pr\,\delta\,\partial_t\theta \quad = \quad & 3\frac{(1-\theta-Bh\theta)}{h^2} \\
& + Pr\,\delta\left[\frac{7}{40}(1-\theta)\frac{\nabla\cdot\mathbf{q}}{h} - \frac{27}{20}\frac{\mathbf{q}\cdot\nabla\theta}{h}\right] \\
& + \eta\left[\nabla^2\theta + \frac{\nabla h\cdot\nabla\theta}{h} + (1-\theta)\frac{\nabla^2 h}{h}\right. \\
& \left. + \left(1-\theta-\frac{3}{2}Bh\theta\right)\frac{\nabla h\cdot\nabla h}{h^2}\right]
\end{aligned}
$$

mass balance :

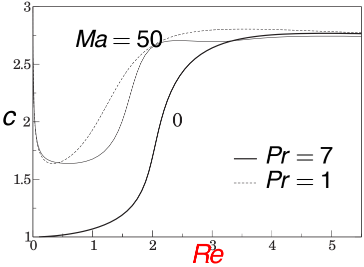
$$
\partial_t h = -\nabla\cdot\mathbf{q}
$$

coherent model at $O(\varepsilon)$ [$O(\varepsilon^2)$ for diffusion terms]

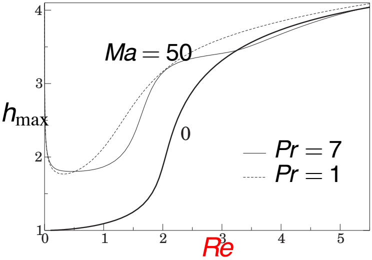
- extension to $O(\varepsilon^2)$ is possible
- intoduction of secondary fields to account for departures from parabolic velocity profile and linear temperature distribution: 9 scalar equations
- Padé-like regularization technique $(\mathscr{R}_0 = \mathscr{G}^{-1}\mathscr{F})$: coherent $O(\varepsilon^2)$ model in terms of 4 equations

hierarchy of models in terms of complexity and accuracy

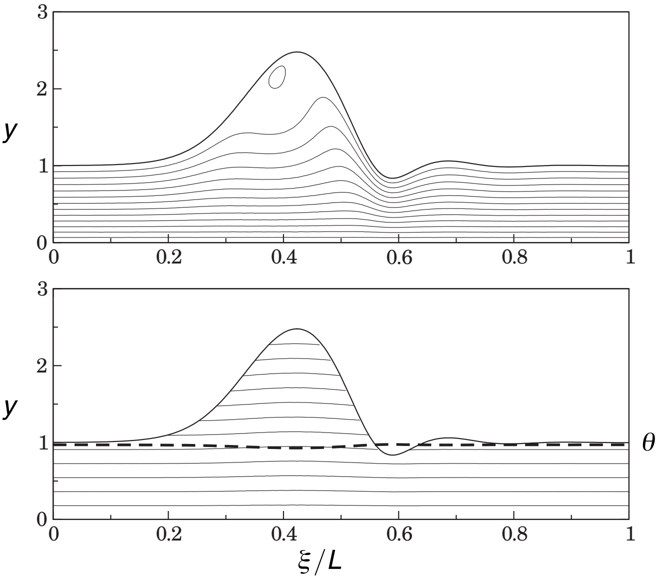
2D flows : Solitary wave solutions



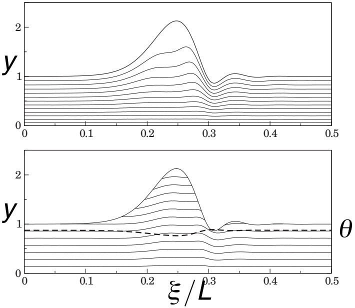
speed

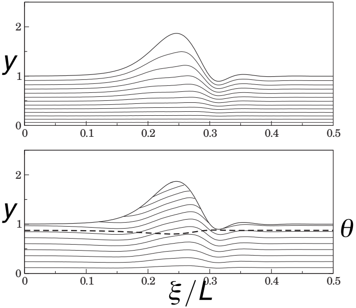


maximum height

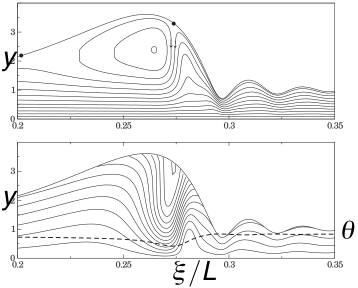


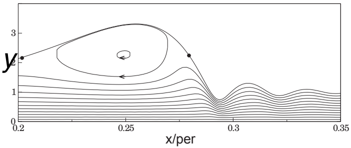
streamlines in moving frame (above) and isotherms (below)
$(Re, Ma) = (0.01, 50)$, $Bi = 0.1$, $Pr = 7$.





$(Re, Ma) = (1, 50)$, $Bi = 0.1$, $Pr = 1$ $\qquad$ $Pr = 7$





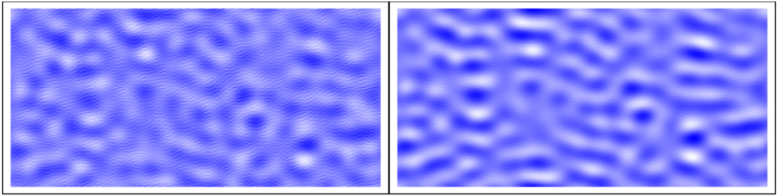
$(Re, Ma) = (3, 50)$, $Pr = 1$ $\qquad$ $(Re, Ma) = (3, 0)$

formation of a thermal boundary layer

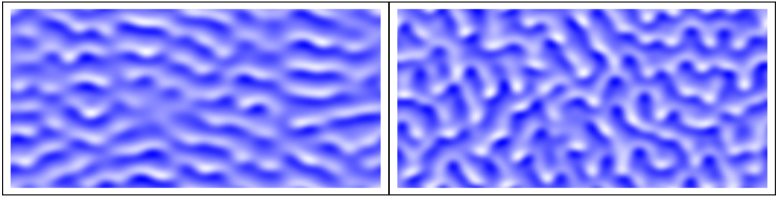
spurious behaviour: free-surface temperature lower than air temperature

3D flows: simulations in a periodic domain

Re=0.5, *Ma*=25, *Bi*=0.1, *Pr*=7 and Γ=3375 (water)

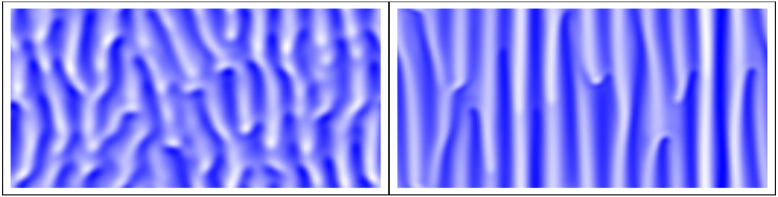


$t = 450 - \{0.998, 1.002\}$ $t = 950 - \{0.991, 1.009\}$



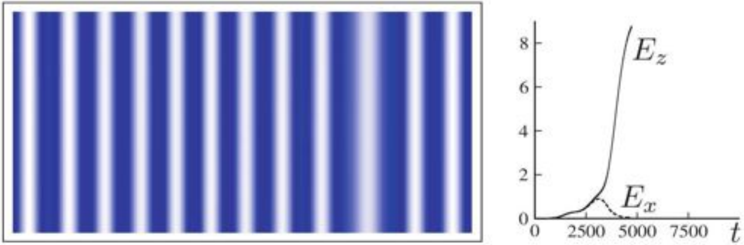
$t = 1450 - \{0.957, 1.044\}$

$t = 1950 - \{0.931, 1.071\}$

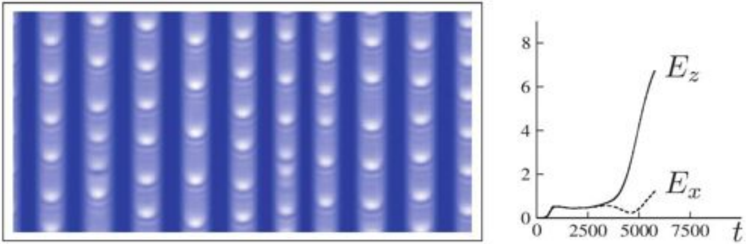


$t = 2450$ – $\{0.899, 1.101\}$ $t = 3450$ – $\{0.692, 1.334\}$

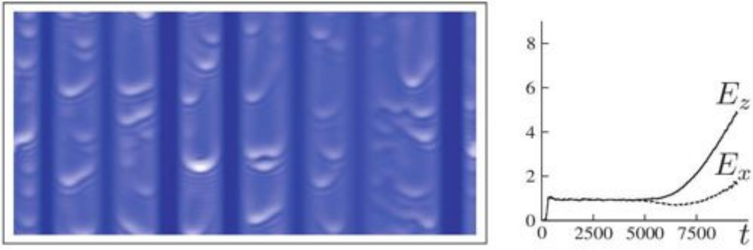
Competition between Marangoni instability (isotropic) and Kapitza insatbility (aligned with flow) leads to channeling phenomena



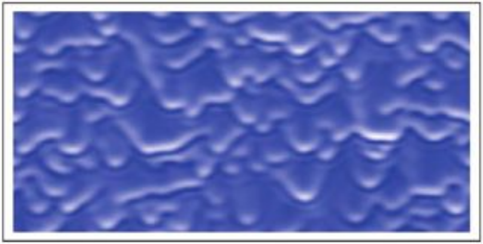
(a) I: $Re = 0.5$, $t = 4552$ – {0.034,2.355}

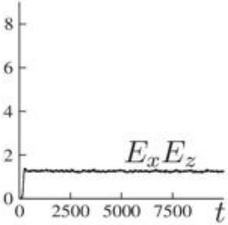


(b) $\mathrm{II_r}$: $Re = 2$, $t = 6120$ – {0.004,2.799}



(c) $\mathrm{II_m}$: $Re = 4$, $t = 9510$ – {0.067,3.371}

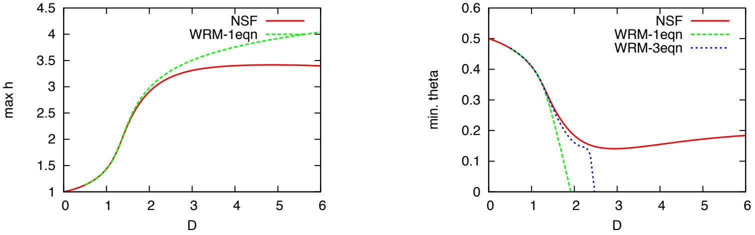




(d) III: $Re = 5$, $t = 20000$ – {0.752,1.612}

Modelling: initial 3eqn formulation

- problem: divergence of free-surface temperature !



$$Bi = 1, \; Pr = 7$$

Modelling: Saint-Venant

- solution: convection terms rewritten to assure compatibility with limit $Pe \gg 1$
- simplest model (3 variables)
- more complex model (4 variables): height (h), flow rate (q), free-surface temperature (θ), wall flux (φ/h)

Modelling: 3eqn model

- At $Pr\delta \ll 1$ we shall have $\partial_t \theta + u(y = h)\partial_x \theta = O(1/(Pr\delta))$
- modification of convection terms

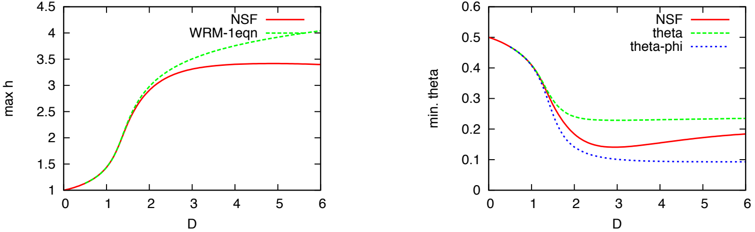
$$\frac{99}{30} Pr\delta \left[\partial_t + \frac{3q}{2h}\partial_x\right)\left(\theta - \frac{7}{22}\ln\theta\right) =$$

$$3\frac{[1-(1+B\,h)\theta}{h^2} + \eta\left\{\left(1-\theta-\frac{3}{2}Bi\,h\theta\right)\left(\frac{\partial_x h}{h}\right)^2\right.$$

$$\left. + \frac{\partial_x h \partial_x \theta}{h} + (1-\theta)\frac{\partial_{xx} h}{h} + \partial_{xx}\theta\right\}$$

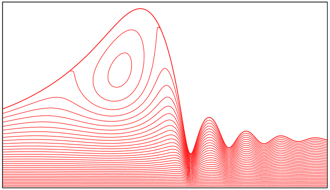
Modelling: 3eqn model

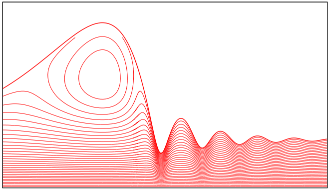
- no more divergence of the temperature !



$$Bi = 1, \; Pr = 7$$

Modelling: 3eqn model





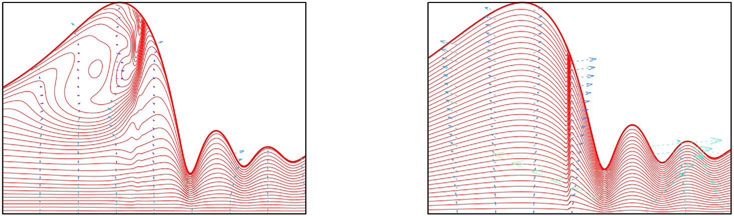
$Re = 15$, $Pe = 460$, $Bi = 0.1$, $f = 9.4$ Hz

steamlines (moving frame)

DNS

model

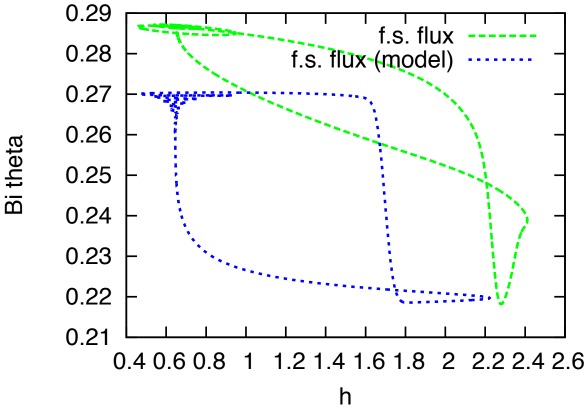
Modelling: 3eqn model



$Re = 15$, $Pe = 460$, $Bi = 0.1$, $f = 9.4$ Hz
isotherms and heat flux density

DNS — model (θ)

Modelling: 3eqn model



$Re = 15$, $Pe = 460$, $Bi = 0.1$, $f = 9.4$ Hz
heat flux density (temperature) at free surface
1 eqn for θ

Modelling: 3eqn model

- impossibility to capture correctly the onset of thermal sublayer at free surface (competition convection–heat transfer)
- Pb: critical temperature $\theta_c = 7/22 \approx 0.32$ at which convection terms (unphysically) disappears. . .
- crude representation of temperature field
- solution: add more fields. . .

Modelling: 4eqn model

- introduce $\varphi = h\partial_y T|_{y=0}$
- thus $(\partial_t + u(y=h)\partial_x)\theta = O(1/Pe)$ and $\partial_t(\varphi/h) = O(1/Pe)$ at $Pe \ll 1$
- $$T = T^{(0)} + \left(\theta - T^{(0)}|_{y=h}\right) f_1(\bar{y}) + \left(\varphi - h\partial_y T^{(0)}|_{y=0}\right) f_2(\bar{y}) + h.o.t.$$

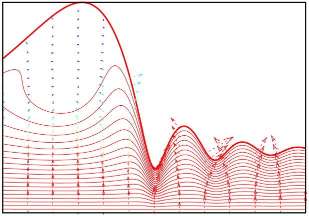
  with $\bar{y} = y/h$, $f_1'(0) = 0$ and $f_2(1) = 0$
- weights $w_1 = \bar{y}$ and $w_2 = 1 - \bar{y}$ are determined so that h.o.t. need not to be determined to assure consistancy at $O(\varepsilon)$

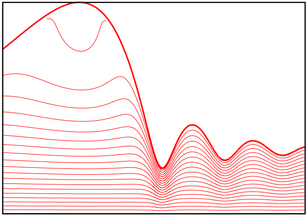
$$T = 1 + \left(\frac{1}{1+Bi\,h} - 1\right)\bar{y} + \left(\theta - \frac{1}{1+Bi\,h}\right)\bar{y}^2 + \left(\varphi + \frac{Bi\,h}{1+Bi\,h}\right)\left(\bar{y} - \frac{3}{2}\bar{y}^2 + \frac{1}{2}\bar{y}^3\right)$$

Modelling: 4eqn model

- $R_i = \langle \text{heat} | w_i \rangle = 0$ contain convection terms $\propto Pe$, say $R_i^{(conv)}$
- $R_i^{(conv)}$ are rewritten as l.c. of $\partial_t + (3q/(2h)\partial_x)\theta$ and $\partial_t(\varphi/h)$

Modelling: 4eqn model



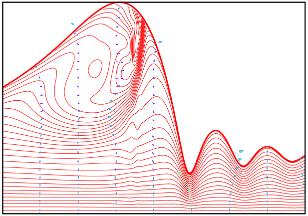


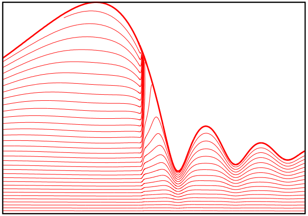
$Re = 15$, $Pe = 460$, $Bi = 100$, $f = 12.6$ Hz

isotherms and heat flux density

DNS model (θ and $\phi = h\partial_y T(y=0)$)

Modelling: 4eqn model





$Re = 15$, $Pe = 460$, $Bi = 0.1, f = 9.4$ Hz

isotherms and heat flux density

DNS model (θ and ϕ)

Conclusions

- simple models enable to capture hydrodynamics (amplitude, form, wave speed)
- reasonable representation of free-surface temperature
- low numeraical cost enable to silulate 3D flows on complex or large domains

Thank you ☺.