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CALL FOR CONTRIBUTIONS: Towards numerical benchmark solutions for 3D mixed convection flows in rectangular channels heated from below

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Abstract

The main purpose of this paper is to propose two three-dimensional Poiseuille–Rayleigh–Bénard flows (mixed convection flows in horizontal rectangular channels heated from below), covering two different flow ranges, as benchmark problems and to solicit numerical comparisons between various contributors in order to obtain two benchmark solutions for the validation of numerical codes. The second objective is to identify the less perturbing outflow boundary conditions for this flow type. The first test case is a steady longitudinal roll flow in a large aspect ratio channel ($A = L/H = 50$, $B = l/H = 10$) at moderate Reynolds number $Re = 50$, Rayleigh number $Ra = 5000$ and Prandtl number $Pr = 0.7$. The second one is a fully-established space and time periodic transversal roll flow in a small aspect ratio channel ($A = 25$, $B = 4$) at small Reynolds number $Re = 0.1$, $Ra = 2500$ and $Pr = 7$. The model equations are the incompressible Navier–Stokes equations under the Boussinesq approximation.

Keywords: Comparison exercise; Numerical benchmark; Poiseuille–Rayleigh–Bénard flow; Mixed convection; Rectangular channel; Outflow boundary conditions

1. Objectives

The purpose of this paper is to propose two three-dimensional Poiseuille–Rayleigh–Bénard (PRB) flows as benchmark problems and to solicit interested groups to submit numerical solutions for comparison. The main objective is to obtain a numerical benchmark solution to validate numerical codes for the computation of thermoconvective instabilities in open channels. The second objective is to evaluate the influence of the outflow boundary conditions on the bulk solutions and to identify the less perturbing outflow boundary conditions for two different flow classes. The third objective is to identify the most efficient numerical methods in terms of CPU time and computational cost to deal with this type of problems.

2. Governing equations

The two flows proposed as benchmark cases are PRB flows in horizontal rectangular channels (cf. Fig. 1). A Poiseuille flow

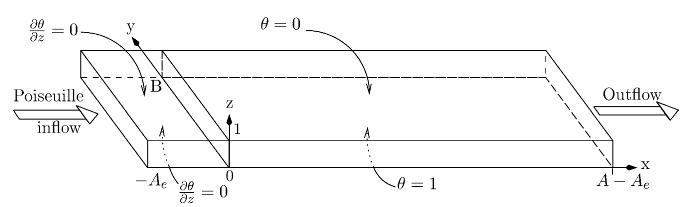

Fig. 1. Geometry and top and bottom thermal boundary conditions (the vertical lateral walls are adiabatic).

is imposed at the channel entrance and the incoming fluid is cold. After an entrance zone over which a zero heat flux is imposed on the four walls, the top horizontal wall is maintained at a cold temperature T_c and the bottom wall is maintained at a higher temperature T_h. The vertical lateral walls are adiabatic. Let A and B represent the streamwise and spanwise aspect ratios of the computational domain and A_e the streamwise entrance aspect ratio. The working fluid is Newtonian and the flows are governed by the 3D incompressible Navier–Stokes equations under the Boussinesq assumption. Using the channel height H, the mean flow velocity U_{mean}, ρU_{mean}^2 and H/U_{mean} as reference quantities for lengths, velocities, pressure and time, respectively, and using the reduced temperature

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$\theta = (T - T_c)/(T_h - T_c)$, the governing equations take the following dimensionless form:

$$\begin{cases} \nabla.\vec{v} = 0 \\ \frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v} = -\nabla p + \frac{1}{Re}\nabla^2 \vec{v} + \frac{Ra}{Pr\,Re^2}\theta \vec{k} \\ \frac{\partial \theta}{\partial t} + \vec{v}.\nabla\theta = \frac{1}{Pr\,Re}\nabla^2\theta \end{cases} \tag{1}$$

where x, y, z, t, $\vec{v} = (u, v, w)$ and p are the dimensionless streamwise, spanwise and vertical coordinates, time, velocity vector and pressure, $\vec{k}$ is the upward unit vector, Pr is Prandtl number ($= \nu/\alpha$), Re is Reynolds number ($= U_{\text{mean}}H/\nu$) and Ra is Rayleigh number ($= g\beta(T_h - T_c)H^3/(\nu\alpha)$). For the two test cases, the boundary and initial conditions for u, v, w and θ are:

- at $z = 0$, $\vec{v} = \vec{0}$; for $x \in [-A_e, 0]$, $\partial\theta/\partial z = 0$; for $x \in [0, A - A_e]$, $\theta = 1$;
- at $z = 1$, $\vec{v} = \vec{0}$; for $x \in [-A_e, 0]$, $\partial\theta/\partial z = 0$; for $x \in [0, A - A_e]$, $\theta = 0$;
- at $y = 0$ and B, $\vec{v} = \vec{0}$ and $\partial\theta/\partial y = 0$;
- at $x = -A_e$, $u = u_{\text{Pois}}(y, z)$, $v = w = 0$ and $\theta = 0$, where $u_{\text{Pois}}(y, z)$ is given either directly by an approximate solution of the Poisson equation $\frac{\partial^2 u_{\text{Pois}}}{\partial y^2} + \frac{\partial^2 u_{\text{Pois}}}{\partial z^2} = Re\frac{\partial p}{\partial x}$, with no-slip boundary conditions at $y = 0$ and B and at $z = 0$ and 1, or by the analytical solution of this equation given in [1];
- at $x = A - A_e$, an outflow non-reflective boundary condition is imposed. The choice of this boundary condition is left free: one will just try to impose a boundary condition that perturbs the outflow the least;
- at $t = 0$, $\forall x \in [-A_e, A - A_e]$, $u = u_{\text{Pois}}(y, z)$, $v = w = 0$ and $\theta = 0$.

3. Definition of the two test cases

The first test case is a steady longitudinal roll flow defined by: $Re = 50$; $Ra = 5000$; $Pr = 0.7$; $A = 50$; $B = 10$; $A_e = 2$. The second test case is a space and time periodic transversal roll flow defined by: $Re = 0.1$; $Ra = 2500$; $Pr = 7$; $A = 25$; $B = 4$; $A_e = 5$. These two flows are symmetrical about the median longitudinal vertical plane and they are obtained by starting from the initial conditions given in Section 2 after an intermittent phase which will not be exploited here.

4. Analysis of the results

4.1. Analysis of the first test case

Calculate the four dimensionless heat fluxes defined in Table 1, where S_i, S_o, S_t and S_b are the inlet, outlet, top and bottom surfaces of the channel respectively.

Calculate the 18 dimensionless momentum fluxes defined in Table 2, where S_f and S_r designate the front and rear surfaces at $y = 0$ and $y = B$ respectively, and calculate the integral of the buoyancy term, $\frac{-Ra}{Re^2 Pr}\theta$, on the whole computational domain.

By denoting ϕ the four dimensionless fields θ, u, v and w, and by denoting $Nu_t(x, y)$ and $Nu_b(x, y)$ the local Nusselt numbers on the top and bottom walls, with:

$$Nu_{t,b}(x, y) = -\frac{H(\frac{\partial T}{\partial Z})_{Z=H,Z=0}}{T_h - T_c} = -\left(\frac{\partial\theta}{\partial z}\right)_{z=1,z=0} \tag{2}$$

compute and display the following longitudinal and transversal profiles:

(1) $\phi_{y-z}(x)$ at $(y, z) = (2, 0.2)$ and $(5, 0.5)$ ($= 8$ profiles);
(2) $Nu_{t,b_y}(x)$ at $y = 2$ and 5 ($= 4$ profiles);
(3) $\phi_{x-z}(y)$ at $x = 10, 30$ and 48 and at $z = 0.2$ and 0.5 ($= 24$ profiles);
(4) $Nu_{t,b_x}(y)$ at $x = 10, 30$ and 48 ($= 6$ profiles);

and determine their minimum and maximum values ($\phi_{y-z}^{\min,\max}$, $\phi_{x-z}^{\min,\max}$, $Nu_{t,b_y}^{\min,\max}$, $Nu_{t,b_x}^{\min,\max}$) and their locations $(x, y, z)^{\min,\max}$. Consequently, 42 profiles and 84 extrema and their coordinates have to be calculated.

4.2. Analysis of the second test case

4.2.1. Spatial analysis at a fixed time t°

This flow is first analyzed at the fixed time $t^\circ > t^*$, where t^* is the time necessary to get the fully-established periodic

Table 1

$\Phi_{\theta,i}$	$\Phi_{\theta,o}$	$\Phi_{\theta,b}$	$\Phi_{\theta,t}$
$\iint_{S_i}(-Re\,Pr\,u\theta + \frac{\partial\theta}{\partial x})\,dy\,dz$	$\iint_{S_o}(Re\,Pr\,u\theta - \frac{\partial\theta}{\partial x})\,dy\,dz$	$\iint_{S_b}\frac{\partial\theta}{\partial z}\,dx\,dy$	$\iint_{S_t} -\frac{\partial\theta}{\partial z}\,dx\,dy$

Table 2

	Φ_u	Φ_v	Φ_w
S_i	$\iint_{S_i}\left(p - \frac{2}{Re}\frac{\partial u}{\partial x} + u^2\right)dy\,dz$	$\iint_{S_i}\frac{-1}{Re}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)dy\,dz$	$\iint_{S_i}\frac{-1}{Re}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)dy\,dz$
S_o	$\iint_{S_o}\left(-p + \frac{2}{Re}\frac{\partial u}{\partial x} - u^2\right)dy\,dz$	$\iint_{S_o}\left(\frac{1}{Re}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) - uv\right)dy\,dz$	$\iint_{S_o}\left(\frac{1}{Re}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) - uw\right)dy\,dz$
S_f	$\iint_{S_f}\left(-\frac{1}{Re}\frac{\partial u}{\partial y}\right)dx\,dz$	$\iint_{S_f}\left(p - \frac{2}{Re}\frac{\partial v}{\partial y}\right)dx\,dz$	$\iint_{S_f}\left(-\frac{1}{Re}\frac{\partial w}{\partial y}\right)dx\,dz$
S_r	$\iint_{S_r}\left(\frac{1}{Re}\frac{\partial u}{\partial y}\right)dx\,dz$	$\iint_{S_r}\left(-p + \frac{2}{Re}\frac{\partial v}{\partial y}\right)dx\,dz$	$\iint_{S_r}\left(\frac{1}{Re}\frac{\partial w}{\partial y}\right)dx\,dz$
S_b	$\iint_{S_b}\left(-\frac{1}{Re}\frac{\partial u}{\partial z}\right)dx\,dy$	$\iint_{S_b}\left(-\frac{1}{Re}\frac{\partial v}{\partial z}\right)dx\,dy$	$\iint_{S_b}\left(p - \frac{2}{Re}\frac{\partial w}{\partial z}\right)dx\,dy$
S_t	$\iint_{S_t}\left(\frac{1}{Re}\frac{\partial u}{\partial z}\right)dx\,dy$	$\iint_{S_t}\left(\frac{1}{Re}\frac{\partial v}{\partial z}\right)dx\,dy$	$\iint_{S_t}\left(-p + \frac{2}{Re}\frac{\partial w}{\partial z}\right)dx\,dy$
S_{tot}	0	0	$\iiint_D\left(\frac{-Ra}{Re^2 Pr}\theta\right)dx\,dy\,dz$

Table 3

$\tilde{\phi}^{\max}_{y,z=0.2,0.2}(x) = \sup_{t>t*} \phi(x, y=0.2, z=0.2, t)$	$\tilde{\phi}^{\min}_{y,z=0.2,0.2}(x) = \inf_{t>t*} \phi(x, y=0.2, z=0.2, t)$
$\tilde{\phi}^{\max}_{y,z=2,0.5}(x) = \sup_{t>t*} \phi(x, y=2, z=0.5, t)$	$\tilde{\phi}^{\min}_{y,z=2,0.5}(x) = \inf_{t>t*} \phi(x, y=2, z=0.5, t)$

flow and where t° is the time when the vertical velocity component w reaches a local maximum at the fixed point $(x, y, z) = (15.5, 2, 0.5)$. Compute the four heat fluxes $\Phi_{\theta,i}$, $\Phi_{\theta,o}$, $\Phi_{\theta,t}$ and $\Phi_{\theta,b}$, the eighteen momentum fluxes $\Phi_{u,i}$, $\Phi_{u,o}$, ... and $\Phi_{w,t}$, the volume integral of the buoyancy term given in Tables 1 and 2 (note that, since the flow of the second test case is unsteady, the total fluxes are non-zero) and the following longitudinal, transversal and vertical profiles:

(1) $\phi_{y-z}(x)$ at $(y, z) = (0.2, 0.2)$ and $(2, 0.5)$ (= 8 profiles);
(2) $Nu_{t,b_y}(x)$ at $y = 0.2$ and 2 (= 4 profiles);
(3) $\phi_{x-z}(y)$ at $(x, z) = (15.5, 0.5)$ (= 4 profiles);
(4) $Nu_{t,b_x}(y)$ at $x = 15.5$ (= 2 profiles);
(5) $\phi_{x-y}(z)$ at $x = 20$ and at $y = 0.2$ and 2 (= 8 profiles);

and, for each profile, determine their minimum and maximum values $\phi^{\min,\max}_{y-z}$, $Nu^{\min,\max}_{t,b_y}$, $\phi^{\min,\max}_{x-z}$, $Nu^{\min,\max}_{t,b_x}$ and $\phi^{\min,\max}_{x-y}$ and their locations $(x, y, z)^{\min,\max}$. Therefore, 26 profiles and 52 extrema and their coordinates have to be calculated. Compute the four average dimensionless wavelengths $\bar{\lambda}_{\phi_{y-z}(x)}$, in the interval $7 \leqslant x \leqslant 15$, from the four longitudinal profiles $\phi_{y-z}(x)$ at $(y, z) = (2, 0.5)$.

4.2.2. Temporal analysis

Starting from time $t = t^\circ$, record the following signals $\phi(t)$ and $Nu_{t,b}(t)$:

(1) $\phi_{x-y-z}(t)$ at $x = 0, 5, 15$ and 20, at $y = 2$ and at $z = 0.5$ (= 16 signals);
(2) $Nu_{t,b_{x-y}}(t)$ at $x = 5, 15$ and 20 and at $y = 2$ (= 6 signals).

Determine the minimum and maximum values $\phi^{\min,\max}_{x-y-z}$, $Nu^{\min,\max}_{t,b_{x-y}}$ of these 22 signals and the 16 dimensionless frequencies $f_{\phi_{x-y-z}(t)}$ from the 16 signals $\phi_{x-y-z}(t)$. Finally, compute the 16 envelopes of ϕ defined in Table 3.

5. Deadlines and presentation of the results

The time allotted for completion of this exercise is not fixed yet. However, a presentation of the first results is already scheduled during the congress SFT2006 of the French Heat Transfer Society that will take place at the Ile de Ré, 16–19 May 2006. Therefore, the potential contributors should contact *as soon as possible* the coordinators of this comparison exercise (M. Medale and X. Nicolas) and send them their (first) results before the end of April to permit their compilation with the other results.

Remarks, comments and further information relative to this comparison exercise (required results, data format, presentation of the results, deadlines, first results, ...) will be available on the web page of the French Heat Transfer Society (SFT): www.sft.asso.fr/groupes/simul.html.

References

[1] A. Benzaoui, X. Nicolas, S. Xin, Efficient vectorized finite difference method to solve the incompressible Navier–Stokes equations for 3D mixed convection flows in high aspect ratio channels, Numer. Heat Transfer B 48 (2005) 277–302.