Une approche pseudo-analytique basée sur la formulation *improved lumped* pour la reconstruction transitoire du flux thermique en écoulement dispersé sous conditions de type APRP.

A pseudo-analytical improved lumped approach for transient heat flux reconstruction in dispersed flow under LOCA-like conditions.

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Résumé - Ce travail présente une méthode pseudo-analytique pour l'estimation du flux thermique transitoire à la paroi interne d'un tube chaud soumis à un renoyage, dans des conditions de type APRP. L'approche combine une formulation *improved lumped* avec la résolution de l'équation de la chaleur à variation axiale, en conservant partiellement les effets des gradients radiaux, avec un faible coût de calcul. Le modèle a été évalué avec des données d'écoulements dispersés obtenues dans l'installation COLIBRI. Les profils de flux thermique prédits sont cohérents avec ceux obtenus par une méthode conventionnelle, avec des écarts résiduels principalement liés à l'estimation des valeurs pics. Bien que limitée aux conditions d'écoulements dispersés, cette étude constitue une étape importante vers la reconstruction du flux thermique dans des expériences de renoyage.

Abstract - This work presents a pseudo-analytical method for estimating the transient inner wall heat flux of a hot tube subjected to a reflooding scenario under LOCA-like conditions. The approach combines an improved lumped formulation with the solution of the axially varying heat conduction equation, while partially retaining radial gradient effects and maintaining a low computational cost. The model was evaluated using dispersed flow data obtained from the COLIBRI facility. The predicted heat flux profiles are consistent with those obtained using a conventional method, with remaining discrepancies mainly associated with peak values. Although limited to dispersed flow conditions, this study represents an important step toward heat flux reconstruction in reflooding experiments.

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Nomenclature

A	finite difference coefficient, $1/\mathrm{m}^2$
B	finite difference coefficient, $1/\mathrm{m}^2$
C_1	average temperature coefficient, $\mathrm{K/W}$
C_2	geometric coefficient, $1/\mathrm{m}^2$
C_3	improved lumped constant, $(-)$
C_4	improved lumped constant, $\mathrm{m}^2\,\mathrm{K/W}$
E_d	total dissipated energy, J
F	source term in the averaged energy equation, $\mathrm{K/m}^2$
h_c	convective-radiative heat transfer coefficient, $\mathrm{W/(m^2\,K)}$
k	thermal conductivity, $\mathrm{W/(m\,K)}$
q''	heat flux, $\mathrm{W/m}^2$
Q	volumetric heat generation rate, $\mathrm{W/m}^3$
r	radial coordinate, m
t	time, s
T	temperature, K
z	axial coordinate, m
Δt	time step, s
Δz	axial spatial step, m
Greek symbols	
α	thermal diffusivity, m^2/s
ρ	density, $\mathrm{kg/m}^3$
θ	temperature difference between outer wall and ambient medium, K
Subscripts and superscripts	
av	radial average
e	outer surface
i	inner surface
m	measured value
max	maximum axial position
n	time step index
w	wall
∞	ambient condition

1. Introduction

Flow boiling phenomena are crucial in nuclear safety research, as several postulated accidents in water-cooled reactors are directly related to the coolant capability in the reactor core. One important example is the loss-of-coolant accident (LOCA) in Pressurized Water Reactors (PWRs), which results from breaches in the primary circuit that lead to rapid vessel depressurization. This process causes a significant rise in cladding temperature, which may ultimately result in core melting. To mitigate this scenario, Emergency Core Cooling Systems (ECCS) are designed to inject borated water into the reactor vessel, removing residual decay heat and restoring the core cooling capability.

This period, most commonly known as the reflooding phase, is characterized by four distinct vertical regions [1]: i) single-phase liquid water; ii) a quenching front, characterized by intense boiling; iii) a vapor core with entrained droplets; and iv) single-phase steam. The coexistence of multiple heat transfer and hydrodynamic regimes poses significant challenges to the overall understanding of the phenomenon. In addition, each regime presents its own modeling difficulties, such as thermal and hydrodynamic non-equilibrium between liquid and vapor phases in dispersed flows [2, 3]. Therefore, many aspects of flow boiling during LOCA remain insufficiently understood and require further investigation.

Among the relevant parameters for understanding flow boiling, those associated with thermal measurements play a crucial role, especially from a nuclear safety perspective. Traditional approaches for estimating the heat flux at the inner surface of heated channels rely on external temperature measurements and lumped assumptions for modeling heat conduction through the wall. Peña Carrillo [4, 5] and Luna Valencia [6, 7], for instance, investigated dispersed flows in cylindrical geometries under the assumption of negligible radial temperature gradients based on the Biot number criterion ($\mathrm{Bi} << 0.1$). Although valid under film boiling conditions, the lumped assumption fails to accurately predict the thermal behavior along the full length of flow boiling near LOCA conditions. In the quenching region, high heat transfer coefficients are expected ($\mathrm{Bi} >> 1$), requiring alternative techniques for heat flux estimation.

This paper introduces a reduced-order solution to a transient two-dimensional heat conduction problem as an alternative method for estimating boundary heat flux. The approach combines the improved lumped formulation proposed by Cotta and Mikhaĭlov [8] with a finite difference method. The proposed solution seeks to retain the low computational cost typical of lumped models while preserving information about temperature gradients expected under flow boiling conditions close to LOCA scenarios. Experimental data obtained in the COLIBRI facility were employed for heat flux estimation through the proposed method, and the resulting profiles were then compared with those obtained using the method of Peña Carrillo [4] and Luna Valencia [6]. This represents an important step toward assessing the applicability of the method to future reflooding experiments.

2. Methodology

2.1. Heat conduction problem

Figure 1 presents the physical problem addressed in this work. The test section employed in COLIBRI consists of a 100 mm-long hollow cylinder made of Inconel 625, with internal and external diameters of 11.78 mm and 12.92 mm, respectively. The geometry was designed to represent subchannel flow in a fuel assembly and is based on hydraulic diameter equivalence. Convective and radiative heat losses occur at the external surface and are evaluated using the measured external surface temperature, $T_m(z,t)$. Because the temperature field obtained from the infrared camera is limited to a narrow angular region (i.e., pixels near the centerline), only the heat flux along an axial line is estimated; therefore, an axisymmetric assumption is adopted. At the inner surface, the internal flow is represented by a heat flux profile, $q''_w(z,t)$. Volumetric heat generation, Q, is included to mimic the decay heat expected in fuel rods. Finally, axial temperature gradients at both ends are neglected. This assumption is justified since the analysis is restricted to the central section (60 mm) of the tube, excluding pixels near the boundaries.

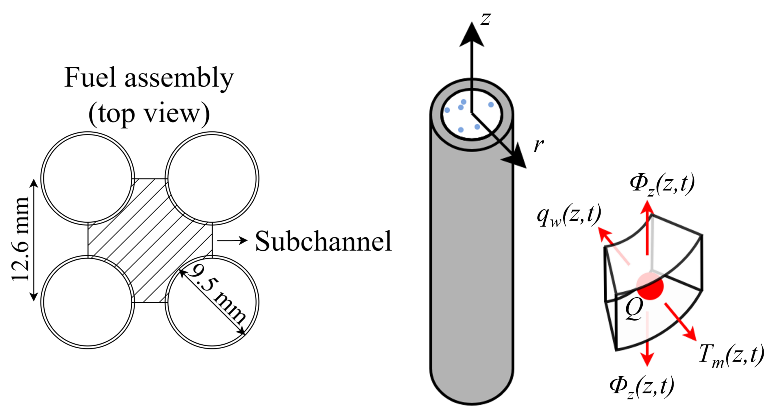

Figure 1 : *Schematic drawing of the subchannel geometry (left) and the heat conduction problem (right)*

Assuming constant thermophysical properties, the governing heat conduction equation is given by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}+\frac{Q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

where $T(r, z, t)$ is the transient temperature field, k is the thermal conductivity, and α is the thermal diffusivity. According to the imposed physical conditions, the following boundary conditions are applied:

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_i} = \frac{q''_w}{k} \tag{2}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_e} = \frac{h_c(T_\infty - T_m)}{k} = -\frac{h_c\theta_m}{k} \tag{3}$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = \left.\frac{\partial T}{\partial z}\right|_{z=z_{max}} = 0 \tag{4}$$

where h_c is the convective-radiative heat transfer coefficient and T_∞ is the surrounding fluid temperature. The initial condition is taken as $T_{initial} = T_m(1)$.

To apply the improved lumped formulation proposed by Cotta and Mikhaĭlov [8], the average radial temperature of the hollow cylinder is first defined as:

$$T_{av} = \frac{2}{(r_e^2 - r_i^2)} \int_{r_i}^{r_e} rTdr \tag{5}$$

This approach integrates one spatial dimension, similar to the classical lumped capacitance model. However, unlike the conventional approach, the improved lumped formulation preserves partial information about temperature gradients in the integrated direction, making it more suitable for high Biot number problems. Applying the operator $\int_{r_i}^{r_e} r\,dr$ to Equation (1) and using Equation (5) leads to the following formulation:

$$C_1 q_w + C_2\theta_m + \frac{\partial^2 T_{av}}{\partial z^2} + \frac{Q}{k} = \frac{1}{\alpha}\frac{\partial T_{av}}{\partial t} \tag{6}$$

where C_1 and C_2 are constants:

$$C_1 = \frac{-2r_i}{(r_e^2 - r_i^2)k} \tag{7}$$

$$C_2 = \frac{-2r_e h_c}{(r_e^2 - r_i^2)k} \tag{8}$$

Equation (6) relates both boundary conditions to the average radial temperature without additional approximations. A second relation between these quantities can be obtained using Hermite integral approximations, as presented by Mennig et al. [9]. The $H_{1,1}$ formulation (corrected trapezoidal rule) is written as:

$$H_{1,1} : \int_0^h y(x)dx \approx \frac{h}{2}[y(0) + y(h)] + \frac{h^2}{12}[y'(0) - y'(h)] \tag{9}$$

Applying this formulation to approximate both the average temperature and the heat flux profiles yields:

$$T_{av} \approx \frac{1}{(r_e + r_i)}(r_e T_m + r_i T_{(r_i)}) + \frac{(r_e - r_i)}{6(r_e + r_i)}\left[T_{(r_i)} - T_m + \frac{(q_w r_i + h_c \theta_m r_e)}{k}\right] \tag{10}$$

$$T_m - T_{(r_i)} = \int_{r_i}^{r_e} \frac{\partial T}{\partial r} dr \approx \frac{(r_e - r_i)}{2k}(q_w - h_c\theta_m) - \frac{(r_e - r_i)^2}{12k}\left(\frac{q_w}{r_i} + \frac{h_c \theta_m}{r_e}\right) \tag{11}$$

where $T_{(r_i)}$ is the inner wall temperature. Combining Equations (10) and (11), a new expression relating the average temperature and the boundary conditions is obtained:

$$T_{av} = T_m + C_3\theta_m + C_4 q_w \tag{12}$$

where C_3 and C_4 are constants:

$$C_3 = \frac{(r_e - r_i)(19r_e^2 + 38r_e r_i - 5r_i^2)h_c}{72r_e(r_e + r_i)k} \tag{13}$$

$$C_4 = \frac{(r_e - r_i)(r_e^2 - 2r_e r_i - 23r_i^2)}{72r_i(r_e + r_i)k} \tag{14}$$

Combining Equations (6) and (12) yields an expression involving only the boundary conditions. However, the resulting relation is a non-homogeneous partial differential equation, first order in time and second order in space, with some mathematical complexity. To overcome this problem, a numerical solution based on the finite difference method was adopted. Initially, both equations were merged to obtain a single equation governing the average temperature:

$$\frac{\partial^2 T_{av}}{\partial z^2} - \frac{1}{\alpha}\frac{\partial T_{av}}{\partial t} + \frac{C_1}{C_4}T_{av} = \frac{C_1}{C_4}T_m + \left(\frac{C_1 C_3}{C_4} - C_2\right)\theta_m - \frac{Q}{k} = F \tag{15}$$

where $F(z,t)$ is a function of the internal heat generation Q and the measured temperature T_m. The spatial derivative is approximated explicitly using a three-point central difference scheme, while the temporal derivative is discretized using a first-order backward scheme. This leads to the following matrix form:

$$\boldsymbol{T}_{av}^{n} = \boldsymbol{M}^{-1}\left(\boldsymbol{F} - \frac{\boldsymbol{T}_{av}^{n-1}}{\alpha \Delta t}\right) \tag{16}$$

where:

$$\boldsymbol{M} = \begin{bmatrix} 1 & -2 & 1 & \cdots & 0 \\ A & B & A & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & A & B & A \\ 0 & \cdots & 1 & -2 & 1 \end{bmatrix} \tag{17}$$

$$A = \frac{1}{\Delta z^2} \tag{18}$$

$$B = \frac{C_1}{C_4} - \frac{2}{\Delta z^2} - \frac{1}{\alpha \Delta t} \tag{19}$$

The boundary conditions were derived from Equation 4 and are given by:

$$\left.\frac{\partial^2 T_{av}}{\partial z^2}\right|_{z=0} = \left.\frac{\partial^2 T_{av}}{\partial z^2}\right|_{z=z_{max}} = 0 \tag{20}$$

The initial average temperature was estimated by assuming a negligible initial heat flux, yielding $T_{av}^1 = T_m(1) + C_3\theta_m(1)$. Finally, the heat flux profile was obtained from Equation 12.

2.2. Experimental data from COLIBRI

As mentioned in the Introduction, the COLIBRI facility was designed to characterize dispersed flows under LOCA-like conditions [4, 6]. The experiments involved the simultaneous injection of vapor and droplets into the heated tube. An infrared camera was used to measure the external surface temperature of the tube, which was subsequently used to estimate the inner surface heat flux. In this methodology, a lumped parameter assumption was adopted, enabling direct estimation of the heat flux. Four experiments conducted by Luna Valencia [6] were used for model validation (Table 1). Using the same experimental conditions as input, the proposed model estimated the heat flux profiles from the measured temperatures. These results were then compared with the heat flux obtained using the conventional approach.

	$\dot{m}_v$	$\dot{m}_d$	Q
	kg/h	kg/h	kW/m^3
Experiment 1	4	8	0
Experiment 2	1.6	7.3	0
Experiment 3	3.1	8	0
Experiment 4	3.9	5.1	0

Table 1 : *Experimental input conditions: vapor mass flow rate, droplet mass flow rate, and internal heat generation rate*

2.3. Model verification

The agreement between the improved lumped formulation and the method used in COLIBRI was evaluated using two normalized statistical error metrics: the Normalized Root Mean Square Error (NRMSE) and the Normalized Mean Absolute Error (NMAE). The NRMSE is based on squared deviations and is more sensitive to large discrepancies, highlighting peak estimation errors. The NMAE, based on absolute differences, represents the average magnitude of the errors without overemphasizing outliers. Both metrics were normalized by the mean experimental heat flux, yielding dimensionless values. Another metric considered was the total dissipated energy at the inner surface:

$$E_d = \int_0^{t_{max}} \int_0^{z_{max}} q_w(z,t)\,dz\,dt \tag{21}$$

Although important for the heat transfer balance, the dissipated energy is directly dependent on the selected integration time and therefore does not constitute a statistical basis for comparison between the two methods. Consequently, the resulting values were used exclusively for comparative purposes. The integration was performed over the total rewetting time of the tube (20 s). For the experimental data, only averaged heat flux values at 5 mm axial intervals were available, requiring spatial discretization of the integral. Finally, it should be noted that the present comparison is based solely on experimental data, and therefore agreement between both models does not necessarily imply their physical correctness, as similar results may arise from compensating errors. To address this limitation, a complementary verification based on synthetic data is currently ongoing, allowing for a more controlled assessment of each model's predictive capability.

3. Results and discussion

Figure 2 compares the average heat flux predicted by the improved lumped formulation with that obtained using the original COLIBRI method. The heat flux was averaged over 5 mm axial segments, resulting in 20 axial points. Profiles at different time instants are presented, enabling a visual comparison of curve shapes and peak locations. Predictions in the central region show good agreement with the experimental data, with only minor discrepancies in peak values. However, significant discrepancies are observed near both axial ends, as expected: although these regions were assumed to be adiabatic, substantial heat flux gradients may develop at the boundaries during rewetting.

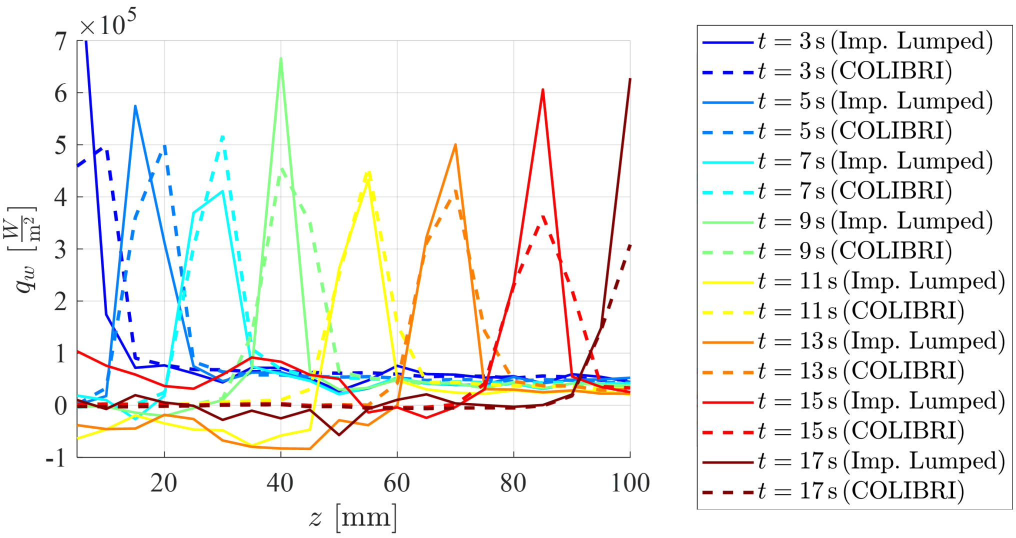

Figure 2 : *Comparison of the average heat flux predicted by the improved lumped formulation and by the original COLIBRI method (Experiment 3)*

Table 2 summarizes the differences between methods in terms of dissipated energy, RMSE, and MAE. The dissipated energy differences were below 20% in all cases. RMSE values reached up to 45%, reflecting discrepancies mainly associated with peak heat flux estimation. In contrast, MAE values were lower, with a maximum of 25%, indicating that the overall heat flux profile is better predicted than the peak values.

4. Conclusion

This work presents a pseudo-analytical approach for estimating the inner wall heat flux under transient flow boiling conditions representative of LOCA scenarios. The method combines

	E_d	e_{RMSE}	e_{MAE}
	%	%	%
Experiment 1	19.39	44.70	23.99
Experiment 2	16.79	38.55	21.69
Experiment 3	6.18	13.46	9.27
Experiment 4	6.28	13.86	11.19

Table 2 : *Error values concerning the dissipated energy, RMSE, and MAE*

an improved lumped formulation with a finite difference solution of the axially varying heat conduction problem, allowing partial preservation of radial temperature gradient effects while maintaining a low computational cost. The proposed model was validated against experimental data from the COLIBRI facility under dispersed flow conditions. The results show good agreement in terms of total dissipated energy, with differences below 20%. Although larger discrepancies are observed in peak heat flux predictions — up to 45% – the overall heat flux distribution along the tube remains reasonably accurate, with discrepancies up to 25%. These findings indicate that the improved lumped formulation provides a physically consistent approach for heat flux estimation. While the comparison is limited to dispersed flow conditions and does not fully validate the method for reflooding experiments, it represents an important step toward the development of a tool capable of reconstructing heat flux in LOCA-like reflooding scenarios.

References

[1] F. D'Auria, *Thermal-hydraulics of water cooled nuclear reactors*, Woodhead Publishing (2017).

[2] D. C. Groeneveld and G. G. J. Delorme, Prediction of thermal non-equilibrium in the post-dryout regime, *Nuclear Engineering and Design*, 36 (1976) 17–26.

[3] Y. Jin, *Investigation of two-phase flow thermal-hydraulic behavior in rod bundle during reflood transients based on the RBHT experimental data*, The Pennsylvania State University (2019).

[4] J. D. Peña Carrillo, *Étude expérimentale du transfert paroi/fluide dans le cas d'un écoulement vertical vapeur/gouttes dans une géométrie tubulaire*, Université de Lorraine (2018).

[5] J. D. Peña Carrillo, A. V. S. Oliveira, A. Labergue, T. Glantz and M. Gradeck, Experimental thermal hydraulics study of the blockage ratio effect during the cooling of a vertical tube with an internal steam-droplets flow, *International Journal of Heat and Mass Transfer*, 140 (2019) 648–659.

[6] J. E. Luna Valencia, *Étude du refroidissement d'un assemblage combustible par un écoulement vertical vapeur/gouttes à l'échelle d'un sous-canal*, Université de Lorraine (2023).

[7] J. E. Luna Valencia, A. V. S. Oliveira, T. Glantz, A. Labergue and M. Gradeck, Evaluation of heat dissipation phenomena in dispersed flow film boiling using a mechanistic model and different correlations for droplet impact heat transfer, *International Journal of Heat and Mass Transfer*, 245 (2025) 126955.

[8] R. M. Cotta and M. D. Mikhailov, *Heat conduction: lumped analysis, integral transforms, symbolic computation*, (1997).

[9] J. Mennig, T. Auerbach and W. Hälg, Two point Hermite approximations for the solution of linear initial value and boundary value problems, *Computer Methods in Applied Mechanics and Engineering*, 39 (1983) 199–224.