Utilisation de l'environnement EDStaR pour la conception des optiques de concentration et des récepteurs

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Collaboration avec LAPLACE, PROMES

ECOLE DES MINES D'ALBI
C A R M A U X

Plan

1. Introduction

2. Monte Carlo et formulation intégrale

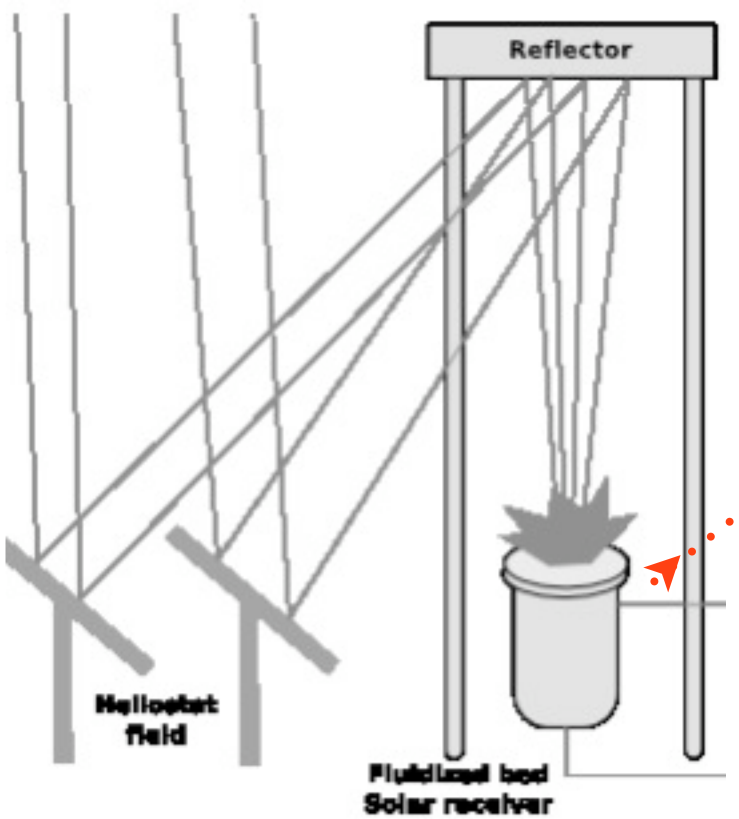
3. Environnement EDStaR

   3.1 Récepteur de type lit fluidisé

   3.2 Miroirs de Fresnel

4. Conclusion & Perspectives

Système concentration solaire



- Récépteur soumis à une haute densité de flux
    - Lit fluidisé
    - Photobioréacteur
    - Pyro-Gazeification

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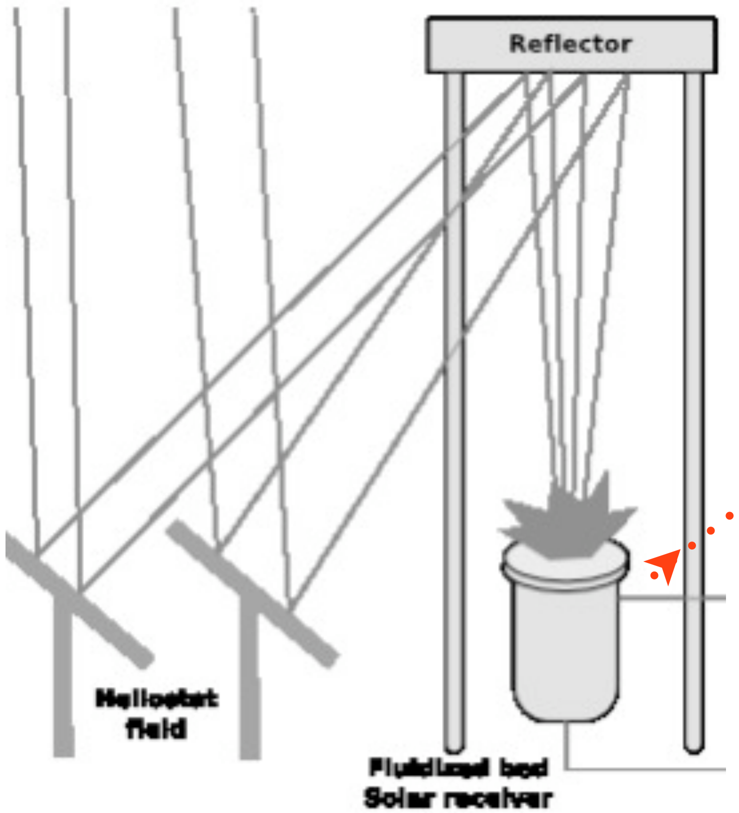

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Objectif :Augmenter l'efficacité énergétique

Contexte

Interaction entre le rayonnement et matière

- Géométries complexes

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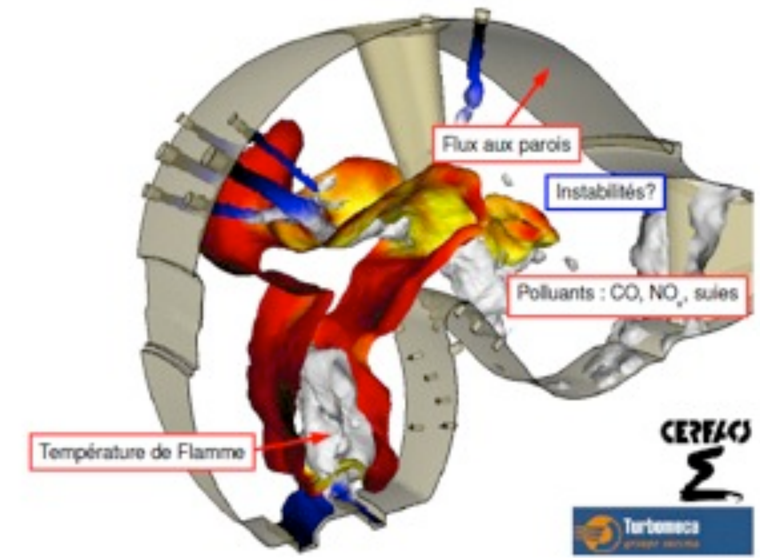

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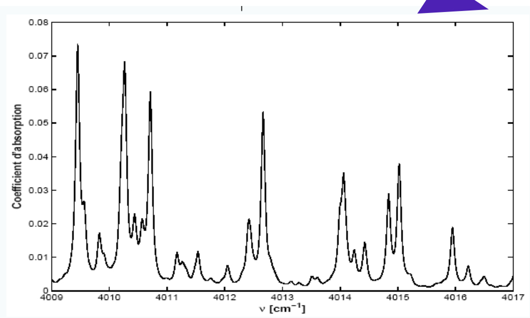

Exemple de spectre : $x_{H_2O} = 0.4$, $x_{air} = 0.6$, $T = 2550$K, $P = 1$atm

Caractérisation

(Formes, indices optiques, hétérogénéité, densité, distribution de taille)

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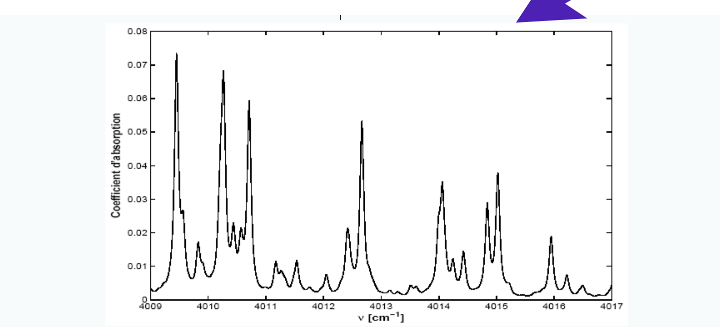

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Caractérisation

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- Modéliser des phénomènes complexes (diffusion multiple, reflexions multiples)

Monte Carlo et formulation intégrale

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collaboration avec le LAPLACE (Toulouse)

Algorithme de Monte Carlo	↔	Formulation intégrale
Simulation suivi de photons		Evaluation des flux radiatifs
Transport corpusculaire		des Puissances Nettes Echangées...

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A : $\tilde{a}_N$

N sampled events $w_1, w_2 \ldots w_N$

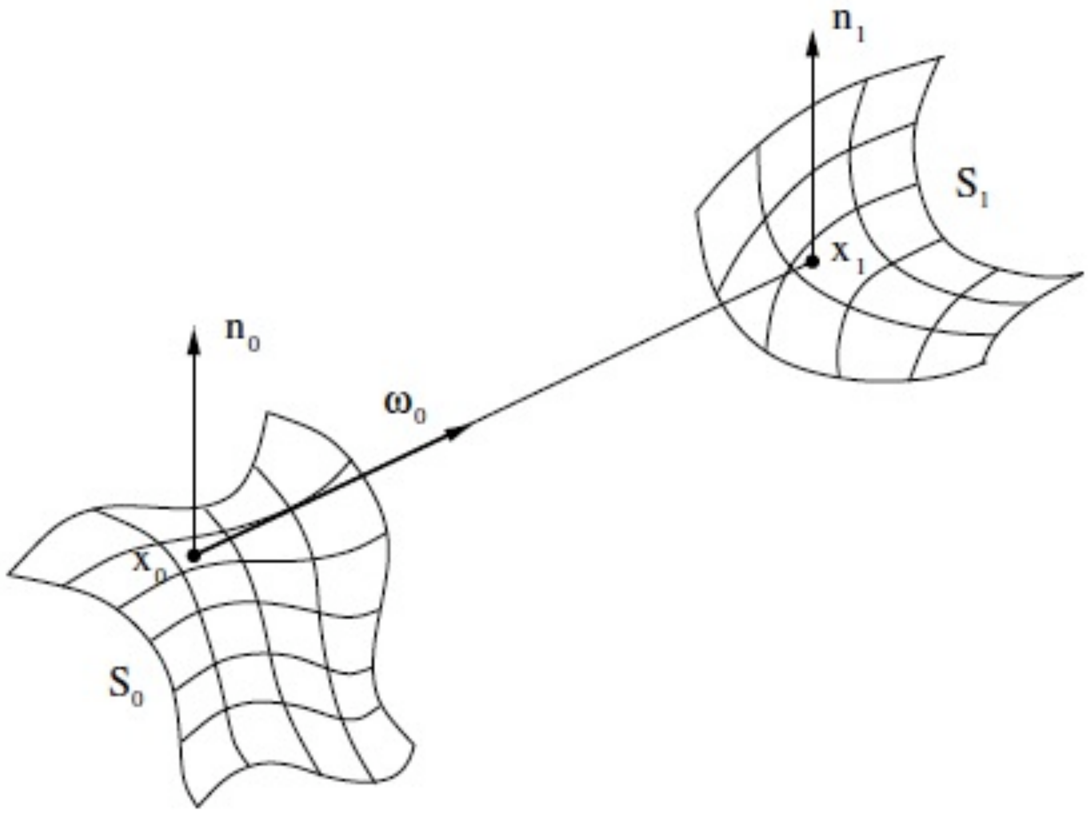
are N independant events of a random variable W

$$\tilde{a}_N = \frac{1}{N} \sum_{i=1}^{N} w_i$$

$$\tilde{\sigma}_N = \frac{1}{\sqrt{N-1}} \sqrt{\left(\frac{1}{N} \sum_{i=1}^{N} w_i^2 \right) - \tilde{a}_N^2}$$

Exemple :calcul du facteur de forme

F_{01}= fraction of the radiation emitted by S_0 that impacts S_1



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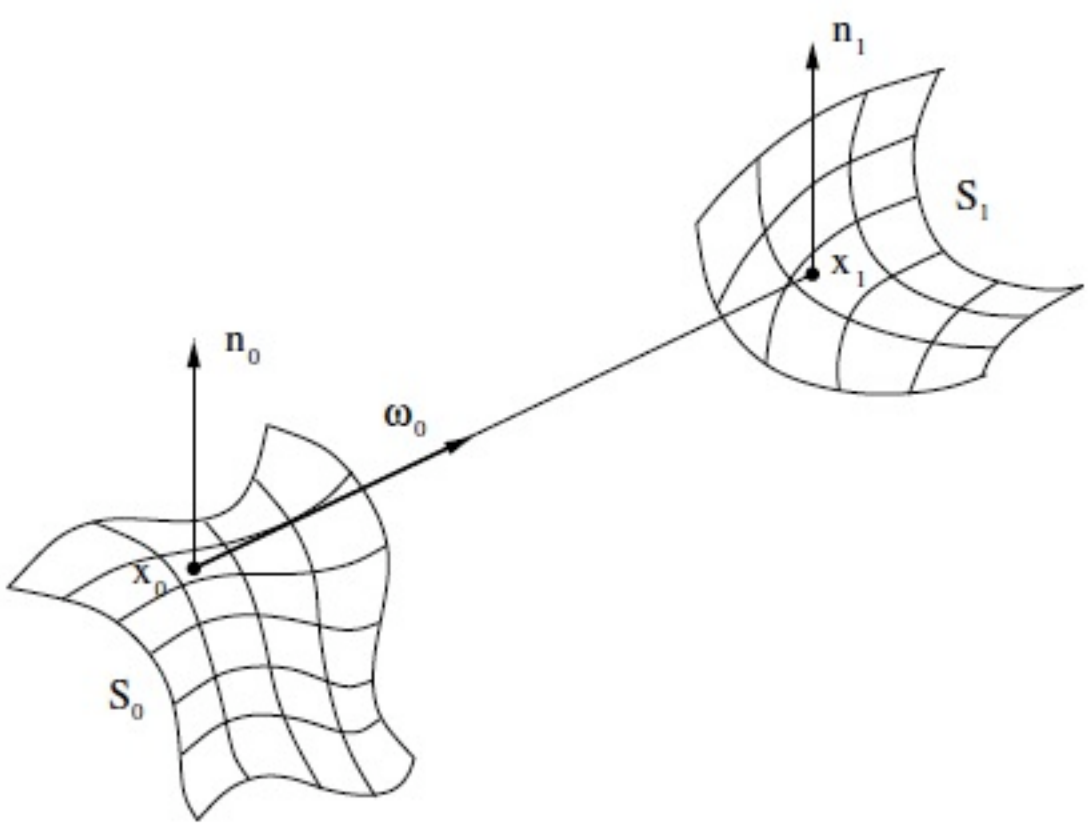

$\tilde{a}_N = 0;$
foreach *event i* **do**
  Uniform sampling of $\mathbf{x_0}$;
  Lambert sampling of ω_0 at $\mathbf{x_0}$;
  if $\mathbf{y_1} \in S_1$ **then**
    $w_i = 1;$
  else
    $w_i = 0;$
  end
  $\tilde{a}_N = \tilde{a}_N + w_i;$
end
$\tilde{a}_N = \frac{1}{N}\tilde{a}_N;$

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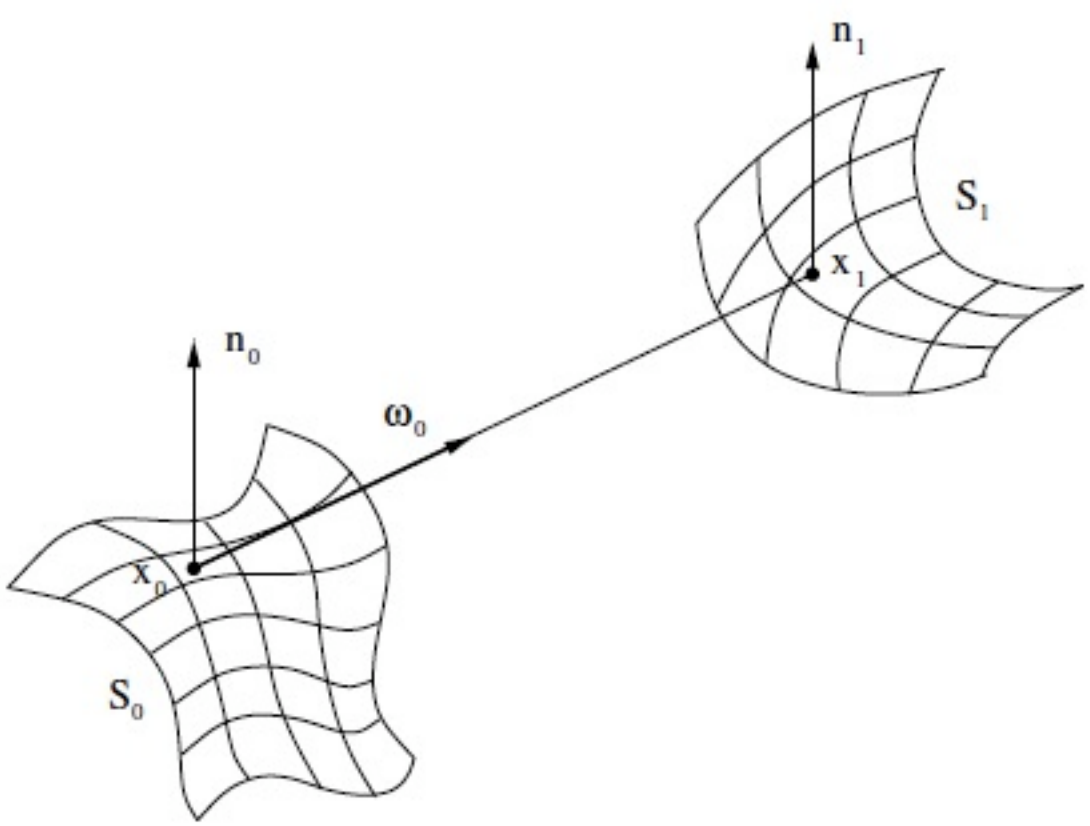

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$$F_{01} = \int_{S_0} p_{\mathbf{X_0}}(\mathbf{x_0}) d\mathbf{x_0} \int_{\mathcal{H}_0(\mathbf{x_0})} p_{\Omega_0}(\omega_0|\mathbf{x_0}) d\omega_0 \hat{w}(\mathbf{x_0}, \omega_0)$$

with

$$p_{\mathbf{X_0}}(\mathbf{x_0}) = \frac{1}{S_0}$$

$$p_{\Omega_0}(\omega_0|\mathbf{x_0}) = \frac{\omega_0.\mathbf{n_0}(\mathbf{x_0})}{\pi}$$

$$\hat{w}(\mathbf{x_0}, \omega_0) = H(\mathbf{y_1} \in S_1)$$

Calcul du facteur de forme

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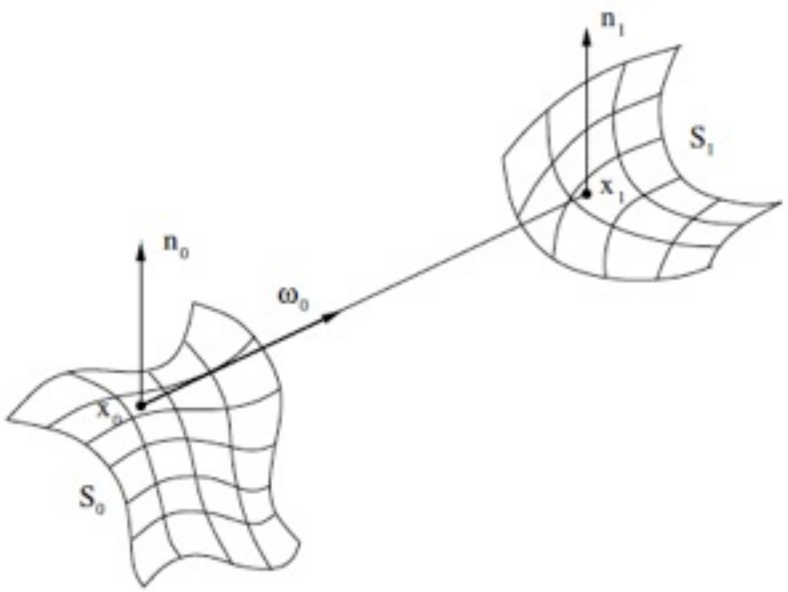

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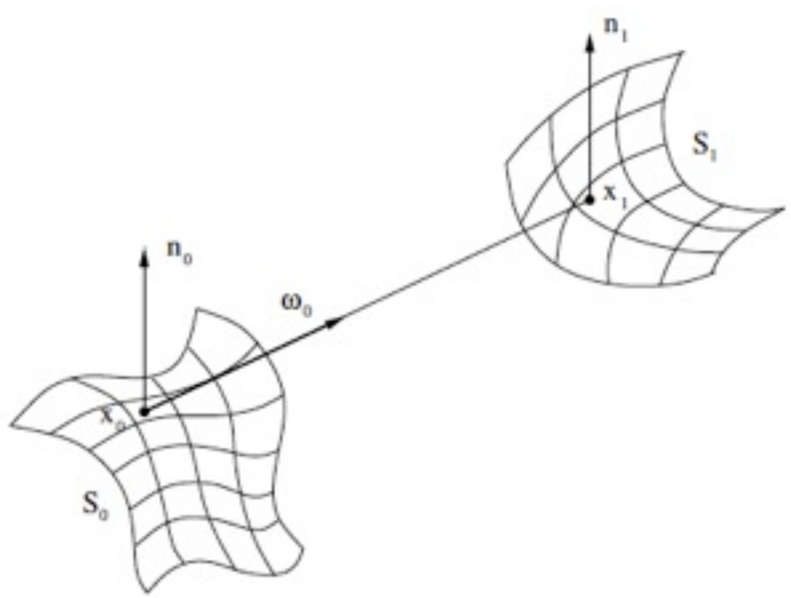

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  $\tilde{a}_N = \tilde{a}_N + w_i$;

end

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A quoi sert la formulation intégrale?

- Accélération de la convergence des algorithmes
    - Echantillonnage préférentiel ;
    - Re-formulation intégrale;
    - Zéro-variance(*)

(*) **Assaraf and al.** Zero-variance principle for Monte Carlo algorithms. Physical Review Letters, 1999
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- Calcul des sensibilités moindre coût CPU

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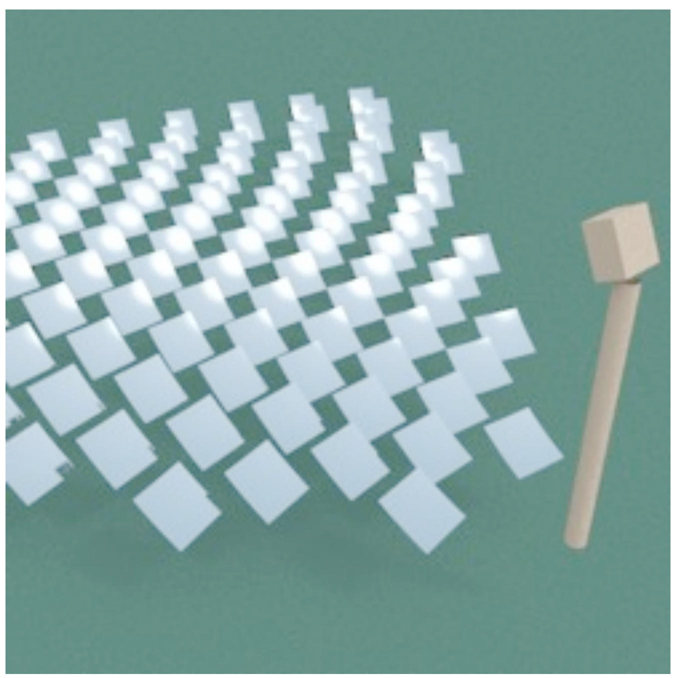
Quelques exemples de mise en oeuvre

MCM3D

EDStaR «Environnement de Développement Statistique radiative» = transport de particules...

http://wiki-energetique.laplace.univ-tlse.fr/wiki/index.php/Edstar



Rendu géométrie centrale à tour EDStaR

MCM3D

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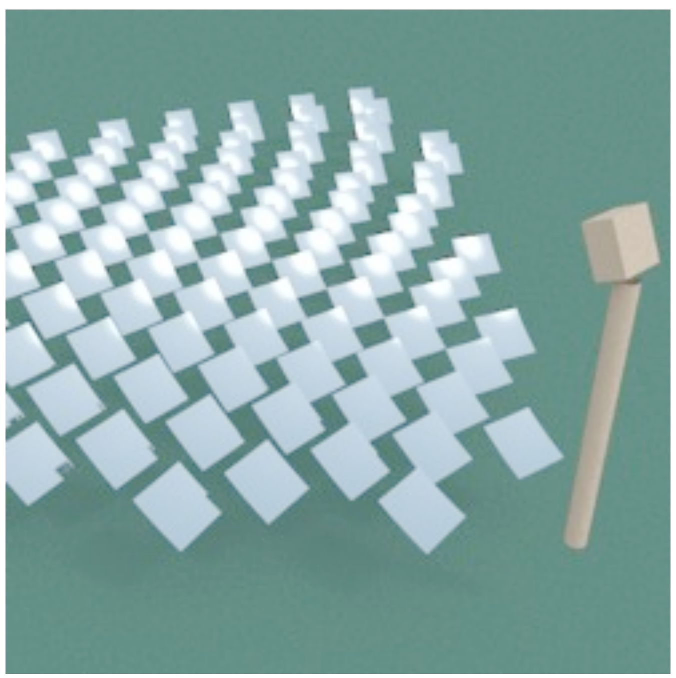
MCM3D : extrait de PBRT (*), C++

Objet «mcm»



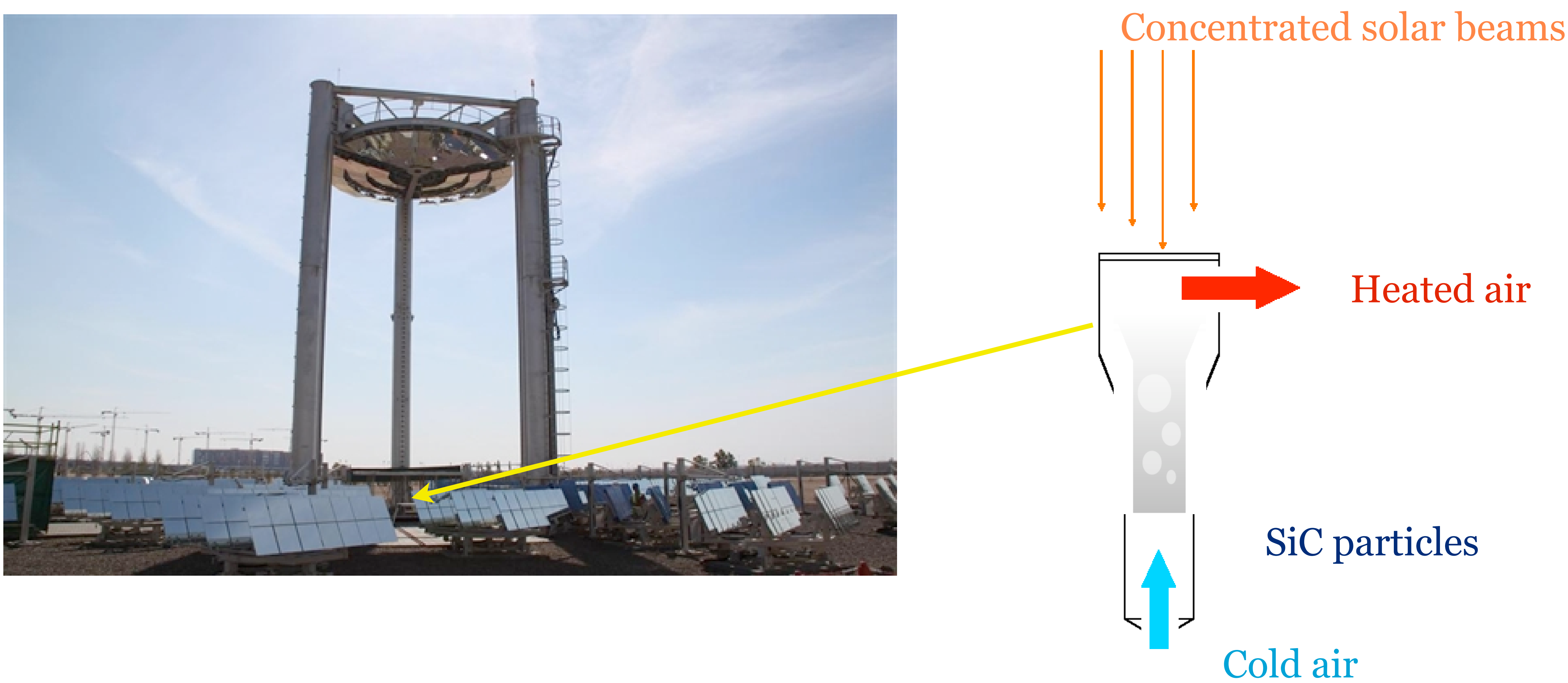
Rendu géométrie centrale à tour EDStaR

- Générateur aléatoire indépendant
- Calculs statistiques (moyenne, écart type) et leurs sensibilités associées
- MPI pour le calcul parallèle

(*) Pharr M., Humphreys G., 2004. Physically based rendering : from theory to implementation, Elvesier.

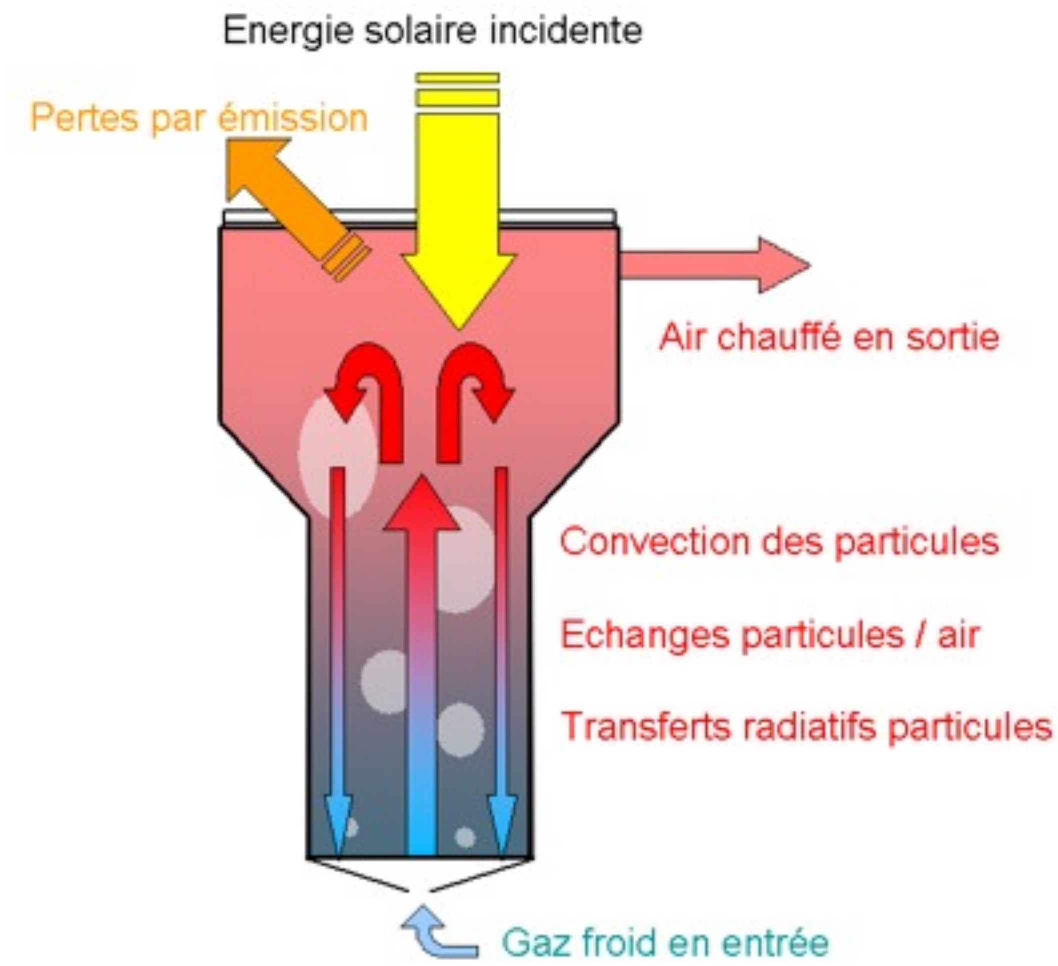
The Fluidized bed receiver in the beam down system

G. Baud Phd thesis



Thermodynamic cycle : electricity production

The heat transfer in the receiver

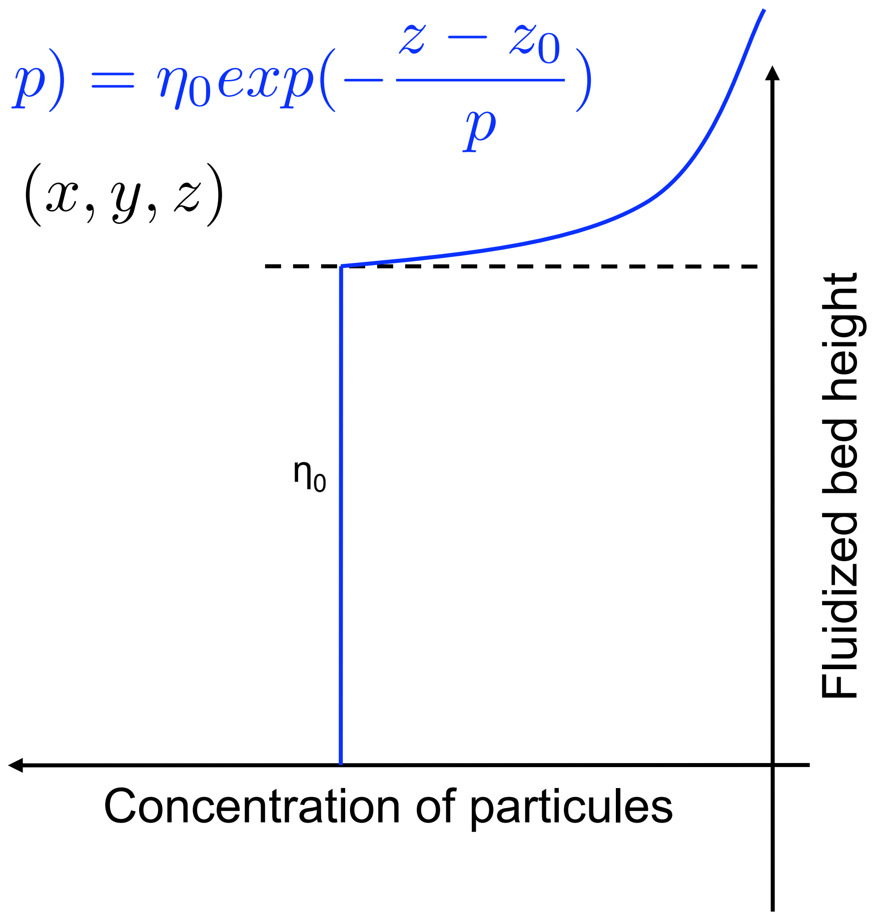


- Incident radiation by solar energy
- Radiative heat loss by particles through the window
- Radiative exchanges inside the fluidized bed
- Convection due to air movement
- Convection due to particle movement
- Convective flux between air and particles

Exemple de calcul de sensibilité

$$\eta(\mathbf{x}, p) = \eta_0 exp(-\frac{z - z_0}{p})$$

$$\mathbf{x} = (x, y, z)$$



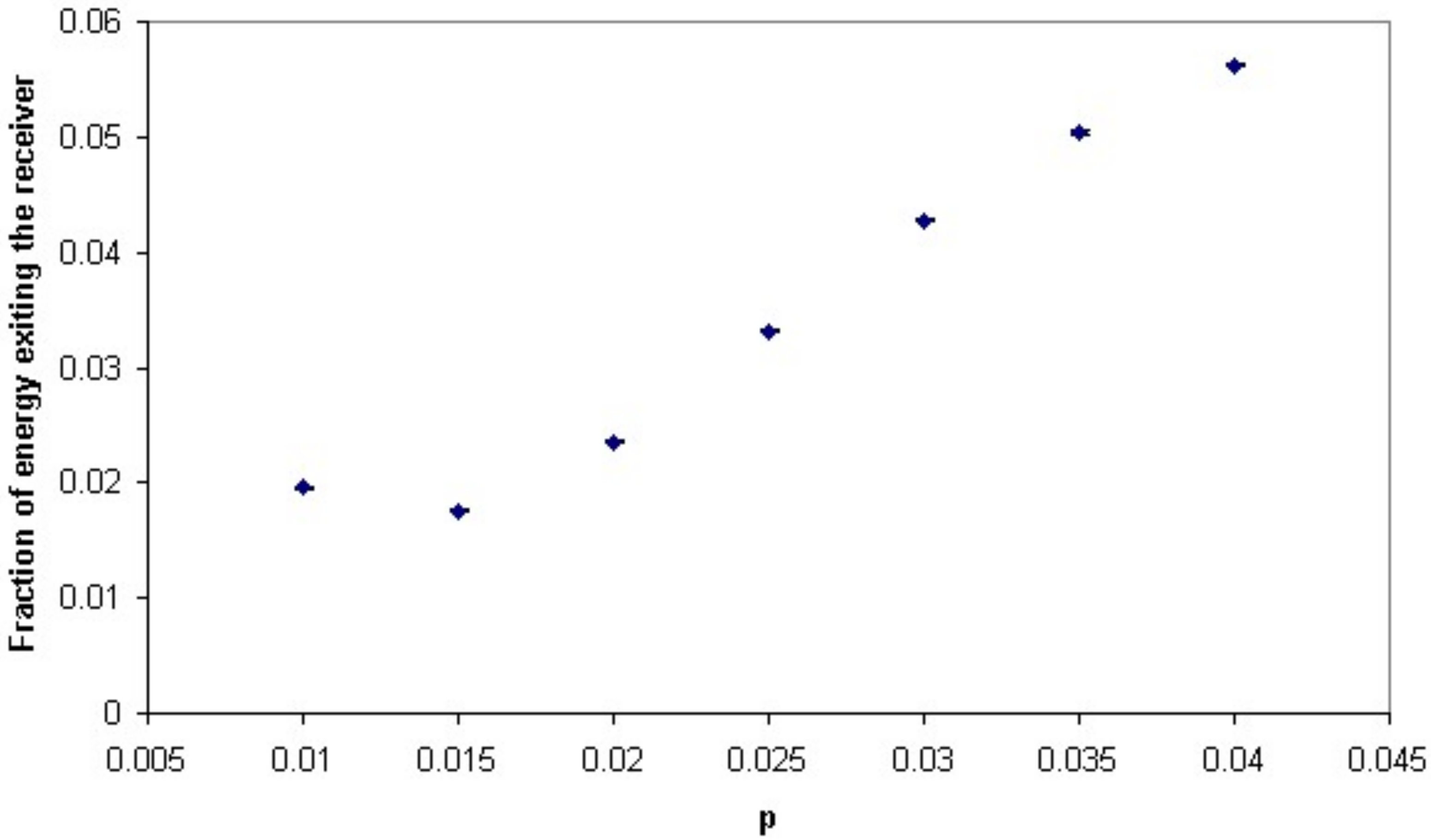
$$\eta(\mathbf{x}, p) = \eta_0 exp(-\frac{z - z_0}{p})$$

dérivée par rapport à p

Fraction of radiative heat loss
Sensitivity to «p» and validation

$\rho^{\mathcal{G}} = 0,1 \qquad \eta_0 = 26.10^3 cm^{-3}$

$\rho^{\mathcal{R}} = 0,4 \qquad d_p = 280 \mu m$



number of realizations 10^6 few mn on

AMD Opteron Processor 246 with 2Gb ram

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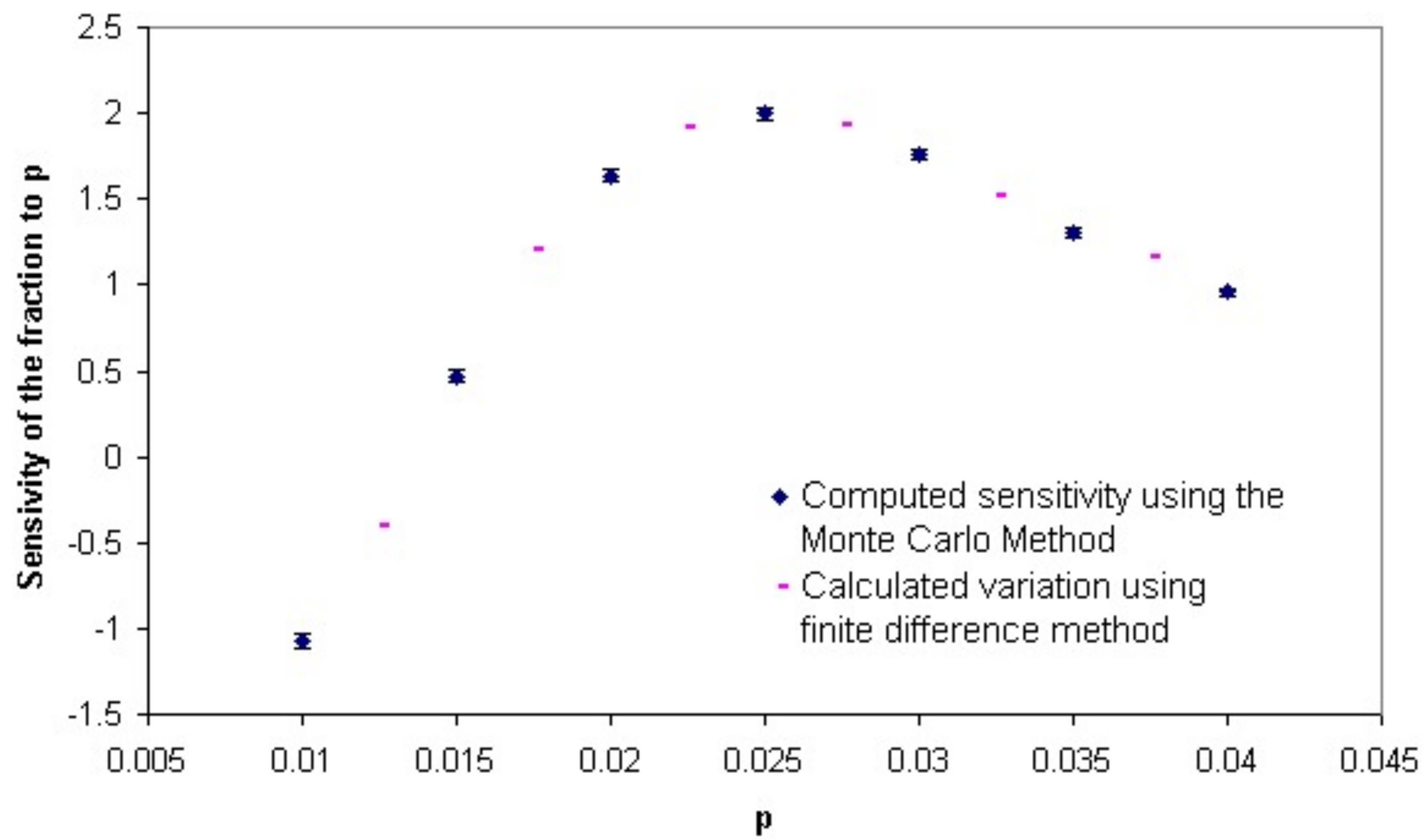
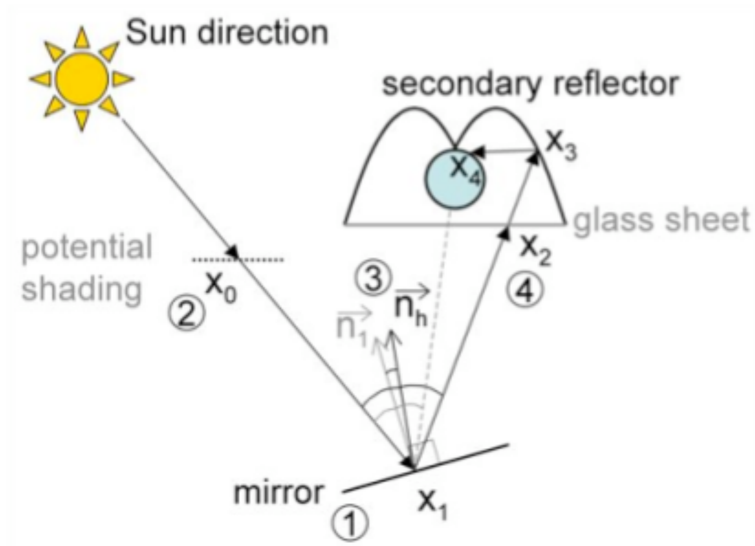
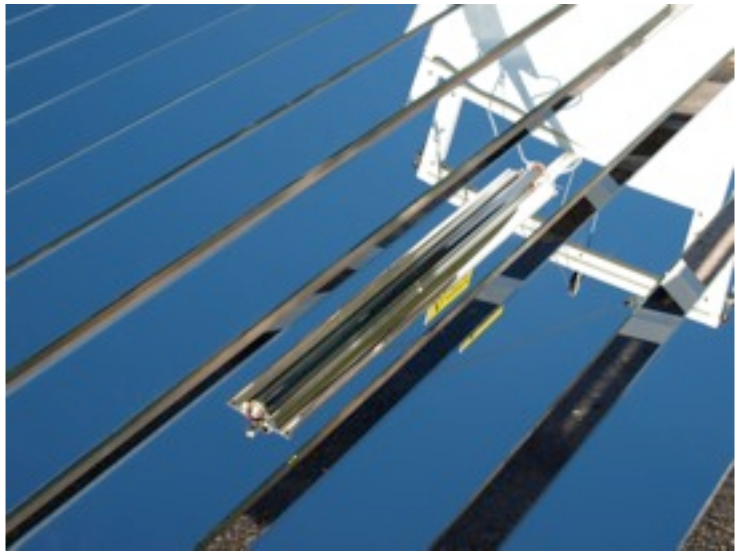
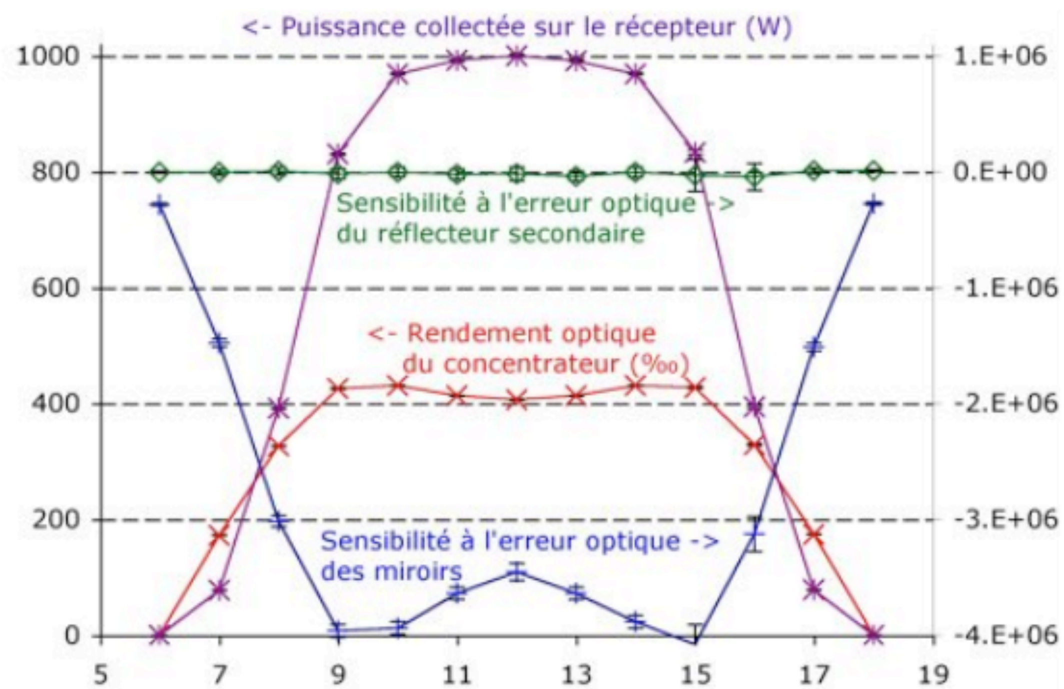


Collecteurs à miroirs linéiques de FRESNEL (Thèse F. Veynandt)







➔ Peu d'influence erreur optique miroir secondaire

Conclusion

- MCM dans EDSTAR géométries complexes, diffusion multiple ...
- Techniques de réduction de la variance
- Calcul des sensibilités paramétriques- géométriques

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- Couplage «multi-physique» dans les récepteurs volumiques CFD, thermique : niveau de précision...

Perspectives

- Chantiers en cours
    - Convergence des sensibilités
    - Taille de la matrice jacobienne sensibilités géométriques
    - Modèle de sensibilité géométrique

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- Thèse en collaboration avec Odeillo (démarrage sept. 2012)
  - Interaction biomasse - rayonnement solaire concentré

The Monte Carlo algorithm

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- The radiative heat loss :

$$E = \int_{\mathcal{G}} p_{\mathbf{X}_0}(\mathbf{x}_0) d\mathbf{x}_0 \int_0^{\infty} p_{\kappa_0}(\kappa_0) d\kappa_0 \left\{ \begin{array}{l} H(\mathbf{x}_1 \in \mathcal{R}) \int_{2\pi} p^{\mathcal{R}}_{\Omega_1}(\omega_1) d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1) d\kappa_1 \, \mathcal{I}_1 \\ +H(\mathbf{x}_1 \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\Omega_1}(\omega_1|\omega_0) d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1) d\kappa_1 \, \mathcal{I}_1 \end{array} \right\}$$

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fluidized bed

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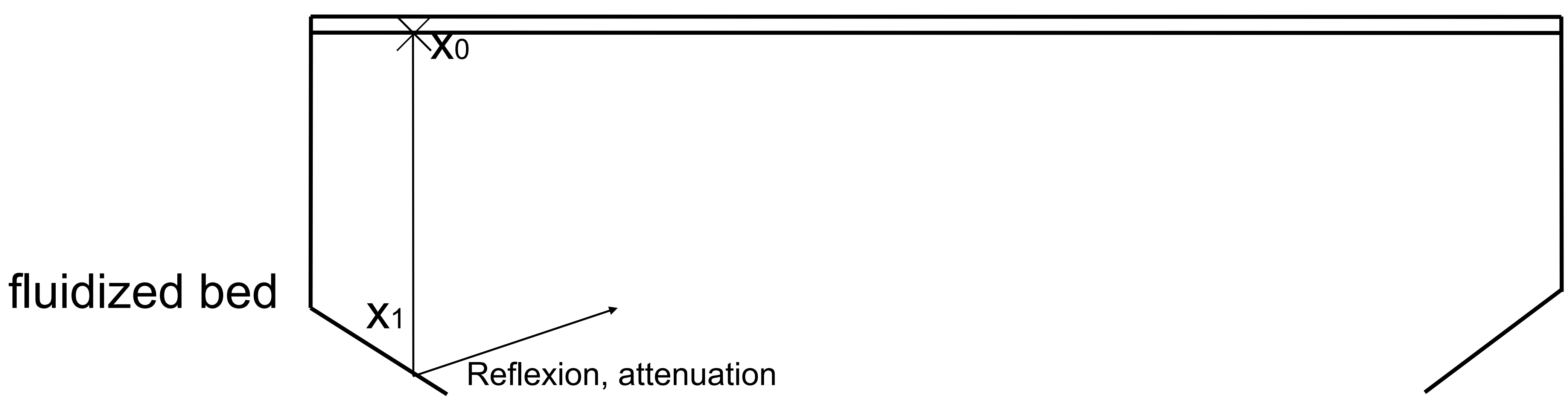

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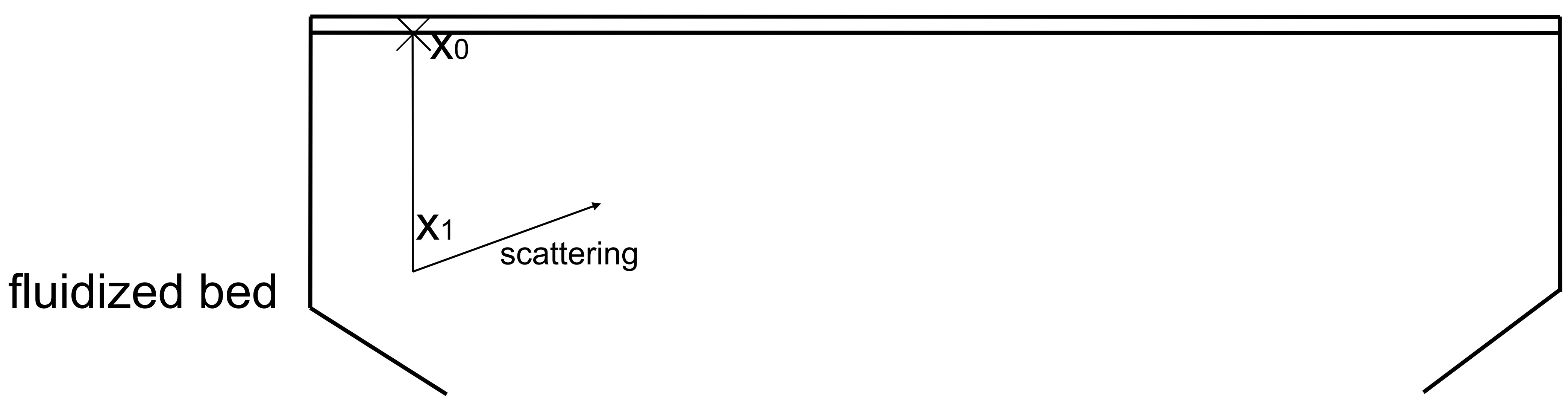

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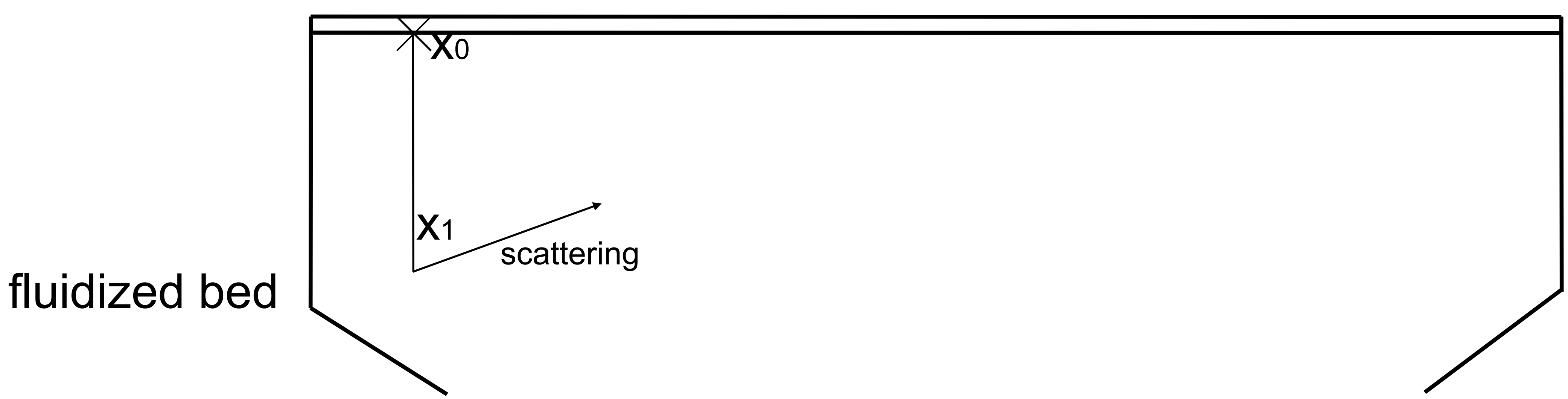

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The recursive term:

$$\begin{aligned} \mathcal{I}_j &= H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R^{\mathcal{G}}_{j+1}}(r^{\mathcal{G}}_{j+1})dr^{\mathcal{G}}_{j+1} \left\{ \begin{array}{l} H(r^{\mathcal{G}}_{j+1} > \rho^{\mathcal{G}}) \, \hat{\omega}_{j+1} \\ +H(r^{\mathcal{G}}_{j+1} \leq \rho^{\mathcal{G}}) \int_{2\pi} p^{\mathcal{G}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{\omega}_j \leq \epsilon) \int_0^1 p_{R^{\mathcal{R}}_{j+1}}(r^{\mathcal{R}}_{j+1})dr^{\mathcal{R}}_{j+1} \times \ldots \\ \left\{ \begin{array}{l} H(r^{\mathcal{R}}_{j+1} > \rho^{\mathcal{R}}) \times 0 \\ +H(r^{\mathcal{R}}_{j+1} \leq \rho^{\mathcal{R}}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{\omega}_j > \epsilon) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{aligned}$$

fluidized bed

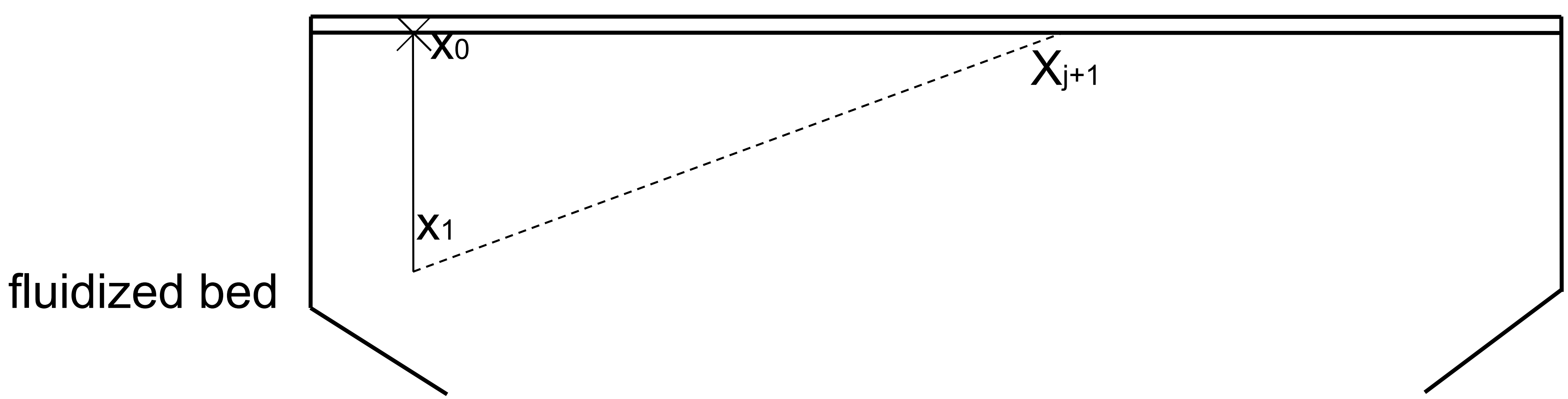
The Monte Carlo algorithm

- The radiative heat loss :

$$E = \int_{\mathcal{G}} p_{\mathbf{X}_0}(\mathbf{x}_0)d\mathbf{x}_0 \int_0^\infty p_{\kappa_0}(\kappa_0)d\kappa_0 \left\{ \begin{array}{l} H(\mathbf{x}_1 \in \mathcal{R}) \int_{2\pi} p_{\Omega_1}^{\mathcal{R}}(\omega_1)d\omega_1 \int_0^\infty p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \\ +H(\mathbf{x}_1 \in \mathcal{V}) \int_{4\pi} p_{\Omega_1}^{\mathcal{V}}(\omega_1|\omega_0)d\omega_1 \int_0^\infty p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \end{array} \right\}$$

The recursive term:

$$\begin{aligned} \mathcal{I}_j \; &= H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R^{\mathcal{G}}_{j+1}}(r^{\mathcal{G}}_{j+1})dr^{\mathcal{G}}_{j+1} \left\{ \begin{array}{l} H(r^{\mathcal{G}}_{j+1} > \rho^{\mathcal{G}}) \, \hat{\omega}_{j+1} \\ +H(r^{\mathcal{G}}_{j+1} \leq \rho^{\mathcal{G}}) \int_{2\pi} p^{\mathcal{G}}_{\Omega_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{\omega}_j \leq \epsilon) \int_0^1 p_{R^{\mathcal{R}}_{j+1}}(r^{\mathcal{R}}_{j+1})dr^{\mathcal{R}}_{j+1} \times \ldots \\ \left\{ \begin{array}{l} H(r^{\mathcal{R}}_{j+1} > \rho^{\mathcal{R}}) \times 0 \\ +H(r^{\mathcal{R}}_{j+1} \leq \rho^{\mathcal{R}}) \int_{2\pi} p^{\mathcal{R}}_{\Omega_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{\omega}_j > \epsilon) \int_{2\pi} p^{\mathcal{R}}_{\Omega_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\Omega_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{aligned}$$



The Monte Carlo algorithm

- The radiative heat loss :

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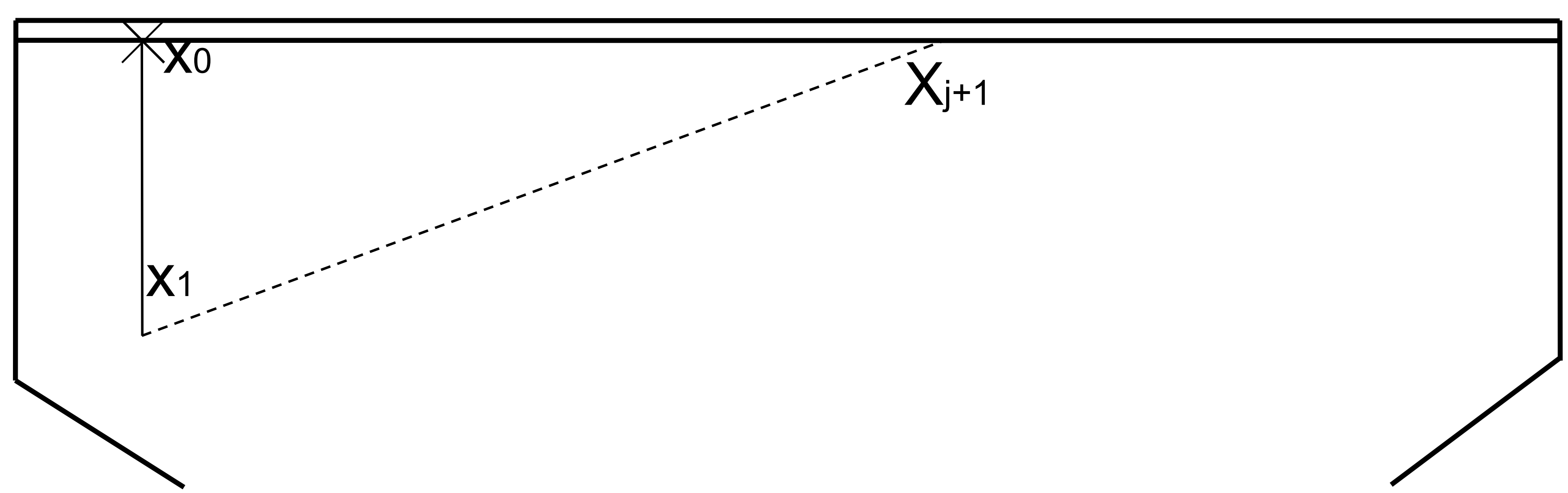
The recursive term:

$$\begin{aligned} \mathcal{I}_j = & \; H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R_{j+1}^{\mathcal{G}}}(r_{j+1}^{\mathcal{G}}) dr_{j+1}^{\mathcal{G}} \left\{ \begin{array}{l} H(r_{j+1}^{\mathcal{G}} > \rho^{\mathcal{G}}) \, \hat{w}_{j+1} \\ +H(r_{j+1}^{\mathcal{G}} \leq \rho^{\mathcal{G}}) \int_{2\pi} p_{\Omega_{j+1}}^{\mathcal{G}}(\omega_{j+1}|\omega_j) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ & +H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{w}_j \leq \epsilon) \int_0^1 p_{R_{j+1}^{\mathcal{R}}}(r_{j+1}^{\mathcal{R}}) dr_{j+1}^{\mathcal{R}} \times \ldots \\ \quad \left\{ \begin{array}{l} H(r_{j+1}^{\mathcal{R}} > \rho^{\mathcal{R}}) \times 0 \\ +H(r_{j+1}^{\mathcal{R}} \leq \rho^{\mathcal{R}}) \int_{2\pi} p_{\Omega_{j+1}}^{\mathcal{R}}(\omega_{j+1}) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{w}_j > \epsilon) \int_{2\pi} p_{\Omega_{j+1}}^{\mathcal{R}}(\omega_{j+1}) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ & +H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p_{\Omega_{j+1}}^{\mathcal{V}}(\omega_{j+1}|\omega_j) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{aligned}$$

Contribution to heat loss*

fluidized bed



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The Monte Carlo algorithm

- The radiative heat loss :

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The recursive term:

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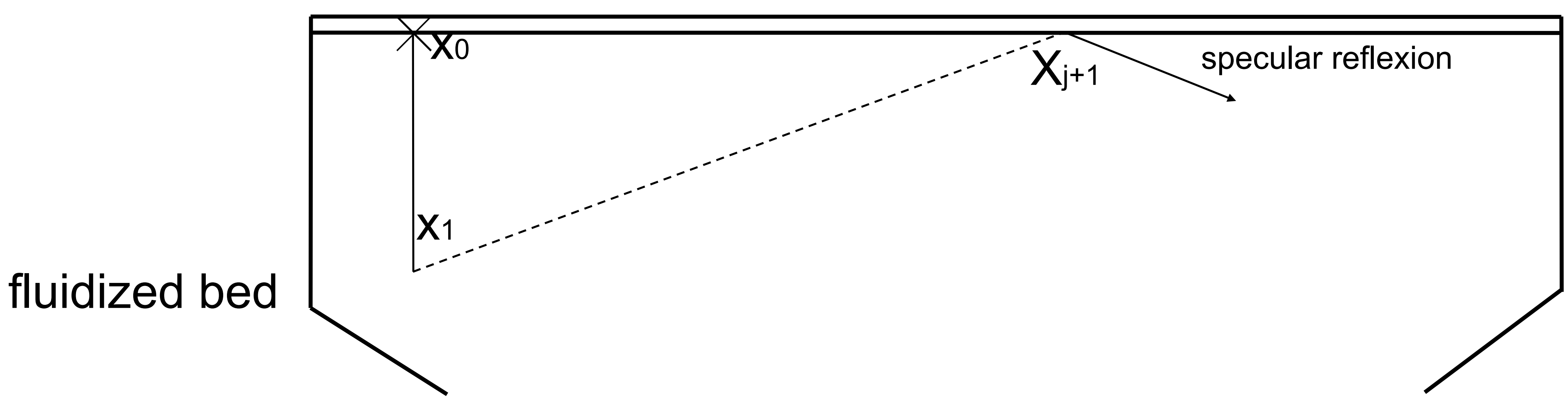

*Contribution , weight =(attenuation by the medium + and by the walls)

The Monte Carlo algorithm

- The radiative heat loss :

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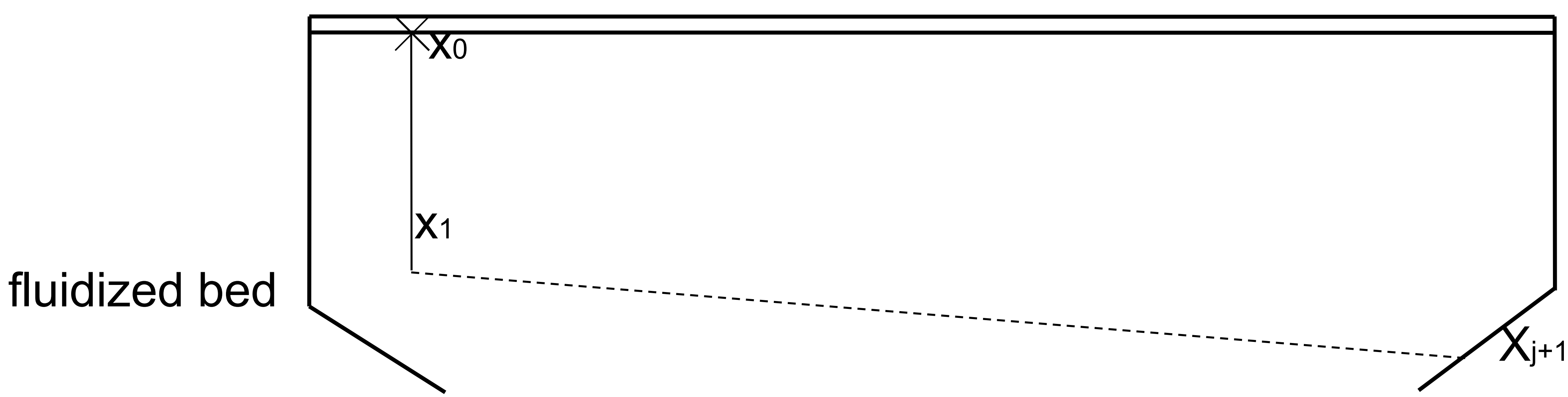
The recursive term:

$$\begin{aligned} \mathcal{I}_j \; &= H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R_{j+1}^{\mathcal{G}}}(r_{j+1}^{\mathcal{G}})dr_{j+1}^{\mathcal{G}} \left\{ \begin{array}{l} H(r_{j+1}^{\mathcal{G}} > \rho^{\mathcal{G}}) \, \hat{w}_{j+1} \\ +H(r_{j+1}^{\mathcal{G}} \le \rho^{\mathcal{G}}) \int_{2\pi} p_{\mathbf{\Omega}_{j+1}}^{\mathcal{G}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{w}_j \le \epsilon) \int_0^1 p_{R_{j+1}^{\mathcal{R}}}(r_{j+1}^{\mathcal{R}})dr_{j+1}^{\mathcal{R}} \times \ldots \\ \left\{ \begin{array}{l} H(r_{j+1}^{\mathcal{R}} > \rho^{\mathcal{R}}) \times 0 \\ +H(r_{j+1}^{\mathcal{R}} \le \rho^{\mathcal{R}}) \int_{2\pi} p_{\mathbf{\Omega}_{j+1}}^{\mathcal{R}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{w}_j > \epsilon) \int_{2\pi} p_{\mathbf{\Omega}_{j+1}}^{\mathcal{R}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p_{\mathbf{\Omega}_{j+1}}^{\mathcal{V}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{aligned}$$



*Contribution , weight =(attenuation by the medium + and by the walls)

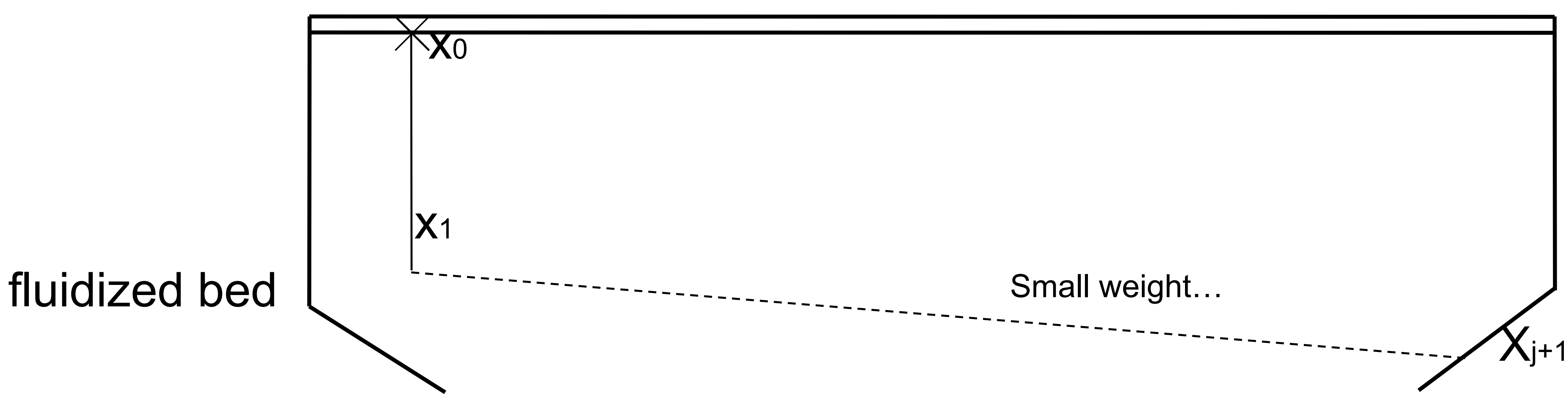
The Monte Carlo algorithm

- The radiative heat loss :

$$E = \int_{\mathcal{G}} p_{\mathbf{X}_0}(\mathbf{x}_0)d\mathbf{x}_0 \int_0^{\infty} p_{\kappa_0}(\kappa_0)d\kappa_0 \left\{ \begin{array}{l} H(\mathbf{x}_1 \in \mathcal{R}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_1}(\omega_1)d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \\ +H(\mathbf{x}_1 \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_1}(\omega_1|\omega_0)d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \end{array} \right\}$$

The recursive term:

$$\begin{aligned} \mathcal{I}_j \; &= H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R^{\mathcal{G}}_{j+1}}(r^{\mathcal{G}}_{j+1})dr^{\mathcal{G}}_{j+1} \left\{ \begin{array}{l} H(r^{\mathcal{G}}_{j+1} > \rho^{\mathcal{G}}) \; \hat{w}_{j+1} \\ +H(r^{\mathcal{G}}_{j+1} \leq \rho^{\mathcal{G}}) \int_{2\pi} p^{\mathcal{G}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{w}_j \leq \epsilon) \int_0^1 p_{R^{\mathcal{R}}_{j+1}}(r^{\mathcal{R}}_{j+1})dr^{\mathcal{R}}_{j+1} \times \ldots \\ \left\{ \begin{array}{l} H(r^{\mathcal{R}}_{j+1} > \rho^{\mathcal{R}}) \times 0 \\ +H(r^{\mathcal{R}}_{j+1} \leq \rho^{\mathcal{R}}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{w}_j > \epsilon) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{aligned}$$



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- The radiative heat loss :

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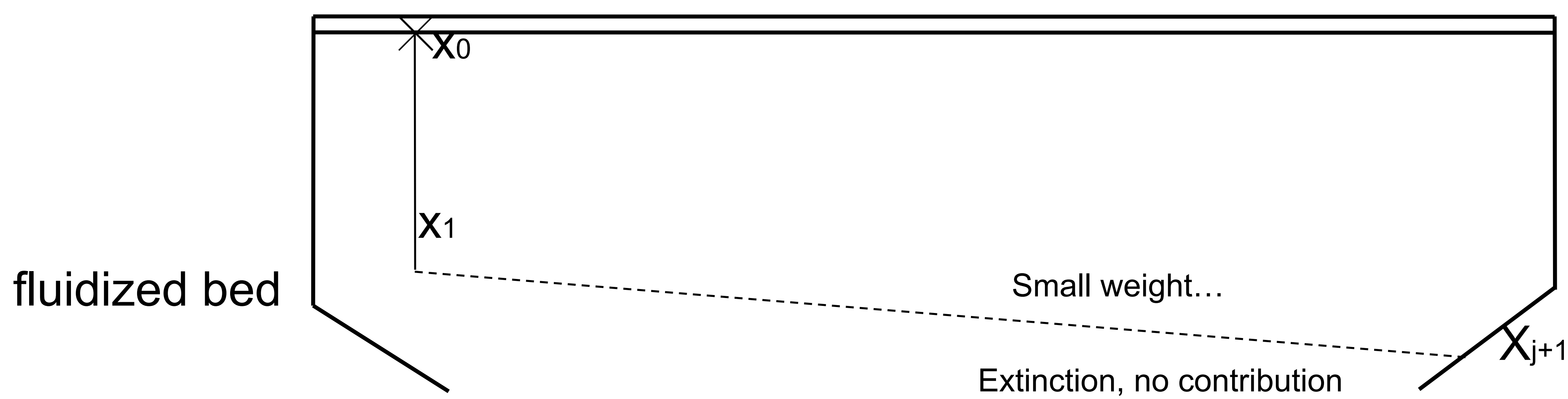

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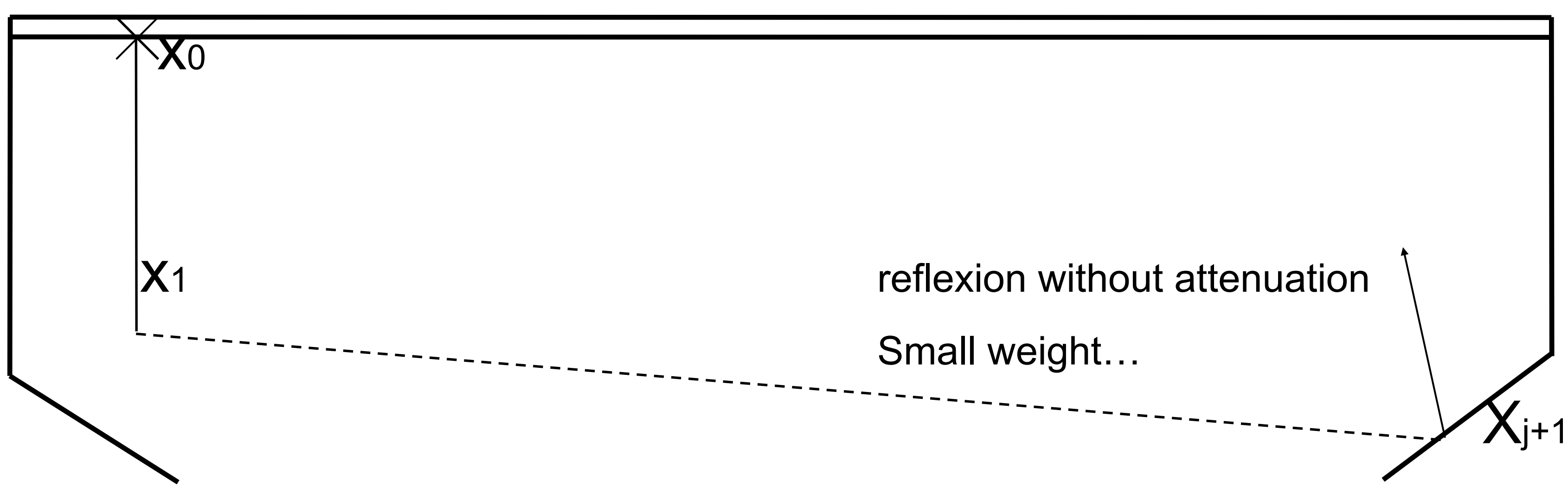

fluidized bed

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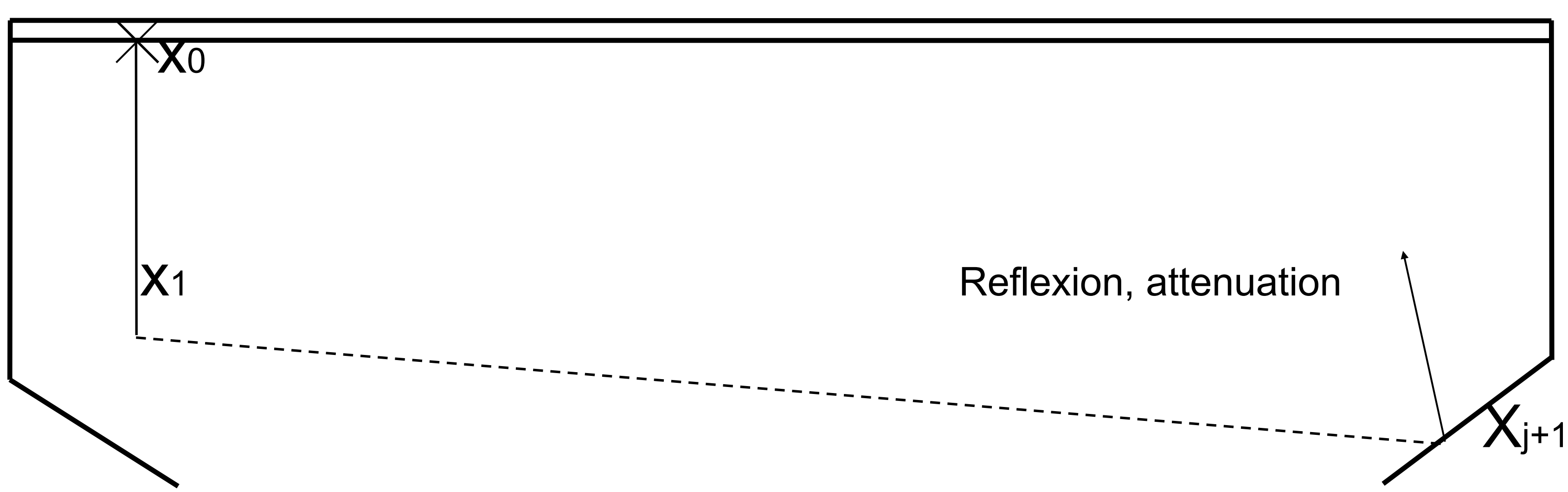
$$E = \int_{\mathcal{G}} p_{\mathbf{X}_0}(\mathbf{x}_0)d\mathbf{x}_0 \int_0^{\infty} p_{\kappa_0}(\kappa_0)d\kappa_0 \left\{ \begin{array}{l} H(\mathbf{x}_1 \in \mathcal{R}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_1}(\omega_1)d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \\ +H(\mathbf{x}_1 \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_1}(\omega_1|\omega_0)d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \end{array} \right\}$$

The recursive term:

$$\begin{aligned} \mathcal{I}_j \; &= H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R^{\mathcal{G}}_{j+1}}(r^{\mathcal{G}}_{j+1})dr^{\mathcal{G}}_{j+1} \left\{ \begin{array}{l} H(r^{\mathcal{G}}_{j+1} > \rho^{\mathcal{G}}) \; \hat{w}_{j+1} \\ +H(r^{\mathcal{G}}_{j+1} \leq \rho^{\mathcal{G}}) \int_{2\pi} p^{\mathcal{G}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{w}_j \leq \epsilon) \int_0^1 p_{R^{\mathcal{R}}_{j+1}}(r^{\mathcal{R}}_{j+1})dr^{\mathcal{R}}_{j+1} \times \ldots \\ \left\{ \begin{array}{l} H(r^{\mathcal{R}}_{j+1} > \rho^{\mathcal{R}}) \times 0 \\ +H(r^{\mathcal{R}}_{j+1} \leq \rho^{\mathcal{R}}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{w}_j > \epsilon) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ &+H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{aligned}$$

fluidized bed



*Contribution , weight =(attenuation by the medium + and by the walls)

The Monte Carlo algorithm

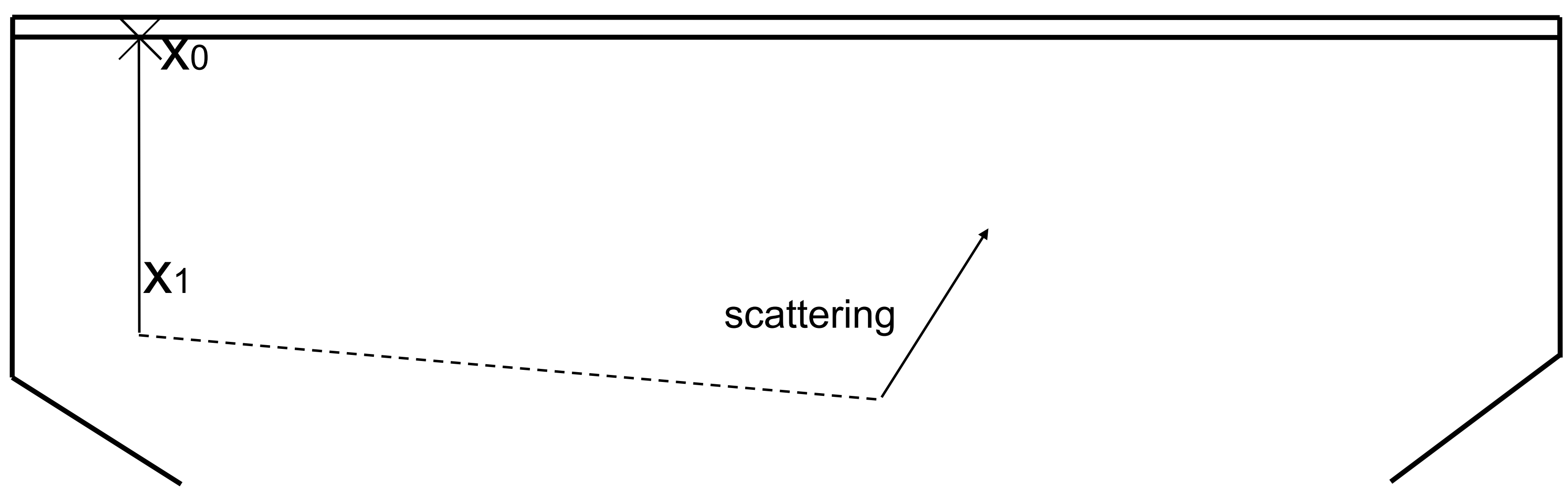
- The radiative heat loss :

$$E = \int_{\mathcal{G}} p_{\mathbf{X}_0}(\mathbf{x}_0)d\mathbf{x}_0 \int_0^\infty p_{\kappa_0}(\kappa_0)d\kappa_0 \left\{ \begin{array}{l} H(\mathbf{x}_1 \in \mathcal{R}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_1}(\omega_1)d\omega_1 \int_0^\infty p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \\ +H(\mathbf{x}_1 \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_1}(\omega_1|\omega_0)d\omega_1 \int_0^\infty p_{\kappa_1}(\kappa_1)d\kappa_1 \, \mathcal{I}_1 \end{array} \right\}$$

The recursive term:

$$\begin{array}{rl} \mathcal{I}_j & = H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R^{\mathcal{G}}_{j+1}}(r^{\mathcal{G}}_{j+1})dr^{\mathcal{G}}_{j+1} \left\{ \begin{array}{l} H(r^{\mathcal{G}}_{j+1} > \rho^{\mathcal{G}}) \, \hat{w}_{j+1} \\ +H(r^{\mathcal{G}}_{j+1} \leq \rho^{\mathcal{G}}) \int_{2\pi} p^{\mathcal{G}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ & +H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{w}_j \leq \epsilon) \int_0^1 p_{R^{\mathcal{R}}_{j+1}}(r^{\mathcal{R}}_{j+1})dr^{\mathcal{R}}_{j+1} \times \ldots \\ \left\{ \begin{array}{l} H(r^{\mathcal{R}}_{j+1} > \rho^{\mathcal{R}}) \times 0 \\ +H(r^{\mathcal{R}}_{j+1} \leq \rho^{\mathcal{R}}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{w}_j > \epsilon) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1})d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ & +H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j)d\omega_{j+1} \int_0^\infty p_{\kappa_{j+1}}(\kappa_{j+1})d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array}$$

fluidized bed



*Contribution , weight =(attenuation by the medium + and by the walls)

The Monte Carlo algorithm

- The radiative heat loss :

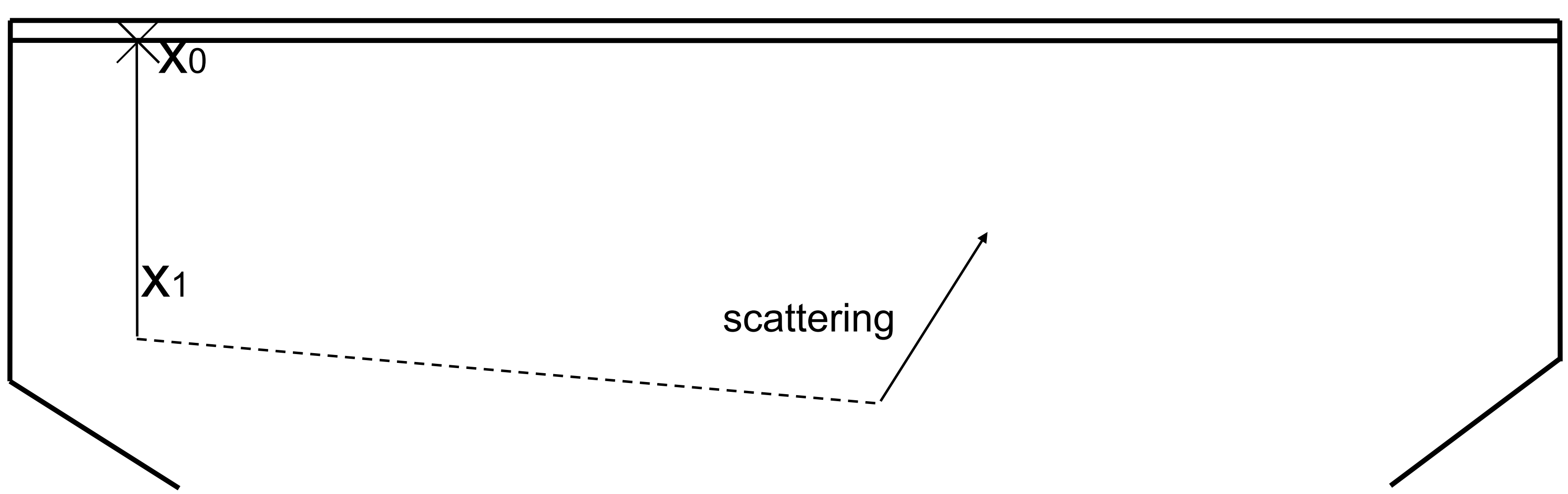
$$E = \int_{\mathcal{G}} p_{\mathbf{X}_0}(\mathbf{x}_0) d\mathbf{x}_0 \int_0^{\infty} p_{\kappa_0}(\kappa_0) d\kappa_0 \left\{ \begin{array}{l} H(\mathbf{x}_1 \in \mathcal{R}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_1}(\omega_1) d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1) d\kappa_1 \, \mathcal{I}_1 \\ +H(\mathbf{x}_1 \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_1}(\omega_1|\omega_0) d\omega_1 \int_0^{\infty} p_{\kappa_1}(\kappa_1) d\kappa_1 \, \mathcal{I}_1 \end{array} \right\}$$

The recursive term:

the weight or its derivative

$$\begin{array}{ll} \mathcal{I}_j & = H(\mathbf{x}_{j+1} \in \mathcal{G}) \int_0^1 p_{R^{\mathcal{G}}_{j+1}}(r^{\mathcal{G}}_{j+1}) dr^{\mathcal{G}}_{j+1} \left\{ \begin{array}{l} H(r^{\mathcal{G}}_{j+1} > \rho^{\mathcal{G}}) \, \boxed{\hat{w}_{j+1}} \\ +H(r^{\mathcal{G}}_{j+1} \leq \rho^{\mathcal{G}}) \int_{2\pi} p^{\mathcal{G}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ & +H(\mathbf{x}_{j+1} \in \mathcal{R}) \left\{ \begin{array}{l} H(\hat{w}_j \leq \epsilon) \int_0^1 p_{R^{\mathcal{R}}_{j+1}}(r^{\mathcal{R}}_{j+1}) dr^{\mathcal{R}}_{j+1} \times \dots \\ \left\{ \begin{array}{l} H(r^{\mathcal{R}}_{j+1} > \rho^{\mathcal{R}}) \times 0 \\ +H(r^{\mathcal{R}}_{j+1} \leq \rho^{\mathcal{R}}) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ +H(\hat{w}_j > \epsilon) \int_{2\pi} p^{\mathcal{R}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array} \right\} \\ & +H(\mathbf{x}_{j+1} \in \mathcal{V}) \int_{4\pi} p^{\mathcal{V}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j) d\omega_{j+1} \int_0^{\infty} p_{\kappa_{j+1}}(\kappa_{j+1}) d\kappa_{j+1} \, \mathcal{I}_{j+1} \end{array}$$

fluidized bed



*Contribution , weight =(attenuation by the medium + and by the walls)

The Monte Carlo algorithm

$$p_{\mathbf{x}_0}(\mathbf{x}_0) = \frac{1}{S_{\mathcal{G}}}$$

$$p_{\kappa_j}(\kappa_j) = \exp(-\kappa_j)$$

$$p^{\mathcal{R}}_{\mathbf{\Omega}_j}(\omega_j) = \frac{\omega_j.\mathbf{n}_j}{\pi}$$

$$p^{\mathcal{V}}_{\mathbf{\Omega}_{j+1}}(\omega_{j+1}|\omega_j) = \frac{1-g^2}{[1+g^2-2g(\omega_{j+1}.\omega_j)]^{3/2}}$$