A goal-based angular adaptivity method for thermal radiation modelling in non gray media

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Motivations

Why angular adaptivity?

- The angular dependence of the radiation field changes with space and optical thickness
  
  thin media: very directional, non local transfer
  
  thick media: close to isotropy, local transfer

Which criteria to use for adapting?

- In the framework of coupled flow/radiation problems, we want to compute accurately the radiative source terms affecting the energy balance

Adapt the angular resolution differently depending on the optical thickness and across space with the radiative source terms as a goal
Numerical methods
Radiative transfer equation

- RTE without scattering at LTE, n=1, homogeneous medium

\[ \Omega \cdot \nabla I(\nu, r, \Omega) = \kappa(\nu) \left( l_b(\nu, T(r)) - I(\nu, r, \Omega) \right) \]  \hspace{1cm} (1)

- Boundary condition for a diffuse opaque gray body

\[ I(\nu, r_w) = \varepsilon l_b(\nu, T(r_w)) + \frac{1 - \varepsilon}{\pi} \int_{\Omega' \cdot n < 0} l(\nu, r_w, \Omega') |\Omega' \cdot n| d\Omega' \]  \hspace{1cm} (2)

- Radiative source term affecting the energy balance

\[ \nabla \cdot q^{rad}(r) = \int_{\nu} \kappa(\nu) \left( 4\pi l_b(\nu, T(r)) - \int_{\Omega} l(\nu, r, \Omega) d\Omega \right) d\nu \]  \hspace{1cm} (3)
Numerical methods
FETCH

**FETCH: Finite element solver for radiation transport**
- Combines CG and DG methods for space discretisation
- Uses arbitrary angular discretisations ($S_N$, $P_N$, wavelets)
- Is implemented matrix free in **parallel**
- Is coupled with FEM flow solver Fluidity
- Has the ability to adapt its resolution in space and angle
Numerical methods
Angular discretisation

Angular adaptivity is based on hierarchical angular basis functions
- It provides an easy estimation of angular error
- No interpolations are required to couple angles across space

Haar wavelets
- \( P_0 \) (piece-wise constant) hierarchical basis functions
- Mapping with discrete ordinate basis functions

Figure: Haar wavelet and \( S_N \) basis functions over a 1D interval, order 3
Figure: Benchmark test case, comparison between Monte Carlo, Haar wavelets and Discrete Ordinates
Numerical methods
Radiative property modelling

Global model

\[ F(k) = \frac{\pi}{\sigma T_{\text{ref}}^4} \int_{\nu/\kappa_\nu(T_{\text{ref}}) \leq k} l_b(\nu, T_{\text{ref}}) d\nu \]  \hspace{1cm} (4)

\[ \Omega \cdot \nabla I_i(r, \Omega) = k_i \left( a_i \frac{\sigma T^4(r)}{\pi} - I_i(r, \Omega) \right) \]  \hspace{1cm} (5)

- computation of model parameters \((k_i, a_i)\) from LBL absorption spectrum

Figure: (Left) Absorption spectrum for 2% H\(_2\)O in air at 294.2 K and incoming flux BC. (Right) Emissivities for different column lengths.
Outline

- Motivations
- Numerical methods
- Goal-based angular adaptivity
- Results
- Conclusion
The adaptivity is based on a goal represented by a functional. Here the functional is the radiative source term affecting the energy balance

$$F_{\nu}(l(\nu, r, \Omega)) = \int_{r} \int_{\Omega} \int_{\nu} \kappa(\nu)(l_{b}(\nu, T(r)) - l(\nu, r, \Omega)) \, d\nu \, d\Omega \, dr \quad (6)$$

Forward \( l \) and adjoint \( l^* \) solutions are required in the goal-based adaptivity algorithm, where the adjoint problem is defined by

$$-\Omega \cdot \nabla l^*(\nu, r, \Omega) = S^*_{\nu} - \kappa(\nu) l^*(\nu, r, \Omega) \quad (7)$$

The volume source is related to the integrand \( f \) of the goal functional \( F \)

$$S^*_{\nu} = -\frac{\partial f_{\nu}}{\partial l} = \kappa(\nu) \quad (8)$$
Goal-based angular adaptivity

Error estimation

- An error estimation in an arbitrary goal can be written as

\[ F(l_{\text{exact}}) - F(l) = - \int R(l)(l_{\text{exact}}^* - l^*) dP \]  

(9)

- This error measure is computed for each DOF (space \(i\), angle \(q\), class \(k\)) and the resolution is increased locally if this error is above a given tolerance

\[ e_{k,q,i} = \frac{R_{k,q,i} \epsilon_{k,q,i}^*}{\Delta F} \]  

(10)

- We use the wavelet hierarchy to approximate the error in the adjoint solution

\[ \epsilon_{k,q,i}^* = \begin{cases} l_{k,q,i}^*, & \text{if } q \text{ belongs to the max order} \\ 0, & \text{otherwise} \end{cases} \]  

(11)
Goal-based angular adaptivity

Adaptivity algorithm

\begin{verbatim}
for a = 1, Na do
    Solve forward and adjoint problem and calculate error field
    for k = 1, Nk do
        for i = 1, Ni do
            for q ∈ M_{ik} do
                if e_{k,q,i} < 0.01 then
                    Remove basis function q from node i
                else if e_{k,q,i} > 1.0 then
                    Add next level basis function to node i
                end if
            end for
        end for
    end for
end for
\end{verbatim}
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Results
Test case configuration

- Street canyon configuration and temperature snapshot obtained from uncoupled simulations at $Ra = 10^8$ and $Re = 5 \times 10^3$
- Discretised with $10^6$ spatial elements, simulations run on 48 cores
- Results published in Soucasse et al., JQSRT 200, 2017

- Accuracy and efficiency of the method
- Analysis of the distribution of the adapted angles
- Use of adapted resolution in coupled calculations
Results
Accuracy and efficiency

Figure: Functional error plot against averaged number of angles (left) ad CPU time (right).

- Gain of a factor 5 in CPU time and in angular resolution with goal-based adapted calculations without compromising the accuracy
Results

Accuracy and efficiency

Figure: Reference radiative source term $W_{4,4}$ (left) and difference with goal based adaptivity solutions $W_{4,4}^{\text{GB}}$

- Patterns of the radiative power field follow the thermal structures
- Local differences between reference and adaptivity solutions are around 2 % at most
Results
Distribution of the adapted angles

Figure: Averaged number of angular basis functions for each k-class

- The averaged angle number vary a lot with the optical thickness
Results
Distribution of the adapted angles

$k_3 = 2.2 \times 10^{-3} \text{ m}^{-1}$

$k_6 = 1.1 \times 10^{-1} \text{ m}^{-1}$

$k_{10} = 2.1 \times 10^1 \text{ m}^{-1}$

Figure: Spatial distribution of angular resolution for three $k$-classes $W_{4,4}^{GB}$

- Thin: very directional but do not contribute much to the power
- Intermediate: very directional and contribute significantly to the power
- Thick: local transfer, focus on thermal gradients
Results
Distribution of the adapted angles

Figure: Angular distribution of the radiative intensity at different spatial points and $k$-class
Results
Coupled calculations

Figure: Difference in the radiative source term between uniform and adapted resolution during a coupled unsteady simulation

- Increase of differences with time
- Changing the adapted resolution with time does not help
Future works

- To enhance the performances of angular adaptivity with load balancing
- To apply the method to heterogeneous media encountered in combustion processes
- To test the adaptivity algorithm for highly directional thermal radiation problems (i.e. solar applications, fires)
- To combine angular adaptivity with spatial adaptivity
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