Traitement des conditions aux limites de spécularité pour la résolution de l'ETR 3D en éléments finis

D. Le Hardy, Y. Favennec, A. Badri, B. Rousseau

27 novembre 2015

CéfoPRam

Plan

1. Objectifs
2. Modèle mathématique
3. Construction du modèle numérique
4. Validation

Caractérisation radiative infrarouge de matériaux semi-transparents

Méthodes pour la caractérisation radiative :

- Théorie de Mie (1D), (Dombrovsky, Milandri,...)
- Méthodes à N flux (1D), (Dombrovsky, Randrianalisoa, Baillis,...)
- Méthodes Numériques (1D), (Pilon, Moura, ...)

Caractérisation radiative infrarouge de matériaux semi-transparents

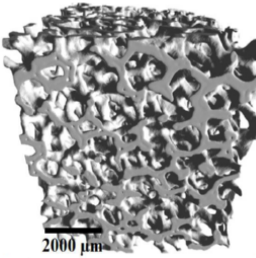
Méthodes pour la caractérisation radiative :

– Théorie de Mie (1D), (Dombrovsky, Milandri,...)

– Méthodes à N flux (1D), (Dombrovsky, Randrianalisoa, Baillis,...)

– Méthodes Numériques (1D), (Pilon, Moura,...)

1D ?



Caractérisation radiative infrarouge de matériaux semi-transparents

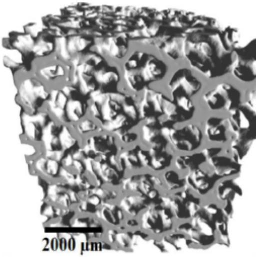
Méthodes pour la caractérisation radiative :

- Théorie de Mie (1D), (Dombrovsky, Milandri,...)
- Méthodes à N flux (1D), (Dombrovsky, Randrianalisoa, Baillis,...)
- Méthodes Numériques (1D), (Pilon, Moura,...)

1~~D~~ ?

$\Downarrow$

3D



Problème inverse

Données expérimentales $\Longrightarrow$

Modèle inverse

κ^N

σ_s^N

Φ^N

Modèle direct

$\cdots$

Modèle adjoint

$\Longrightarrow$

κ

σ_s

Φ

Problème inverse

Données expérimentales $\Longrightarrow$

Modèle inverse

κ^N

σ_s^N

Φ^N

Modèle direct

$\cdots$

Modèle adjoint

$\Longrightarrow$

κ

σ_s

Φ

Plan

1 Objectifs

2 Modèle mathématique

3 Construction du modèle numérique

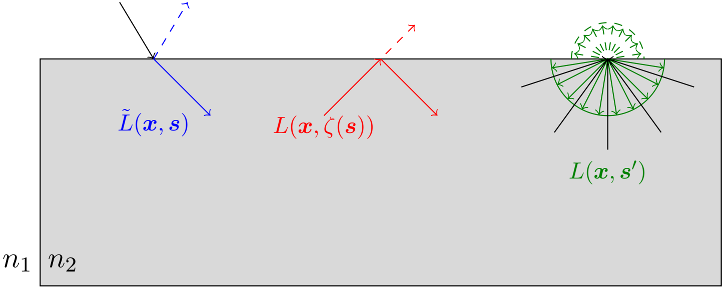
4 Validation

Équation du transfert radiatif

$$\underbrace{\boldsymbol{s} \cdot \nabla L(\boldsymbol{x}, \boldsymbol{s})}_{\text{Transport}} + \underbrace{(\kappa + \sigma_s) L(\boldsymbol{x}, \boldsymbol{s})}_{\text{Pertes par absorption et diffusion}} = \underbrace{\int_{4\pi} \Phi(\boldsymbol{s}' \to \boldsymbol{s}) L(\boldsymbol{x}, \boldsymbol{s}') \, \mathrm{d}\boldsymbol{s}'}_{\text{Gain par diffusion}} + \underbrace{\kappa L_b(T)}_{\text{Emission propre}}$$

Conditions aux limites

$$L(\boldsymbol{x},\boldsymbol{s}) = \underbrace{\tilde{L}(\boldsymbol{x},\boldsymbol{s})}_{\text{Luminance entrante}} + \underbrace{\rho(\boldsymbol{s}\cdot\boldsymbol{n})L(\boldsymbol{x},\zeta(\boldsymbol{s}))}_{\text{Réflexion spéculaire}} + \underbrace{\frac{1-\varepsilon}{\pi}\int_{\boldsymbol{s}'\cdot\boldsymbol{n}>0} L(\boldsymbol{x},\boldsymbol{s}')\boldsymbol{s}'\cdot\boldsymbol{n}\,\mathrm{d}\boldsymbol{s}'}_{\text{Réflexion diffuse}}$$



Réflectivité spéculaire

$$\rho(\boldsymbol{s}\cdot\boldsymbol{n}) = \begin{cases} 1 \text{ si } \theta_i \in]\theta_{cr}, \frac{\pi}{2}[\\ \frac{1}{2}\left[\left(\frac{\sin(\theta_i-\theta_r)}{\sin(\theta_i+\theta_r)}\right)^2 + \left(\frac{\tan(\theta_i-\theta_r)}{\tan(\theta_i+\theta_r)}\right)^2\right] \text{ si } \theta_i \in]0, \theta_{cr}[\\ \left(\frac{n_1-n_2}{n_1+n_2}\right)^2 \text{ si } \theta_i = 0 \end{cases}$$

avec $n_1 \sin\theta_i = n_2 \sin\theta_r$ et $\theta_{cr} = \arcsin(n_2/n_1)$

Plan

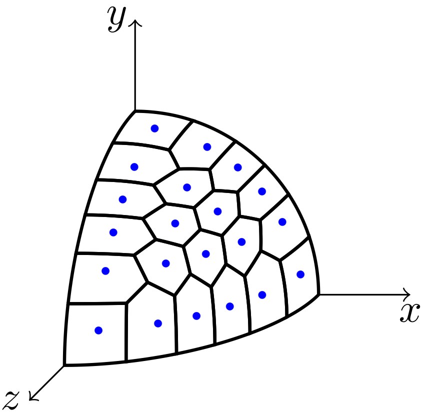
1 Objectifs

2 Modèle mathématique

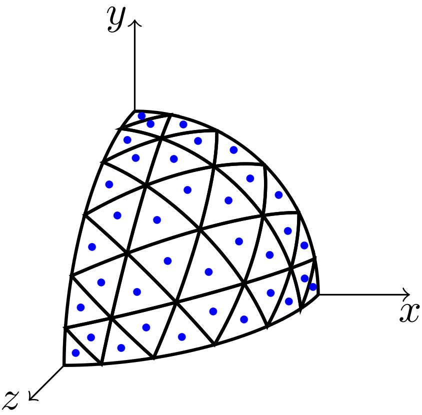
3 Construction du modèle numérique

4 Validation

Quadratures usuelles

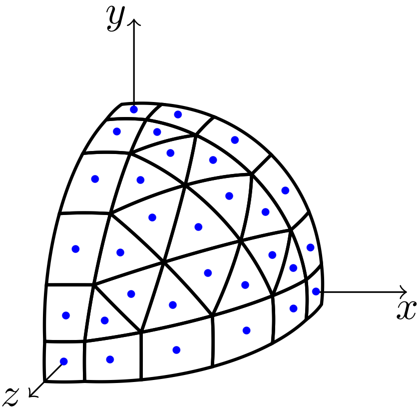


S_{12} [Lee - 1962]

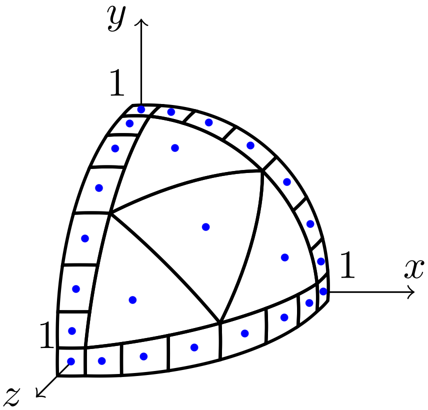


T_6 [Thurgood - 1995]

Nouvelle quadrature $SqT_{n,p}$



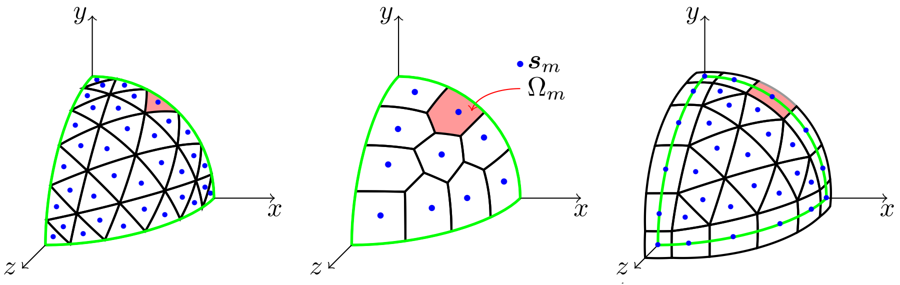
$SqT_{6,4}$



$SqT_{8,2}$

Méthode des ordonnées discrètes

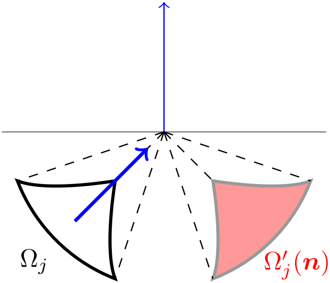
Discrétisations angulaires 3D



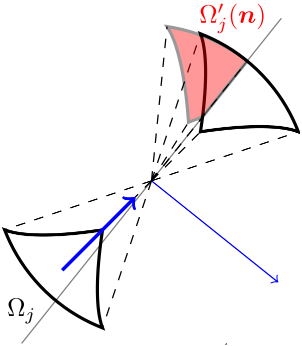
$$(\mathsf{SDS}_m)\ :\ (\boldsymbol{s}_m \cdot \nabla + \beta)\, L(\boldsymbol{x}, \boldsymbol{s}_m) - \sigma_s \sum_{j=1}^{\boldsymbol{N}_d} \omega_j L(\boldsymbol{x}, \boldsymbol{s}_j) \Phi_{m,j} = \kappa L_b(T)$$

avec $\omega_m = \text{mes } \Omega_m$

Différents cas de réflexion spéculaire

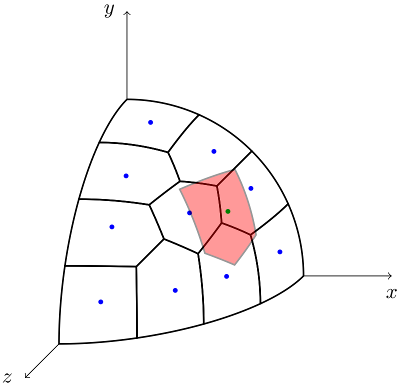


Toutes les directions de l'angle solide Ω_j sont réfléchies



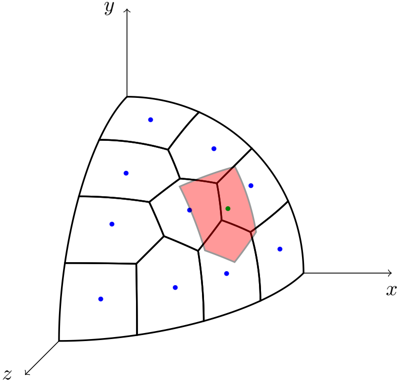
Seule une partie des directions de Ω_j est réfléchie

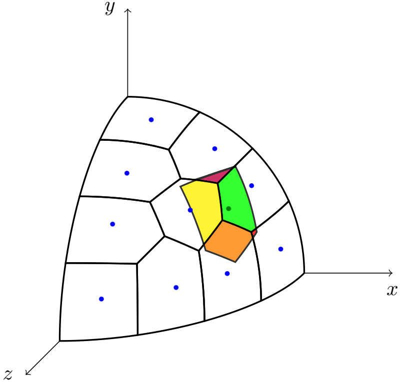
Réflexion spéculaire dans la littérature



Interpolation linéaire [TTSP - Gao - 2012]

Intersection d'angles solides





Distribution **exactement** proportionnelle de l'angle solide

Conditions aux limites discrètes

Formulation continue

$$L(\boldsymbol{x}, \boldsymbol{s}) = \tilde{L}(\boldsymbol{x}, \boldsymbol{s}) + \rho(\boldsymbol{s} \cdot \boldsymbol{n}) L(\boldsymbol{x}, \zeta(\boldsymbol{s}))$$

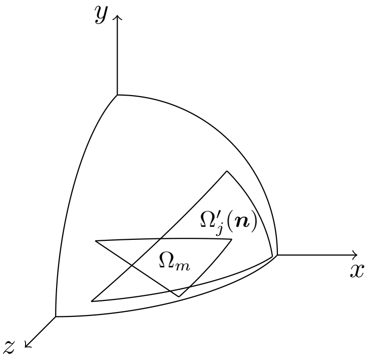
pour $\boldsymbol{s} \cdot \boldsymbol{n} < 0$

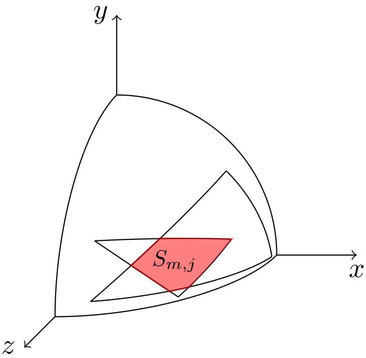
Formulation discrète

$$L_m(\boldsymbol{x}) = \tilde{L}_m + \delta_{m,m}(\boldsymbol{n}) L_m + \sum_{j \neq m} \delta_{m,j}(\boldsymbol{n}) L_j$$

pour $\boldsymbol{s}_m \cdot \boldsymbol{n} < 0$

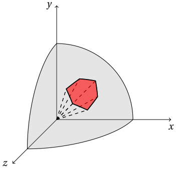
Calcul des coefficients $\delta_{m,j}(\boldsymbol{n})$



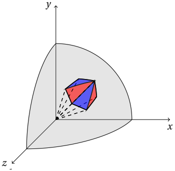


$$\delta_{m,j}(\boldsymbol{n}) = \rho(\boldsymbol{s}_m \cdot \boldsymbol{n}) \frac{S_{m,j}}{\Omega_m} \quad , \quad \delta_{m,m}(\boldsymbol{n}) = \rho(\boldsymbol{s}_m \cdot \boldsymbol{n}) \left[1 - \sum_{j \neq m} \delta_{m,j}(\boldsymbol{n}) \right]$$

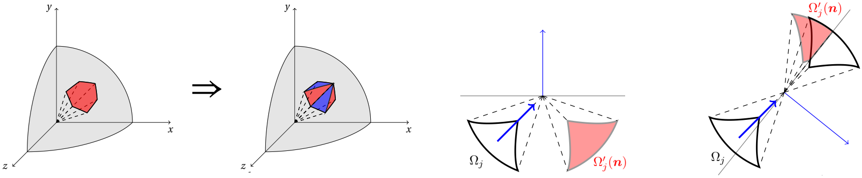
Construction des $S_{m,j}$



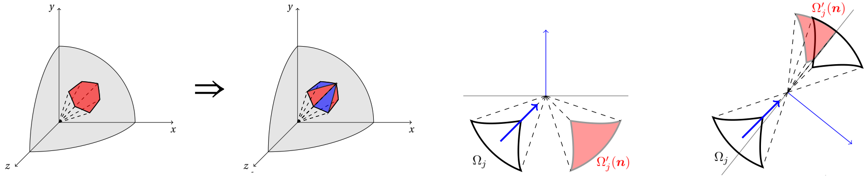
$\Rightarrow$

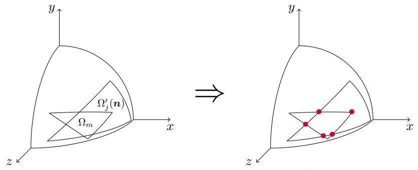


Construction des $S_{m,j}$

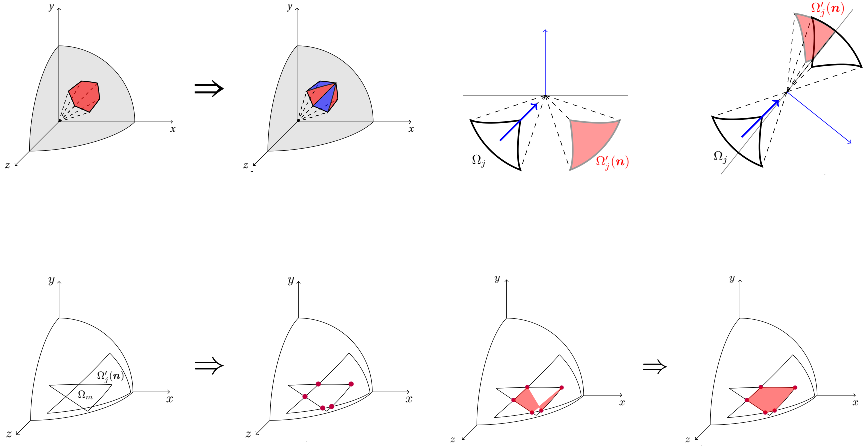


Construction des $S_{m,j}$





Construction des $S_{m,j}$



Formulation faible de (SDS$_m$) par la méthode SUP-G

$$\int_{\mathcal{D}} (\boldsymbol{s}_m \cdot \nabla L_m)(\boldsymbol{s}_m \cdot \nabla v) \,\mathrm{d}\boldsymbol{x} - \int_{\mathcal{D}} \tilde{\beta}_m (\boldsymbol{s}_m \cdot \nabla L_m) v \,\mathrm{d}\boldsymbol{x}$$

$$+ \int_{\partial\mathcal{D}^{m+}} \tilde{\beta}_m L_m v (\boldsymbol{s}_m \cdot \boldsymbol{n}) \,\mathrm{d}\Gamma + \int_{\partial\mathcal{D}^{m-}} \tilde{\beta}_m \delta_{m,m}(\boldsymbol{n}) L_m v (\boldsymbol{s}_m \cdot \boldsymbol{n}) \,\mathrm{d}\Gamma$$

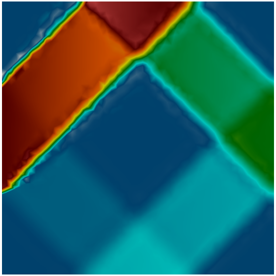
$$- \sum_{j \neq m} \left[\omega_j \Phi_{m,j} \int_{\mathcal{D}} \sigma_s L_j (\boldsymbol{s}_m \cdot \nabla v) \,\mathrm{d}\boldsymbol{x} + \int_{\partial\mathcal{D}^{m-}} \tilde{\beta}_m \delta_{m,j}(\boldsymbol{n}) L_j v (\boldsymbol{s}_m \cdot \boldsymbol{n}) \,\mathrm{d}\Gamma \right]$$

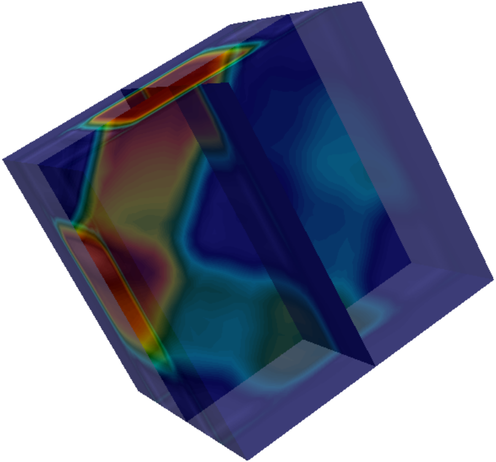
$$= - \int_{\partial\mathcal{D}^{m-}} \tilde{\beta}_m \tilde{L}_m v (\boldsymbol{s}_m \cdot \boldsymbol{n}) \,\mathrm{d}\Gamma + \int_{\mathcal{D}} \kappa L_b (\boldsymbol{s}_m \cdot \nabla v) \,\mathrm{d}\boldsymbol{x}$$

Plan

1. Objectifs
2. Modèle mathématique
3. Construction du modèle numérique
4. Validation

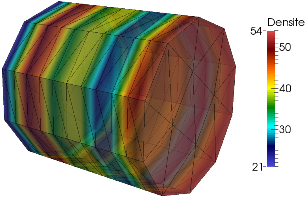
De la 2D à la 3D

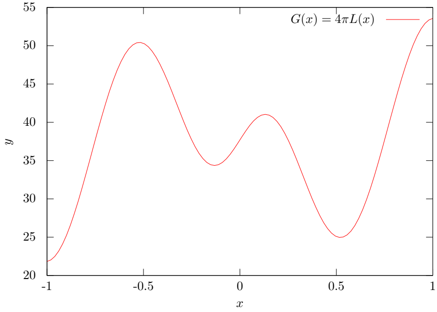


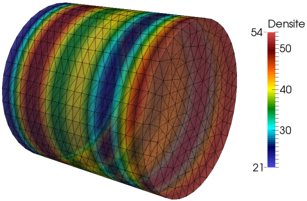


Comparaison analytique

$$\begin{cases} L(\boldsymbol{x}) = \arctan(\pi x)\cos(2\pi x) + 3 \\ \kappa L_b = \boldsymbol{s}\cdot\nabla L(\boldsymbol{x}) + \kappa L(\boldsymbol{x}) \\ G(\boldsymbol{x}) = \int_{4\pi} L(\boldsymbol{x},\boldsymbol{s})\ \mathrm{d}\Omega(\boldsymbol{s}) = 4\pi L(\boldsymbol{x}) \\ \kappa = 0.5\text{cm}^{-1}, \quad \sigma_s = 1\text{cm}^{-1}, \quad g = 0.8 \end{cases}$$

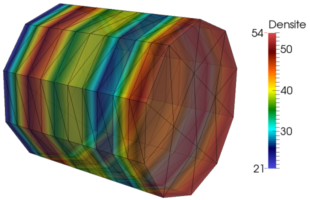


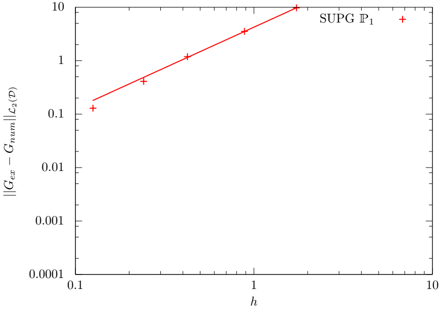


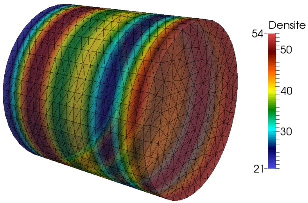


Comparaison analytique

$$\begin{cases} L(\boldsymbol{x}) = \arctan(\pi x)\cos(2\pi x) + 3 \\ \kappa L_b = \boldsymbol{s}\cdot\nabla L(\boldsymbol{x}) + \kappa L(\boldsymbol{x}) \\ G(\boldsymbol{x}) = \int_{4\pi} L(\boldsymbol{x},\boldsymbol{s})\,\mathrm{d}\Omega(\boldsymbol{s}) = 4\pi L(\boldsymbol{x}) \\ \kappa = 0.5\mathrm{cm}^{-1}, \quad \sigma_s = 1\mathrm{cm}^{-1}, \quad g = 0.8 \end{cases}$$

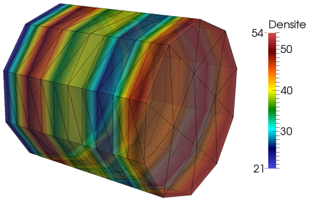


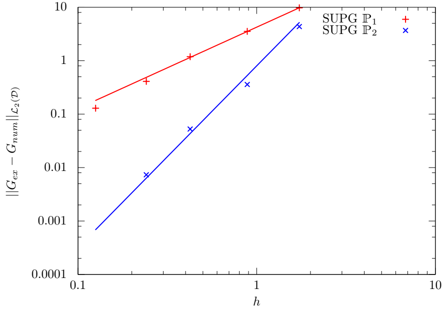


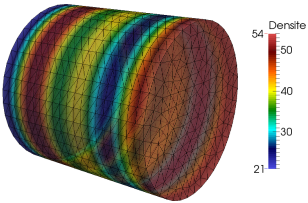


Comparaison analytique

$$\begin{cases} L(\boldsymbol{x}) = \arctan(\pi x)\cos(2\pi x) + 3 \\ \kappa L_b = \boldsymbol{s}\cdot\nabla L(\boldsymbol{x}) + \kappa L(\boldsymbol{x}) \\ G(\boldsymbol{x}) = \displaystyle\int_{4\pi} L(\boldsymbol{x},\boldsymbol{s})\,\mathrm{d}\Omega(\boldsymbol{s}) = 4\pi L(\boldsymbol{x}) \\ \kappa = 0.5\mathrm{cm}^{-1}, \quad \sigma_s = 1\mathrm{cm}^{-1}, \quad g = 0.8 \end{cases}$$







Comparaison sur des grandeurs physiques

avec la méthode Monte-Carlo

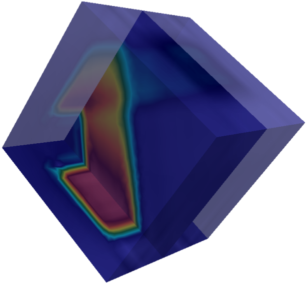
Transmittance normale-hémisphérique FEM

$$T_{nh} = \frac{\displaystyle\sum_{\boldsymbol{s}_m \cdot \boldsymbol{n}_k > 0} \int_{\partial\Gamma_c} (1 - \rho_{21}(\boldsymbol{s}_m))\omega_m L_m (\boldsymbol{s}_m \cdot \boldsymbol{n}_k)}{\displaystyle\int_{\partial\Gamma_{in}} \frac{1}{1 - \rho_{12}(\boldsymbol{s}_{in})} \omega_{in} \tilde{L}_{in} |\boldsymbol{s}_{in} \cdot \boldsymbol{n}_{in}|}$$

Transmittance normale-hémisphérique Monte-Carlo

$$T_{nh} = \frac{\text{Nombre de photons capturé}}{\text{Nombre de photons total}}$$

Source carrée avec une direction de $\frac{\pi}{4}$

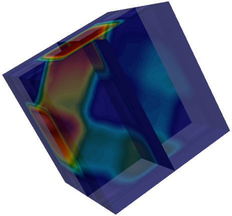


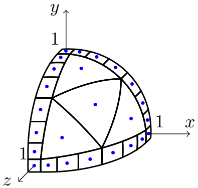
- $\kappa = 0.1\text{cm}^{-1}$
- $\sigma_s = 0.5\text{cm}^{-1}$
- $g = 0$
- $n_2 = 1.4$
- 1 000 000 photons

T_{nh}/R_{nh}	FEM	$E_{\text{MC}}(X)$	$\sigma_{\text{MC}}(X)$
Reflectance	1.34×10^{-1}	1.33×10^{-1}	6.66×10^{-4}
Transmittance	1.12×10^{-1}	1.08×10^{-1}	6.10×10^{-4}
Transmittance Latérale1	7.64×10^{-2}	7.55×10^{-2}	5.18×10^{-4}
Transmittance Latérale2	7.64×10^{-2}	7.50×10^{-2}	5.16×10^{-4}
Transmittance Latérale3	3.47×10^{-1}	3.26×10^{-1}	9.19×10^{-4}
Transmittance Latérale4	7.29×10^{-2}	6.97×10^{-2}	4.99×10^{-4}

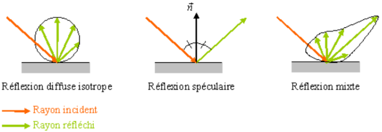
Conclusion

- Résolution de l'**ETR 3D** avec conditions **spéculaires** aux parois par la méthode d'**ordonnées discrètes** combinée avec une méthode **éléments finis**





- 3 **quadratures** angulaires dont une prometteuse pour l'**inversion**



- **Perspectives** : travailler sur les conditions de **réflexion mixte** [image : thèse 2008 Mathilde LORETZ]