Development of a symbolic Monte Carlo model applied to heat transfer in buildings

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Abstract - Buildings account for 30–40% of global energy consumption, making accurate thermal diagnostics essential for energy efficiency improvements. Infrared (IR) thermography is widely used for non-intrusive inspection of building envelopes; however, its quantitative interpretation remains limited due to transient effects and coupled heat transfer mechanisms. This work proposes a Symbolic Monte Carlo (SMC) framework for modeling heat transfer in buildings and generating synthetic IR images. The governing energy balance is reformulated in probabilistic terms, allowing temperature fields to be expressed as transfer functions of thermophysical parameters. The methodology is validated on 1D conduction-convection configuration with available analytical solutions, demonstrating accurate temperature reconstruction and reduced computational cost. The orthogonality between the data characterizing the geometry and the Monte Carlo calculation, keeping the algorithm unchanged make it possible to move easily from 1D to 3D by implementing the CAD of the configuration.

Nomenclature

T	Temperature, K
x	Spatial coordinate, m
h	Convective heat transfer coefficient, $\mathrm{W\,m^{-2}\,K^{-1}}$
λ	Thermal Conductivity, $\mathrm{W\,m^{-1}\,K^{-1}}$
λ_{ref}	Reference thermal conductivity, $\mathrm{W\,m^{-1}\,K^{-1}}$
N	Number of Monte Carlo realizations, –
w_i	Weight of the i-th realization, –
$T(x_{\text{obs}})$	Temperature at observation point, K
s	Statistical error, K
G	Green's function, –
H	Transfer function, –
Greek symbols	
δ	Spatial discretization step, m
δ_b	Boundary spatial discretization step, m
Index and exponent	
i	Monte Carlo realization index
obs	Observation point

1. Introduction

Improving the thermal performance of buildings is a major challenge in the context of energy efficiency and climate mitigation, as the building sector accounts for a significant share of global energy consumption and greenhouse gas emissions [1, 2]. Reliable diagnostic methods are therefore required to identify heat losses and to estimate key thermophysical parameters. In existing buildings, defects in insulation, thermal bridges, and envelope discontinuities significantly reduce energy efficiency and occupant comfort, motivating the development of robust thermal diagnostic tools. Infrared (IR) thermography is widely used for non-intrusive inspection of building envelopes, as it provides spatially resolved surface temperature fields. While effective for the qualitative detection of thermal anomalies, the quantitative interpretation of thermographic data remains challenging due to transient effects, complex boundary conditions, and the coupling of heat transfer mechanisms [3]. Consequently, extracting material properties or boundary conditions from IR measurements generally requires numerical modeling. Deterministic approaches such as finite difference or finite element methods can deliver accurate solutions but often become computationally expensive when applied to inverse problems, uncertainty analysis, or complex geometries. Monte Carlo (MC)

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methods offer an alternative probabilistic framework that naturally handles stochastic boundary conditions and parameter uncertainties while avoiding volumetric meshing [4, 5].

In practice, infrared thermography provides access to surface temperature fields or heat fluxes, while key quantities such as thermal conductivity or boundary conditions remain unknown. These parameters must therefore be estimated through inverse modeling approaches, which aim to reconstruct model inputs from measured observables. This inverse perspective motivates the development of efficient surrogate models capable of rapidly exploring the parameter space.

In this work, a Symbolic Monte Carlo (SMC) formulation [6] is employed to express temperature fields as transfer functions of thermal parameters, with a particular focus on thermal conductivity. The proposed approach enables the evaluation of temperature responses over a wide range of parameter values from a single Monte Carlo simulation, significantly reducing computational cost. The methodology is illustrated on a one-dimensional conduction-convection configuration, for which analytical solutions are available, allowing a direct assessment of accuracy and stability. This study provides a foundation for fast parametric thermal diagnostics and the generation of synthetic thermographic data within a Monte Carlo framework.

2. METHODOLOGY

2.1. Monte Carlo Formulation

Early works such as those by Dunn et al. [7], Hammersley and Handscomb [8], and Kalos et al. [9] provide a comprehensive foundation of the Monte Carlo (MC) method, covering its theoretical basis, algorithmic developments, and applications to complex physical problems.

For clarity, the methodology is first presented in its classical Monte Carlo form before introducing the symbolic extension. The objective is to progressively build the formulation while highlighting the role of thermal conductivity as an uncertain parameter.

The objective of this work is to estimate the temperature at a given observation point as a transfer function of the thermal conductivity using the Symbolic Monte Carlo (SMC) method. Since the thermal conductivity is treated as a parameter of the transfer function, it must not be fixed during the calculation. The approach is first illustrated on an academic 1D steady-state conduction-convection configuration, shown in Fig. 1.

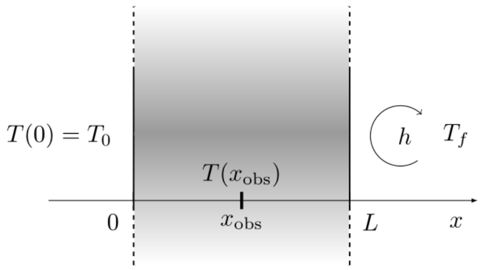

Figure 1 : 1*D steady-state conduction-convection configuration.*

In a first step, a classical Monte Carlo algorithm is developed to estimate the temperature at the observation point, with all physical parameters fixed, including the thermal conductivity. In a second step, this algorithm is reformulated following the Symbolic Monte Carlo formalism [10], in which the propagation paths are preserved, and parameter dependencies are stored symbolically. This reformulation enables the estimation of temperature as a continuous function of thermal conductivity from a single Monte Carlo simulation [4].

For the steady-state conduction problem, the temperature field satisfies the Laplace equation given in eq. (1). The Dirichlet boundary condition with fixed temperature on the left side and the convection Robin boundary condition with the fluid temperature T_f on the right side are given in eq. (2).

$$\frac{d^2T}{dx^2} = 0, \qquad x \in [0, L] \tag{1}$$

$$\begin{cases} T(0) = T_0, \\ \lambda \left.\dfrac{dT}{dx}\right|_{x=L} = -h\big(T(L) - T_f\big) \end{cases} \tag{2}$$

Initially, the Laplacian eq. (1) is discretized using a second-order centered finite difference scheme. By discretizing the conduction equation and the Robin boundary condition at $x = L$ using finite differences, the temperature at the right boundary can be written as a convex combination of the fluid temperature T_f and the temperature in the solid at $L - \delta_b$:

$$T(L) = \underbrace{\frac{\frac{\lambda}{\delta_b}}{h + \frac{\lambda}{\delta_b}}}_{P(\lambda)} T(L - \delta_b) + \underbrace{\frac{h}{h + \frac{\lambda}{\delta_b}}}_{1-P(\lambda)} T_f \tag{3}$$

Equation (3) naturally defines the transition probabilities of the Monte Carlo random walk at the boundary, corresponding respectively to re-injection into the solid and termination at the fluid temperature.

$$T(x_{\text{obs}}) = \begin{cases} \mathcal{H}(x_{\text{obs}} \in \mathcal{D}) \left[\frac{1}{2}T(x_{\text{obs}} + \delta) + \frac{1}{2}T(x_{\text{obs}} - \delta)\right] \\ + \\ \mathcal{H}(x_{\text{obs}} \in \partial\mathcal{D}) \begin{cases} \mathcal{H}(x_{\text{obs}} = 0)T_0 \\ + \\ \mathcal{H}(x_{\text{obs}} = L) \begin{cases} (1 - \mathrm{P}(\lambda))T_f \\ + \\ \mathrm{P}(\lambda)T(L - \delta_b) \end{cases} \end{cases} \end{cases} \tag{4}$$

From eq. (4), the estimated temperature $T(x_{obs})$ can be viewed as the expectation of a random variable W (Monte Carlo Weight) like **??**. The temperature estimate and the associated error s correspond to the eq. (6) and eq. (7)

In a synthetic manner, at each i^{th} realization, the random path starts at the point x_{obs} and continues until a known temperature at the boundary condition is reached. When a known temperature is reached (T_f or T_0), the value of that temperature is recorded in the weight w_i. At the end of an MC realization, an i^{th} realization w_i of the random variable W is given in eq. (5).

The detailed derivation of the Monte Carlo discretization, boundary treatment, transition probabilities, and the complete Symbolic Monte Carlo formulation is not reproduced here for brevity. The interested reader is referred to Chapter 3 of the PhD thesis of Léa Penazzi [10], where the methodology is presented exhaustively.

$$W = \begin{cases} w_i = T_0, & x = 0, \\ w_i = T_f, & x \in \mathcal{D}_f \end{cases} \tag{5}$$

$$T(x_{\text{obs}}) \simeq m = \frac{1}{N}\sum_{i=1}^{N} w_i \tag{6}$$

$$s = \frac{1}{\sqrt{N}} \left[\frac{1}{N}\sum_{i=1}^{N} w_i^2 - \left(\frac{1}{N}\sum_{i=1}^{N} w_i\right)^2\right]^{\frac{1}{2}} \tag{7}$$

2.2. Symbolic Monte Carlo Formulation

Classical Monte Carlo methods require complete resampling when physical parameters (such as thermal conductivity, emissivity, or convective coefficients) are changed. SMC method addresses this limitation by storing propagation paths sampled at reference parameter values and by constructing symbolic expressions that describe how variations in these parameters affect the solution. This produces a transfer function, analogous to the propagators constructed for sources [6], that expresses the quantity of interest (e.g temperature) as an explicit function of parameters (e.g thermal conductivity, convection coefficient), enabling rapid evaluation across parameter space for applications in optimization, sensitivity analysis, and inverse problems. In the studied case, the SMC method aims to estimate the temperature $T(x_{obs}, \lambda)$ as a transfer function dependent on the thermal conductivity λ. Indeed, thermal conductivity λ appears in the probabilities $P(\lambda)$ and $1-P(\lambda)$ when the random walk reaches the right boundary of the domain eq. (3). To perform the Bernoulli test at the right boundary of the wall, the value of λ must therefore be known, hence fixed at the moment of the simulation. This coefficient influences the probabilities P_{ref} and $1 - P_{ref}$ at the domain's right wall. Equation (3) must be reformulated to introduce two arbitrary probabilities with the constant value of h, and the arbitrary probabilities are defined in eq. (8). The equation for the probe temperature is reformulated as presented in eq. (9).

$$\begin{cases} P_{\text{ref}} = \dfrac{\lambda_{\text{ref}}/\delta}{h + \lambda_{\text{ref}}/\delta} \\ 1 - P_{\text{ref}} = \dfrac{h}{h + \lambda_{\text{ref}}/\delta} \end{cases} \tag{8}$$

$$T(x_{obs}, \lambda_{ref}) = (1 - P_{ref}) \left(\frac{1 - P(\lambda)}{1 - P_{ref}} \times T_f \right) + P_{ref} \left(\frac{P(\lambda)}{P_{ref}} \times T(L - \delta) \right) \tag{9}$$

The temperature evolution during the re-injection process can be described by the expression in eq. (10). In this formulation, the parameter j represents the number of re-injection events that occur when the path reaches the right boundary ($x = L$). Each re-injection in the solid medium is governed by the probability P_{ref}. The variable n serves as an indicator function that defines the boundary condition on the left side of the domain. Specifically, $n = 1$ when the path reaches the left boundary ($x = 0$), corresponding to a fixed temperature condition (Dirichlet-type boundary). Conversely, $n = 0$ when the path reaches the right boundary ($x = L$), where the re-injection or exit condition is applied. This distinction allows the model to differentiate between thermal interactions occurring at each boundary; the left boundary acts as a constant temperature source, while the right boundary governs the probability-controlled re-injection process. Consequently, the temperature field derived from this model captures both the stochastic nature of particle movements and the physical constraints imposed by the boundary conditions.

$$T(x_{obs}, \lambda_{ref}) = \left(\frac{P(\lambda)}{P_{ref}} \right)^{j^i} T_0 + \left(\frac{P(\lambda)}{P_{ref}} \right)^{j^i} \left(\frac{1 - P(\lambda)}{1 - P_{ref}} T_f \right) \tag{10}$$

$$T(x_{obs}, \lambda_{ref}) = \left(\frac{P(\lambda)}{P_{ref}} \right)^{j^i} \left[n_i T_0 - n_i \frac{1 - P(\lambda)}{1 - P_{ref}} T_f + \frac{1 - P(\lambda)}{1 - P_{ref}} T_f \right] \tag{11}$$

In the case where the path ends at the left boundary after undergoing j^i re-injections at the left wall, the SMC weight corresponds to the following case, which is given in eq. (12).In the case where the path ends in the fluid after undergoing j^i re-injections at the right wall, the SMC weight corresponds to the eq. (13). When the conductive path is within the domain, the algorithm remains unchanged. On the right wall, the probabilities become $1 - P_{ref}$. Consequently, the SMC weight is different from the weight used in the standard Monte Carlo temperature estimation algorithm.

$$w_i(\lambda) = \left(\frac{P(\lambda)}{P_{\text{ref}}}\right)^{j^i} T_0 \tag{12}$$

$$w_i(\lambda) = \left(\frac{P(\lambda)}{P_{\text{ref}}}\right)^{j^i} \left(\frac{1-P(\lambda)}{1-P_{\text{ref}}}\right) T_f \tag{13}$$

3. ANALYTICAL VALIDATION AND RESULTS

3.1. Analytical Solution

We consider here a simple academic test case designed solely to explain the Symbolic Monte Carlo methodology. For this 1D conduction–convection system, the steady-state temperature distribution can be derived analytically, providing an exact reference solution given by eq. (14).

$$T(x) = \frac{h(T_f - T_0)}{\lambda + hL} x + T_0 \tag{14}$$

This analytical expression will serve as a benchmark to validate the symbolic approach and demonstrate how the transfer function constructed from Monte Carlo paths recovers the exact parameter dependence.

3.2. Monte Carlo Validation

Monte Carlo simulations were performed using different numbers of realizations, $N = 10^4$ and 10^5, and compared with the analytical solution in Fig. 2. For both cases, the MC estimates show excellent agreement with the analytical temperature profile, with a confidence interval of 1%. The associated error bars are very small and become nearly imperceptible on the temperature scale, particularly for $N = 10^5$. For clarity, continuous lines have been preferred over dashed styles in the figures to improve the visibility of both the curves and the confidence intervals.

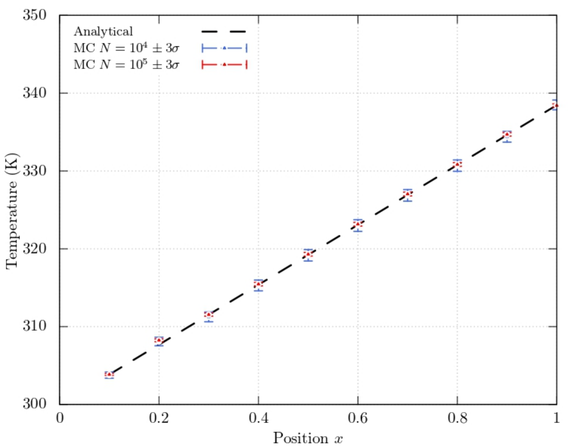

Figure 2 : *Analytical vs. MC solutions for different N realisations.*

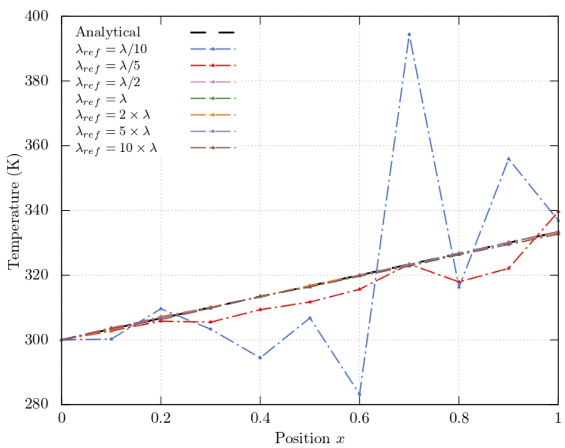

Figure 3 : *Analytical vs. SMC results for $\lambda = 5$ and various λ_{ref}.*

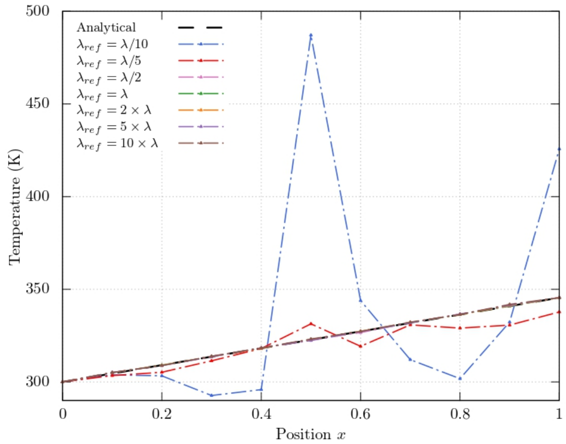

Figure 4 : *Analytical vs. SMC results for* $\lambda = 1$ *and various* λ_{ref}.

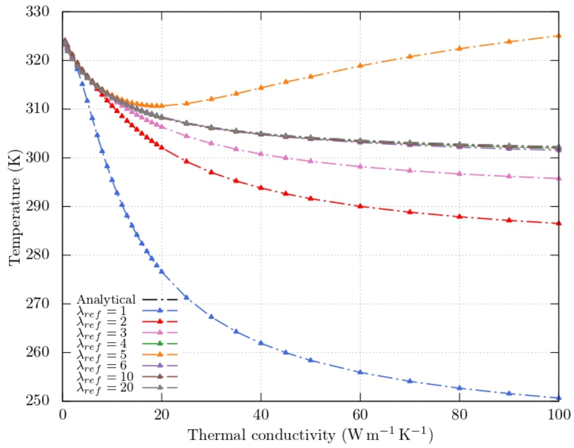

Figure 5 : *Temperature estimates* $T(x_{\text{obs}}, \lambda)$ *for* $\lambda \in [0, 100]$ *and different* λ_{ref}. $x_{\text{obs}} = L/2$, $T_f =$ 350 *K*, $T_0 = 300$ *K*.

The standard deviation of the temperature estimates, also shown in Fig. 2, decreases with increasing N, further confirming the convergence properties of the Monte Carlo approach. For $N = 10^5$, the MC results are virtually indistinguishable from the analytical solution, demonstrating the robustness and reliability of the algorithm and the efficiency of the sampling procedure for the one-dimensional test case.

3.3. Symbolic Monte Carlo Validation

The temperature profiles obtained from the analytical solution and the Symbolic Monte Carlo (SMC) simulations are shown in Fig. 3 and Fig. 4 for $\lambda = 5$ and $\lambda = 1$, respectively. In both cases, the analytical solution exhibits the expected linear temperature distribution along the wall. These results highlight the importance of selecting an appropriate reference conductivity, which remains an open question for practical applications and may require adaptive or multi-reference strategies. The SMC results are in excellent agreement with the analytical solution when the reference conductivity λ_{ref} is of the same order of magnitude as the physical conductivity (e.g., 2λ, 5λ, 10λ).

When λ_{ref} is significantly smaller than λ (e.g., $\lambda/5$ or $\lambda/10$), noticeable deviations appear. This behavior is attributed to a mismatch between the stochastic transition probabilities defined at λ_{ref} and the actual thermal diffusion scale, which leads to biased symbolic weights and degraded temperature reconstruction.

Figure 5 summarizes the SMC transfer functions for multiple λ_{ref} values. For moderate to high reference conductivities ($\lambda_{\text{ref}} = 6, 10, 20$), the symbolic formulation accurately reproduces the smooth and monotonic dependence of temperature on λ, demonstrating that the reweighting procedure remains stable when the reference conductivity is chosen close to the actual physical value. As λ_{ref} decreases, discrepancies progressively increase, indicating a growing imbalance in the symbolic weights and a loss of statistical efficiency. This degradation comes from the increase of the ratio $P(\lambda)/P_{\text{ref}}$ in the weight calculation, which increases variance and makes the sampled paths less representative. An unexpected inversion observed at $\lambda_{\text{ref}} = 5$ highlights the sensitivity of the method to the choice of reference probabilities. Although the exact cause is not yet fully understood, this behavior suggests that an inappropriate reference value may introduce bias by giving too much weight to rare trajectories. These results confirm that the stability and accuracy of the SMC transfer function strongly depend on the selection of λ_{ref}, and that choosing a reference value close to the expected physical range is essential for robust parametric reconstruction.

The range of thermal conductivity values considered in this study $\lambda \in [0, 100]\ \mathrm{W\,m^{-1}\,K^{-1}}$

was intentionally selected to cover a broad spectrum extending beyond typical building materials. In practice, common construction materials exhibit conductivities ranging from approximately $0.02\ \mathrm{W\,m^{-1}\,K^{-1}}$ for insulating materials to about 2 -3 $\mathrm{W\,m^{-1}\,K^{-1}}$ for concrete and other structural components. Higher values included in this study allow us to evaluate the robustness and stability of the Symbolic Monte Carlo formulation over an extended parameter space, ensuring its applicability beyond standard building configurations and facilitating methodological validation.

3.4. Star-lambda and Star-Engine libraries

The Symbolic Monte Carlo algorithm is implemented within the STAR-ENGINE framework, developed by Meso-Star. The Star-Lambda code leverages these libraries to solve both direct and Symbolic Monte Carlo algorithms. To ensure reproducibility of the results presented in this work, the code will be made available through Software Heritage.

4. Conclusion

This paper presented the development and application of a Symbolic Monte Carlo (SMC) method for heat transfer analysis in buildings, focusing on a 1D conduction–convection system. The proposed approach allows the estimation of transient temperature fields as functions of uncertain parameters, such as thermal conductivity, using a single Monte Carlo simulation. The symbolic formulation enables efficient computation of transfer functions, sensitivity analysis, and the generation of synthetic infrared images for building diagnostics. Validation against analytical solutions demonstrates the accuracy of the SMC approach for a range of reference conductivities, confirming its reliability for inverse thermal analysis.

Beyond forward modeling, the SMC method provides a fast surrogate model that enables inversion of thermophysical properties from experimental data. This capability will be leveraged in forthcoming work to estimate material properties from infrared thermography of building facades. A key advantage of the approach lies in its natural extension to three-dimensional geometries without methodological difficulties or computational cost inflation. This scalability stems from the Monte Carlo formalism combined with tools from computer graphics: geometric complexity is handled through stochastic path sampling rather than deterministic mesh refinement, making the method particularly well-suited for realistic building envelope configurations.

The integration of the method within the Stardis simulation platform supports these extensions to complex 3D geometries and practical evaluation of building thermal performance. To sum up, the results show that the SMC method can provide accurate, computationally efficient, and high-resolution temperature estimates, supporting improved diagnostics and energy performance assessment in buildings. Despite these promising results, several limitations remain.

The present study is restricted to a simplified one-dimensional steady-state configuration, and the extension to real infrared measurements will introduce additional challenges, including measurement noise, emissivity uncertainties, and complex boundary conditions. Furthermore, the accuracy of the Symbolic Monte Carlo approach depends on the appropriate selection of reference parameters, which may impact stability and variance. Future work will therefore focus on addressing these challenges and validating the methodology against experimental thermographic data in realistic building configurations.

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