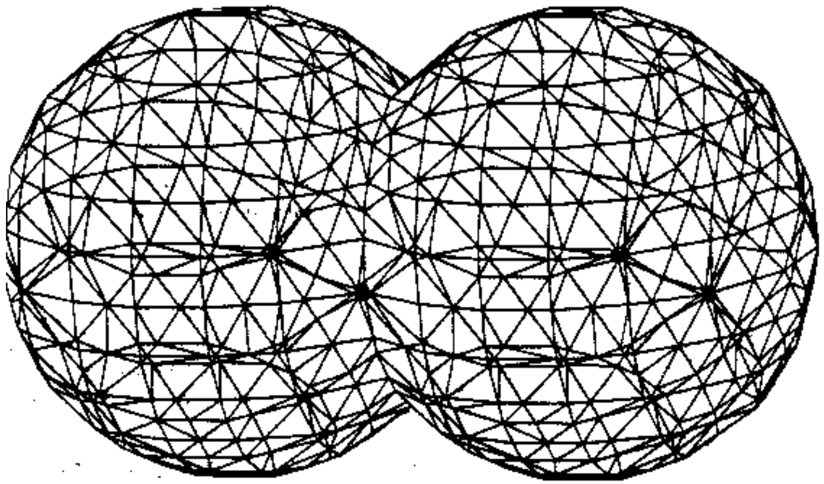
Application d'une méthode de type 'meshless' à la résolution de problèmes de transferts radiatifs

C.A. Wang, H. Sadat, V. LeDez, D. Lemonnier

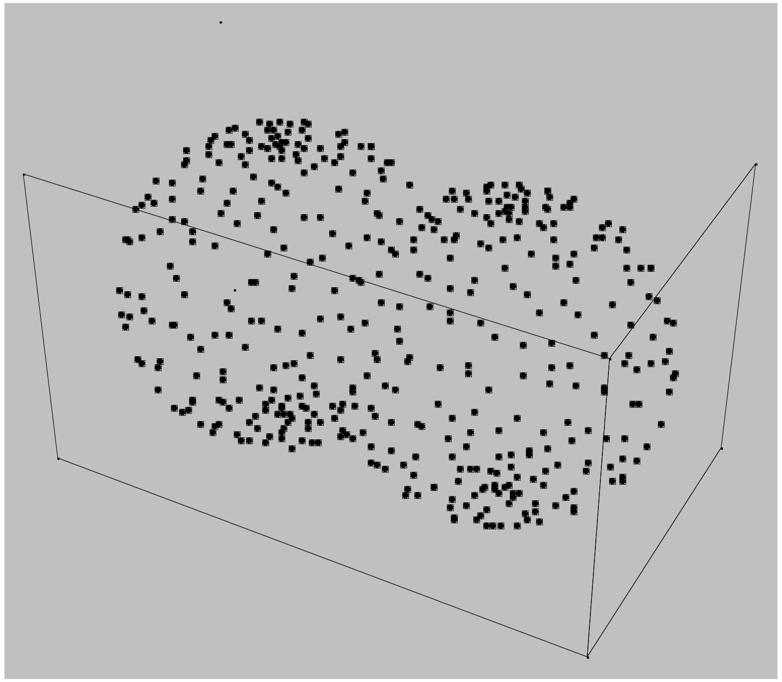
Institut P' • UPR CNRS 3346
SP2MI • Téléport 2
Boulevard Marie et Pierre Curie • BP 30179
F86962 FUTUROSCOPE CHASSENEUIL Cedex

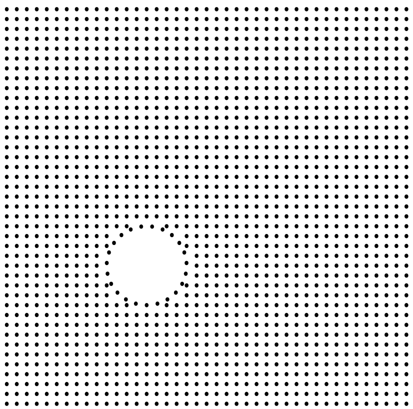
Discretisation

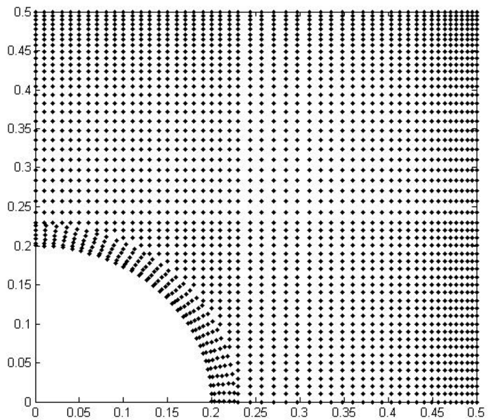
Avec maillage

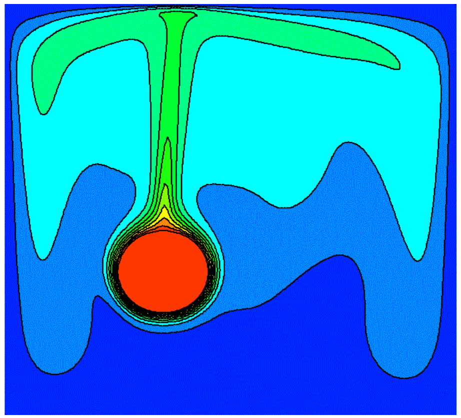


Sans maillage

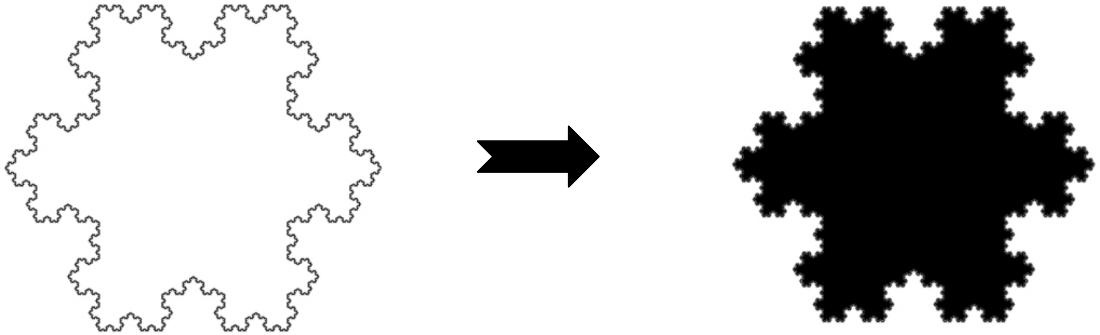


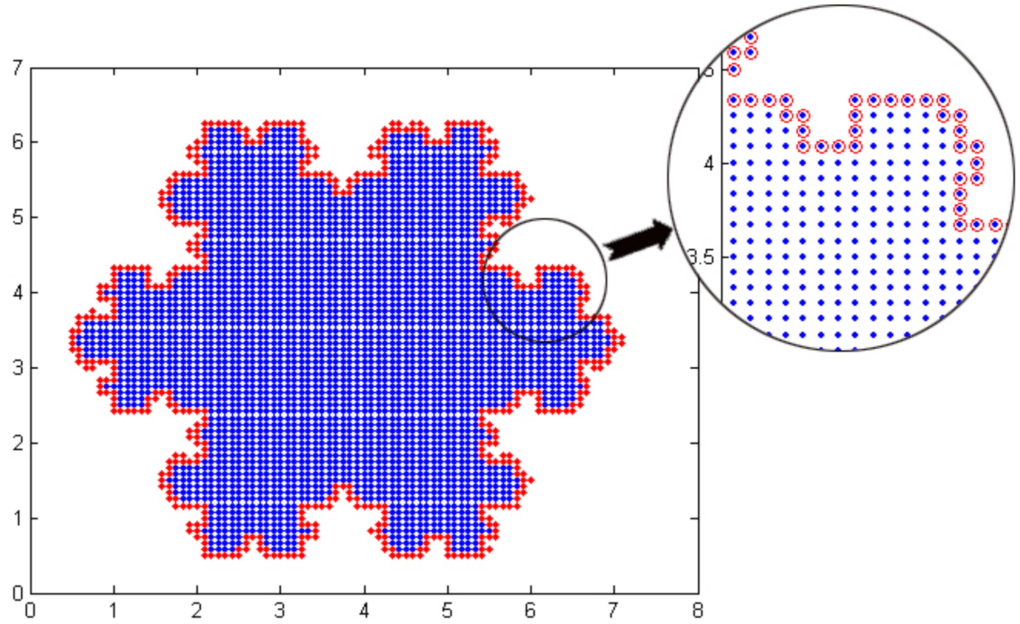




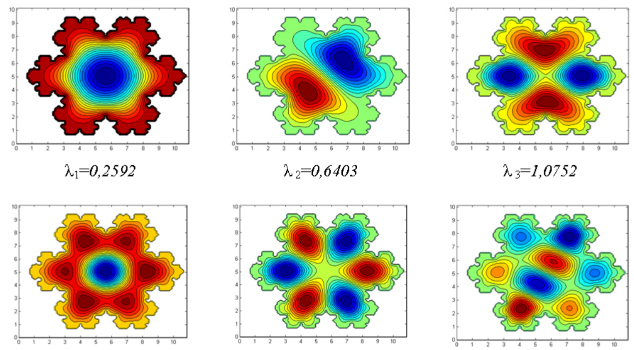


Construction du nuage de points: mailleurs, tir aléatoire, reconstruction de volumes..





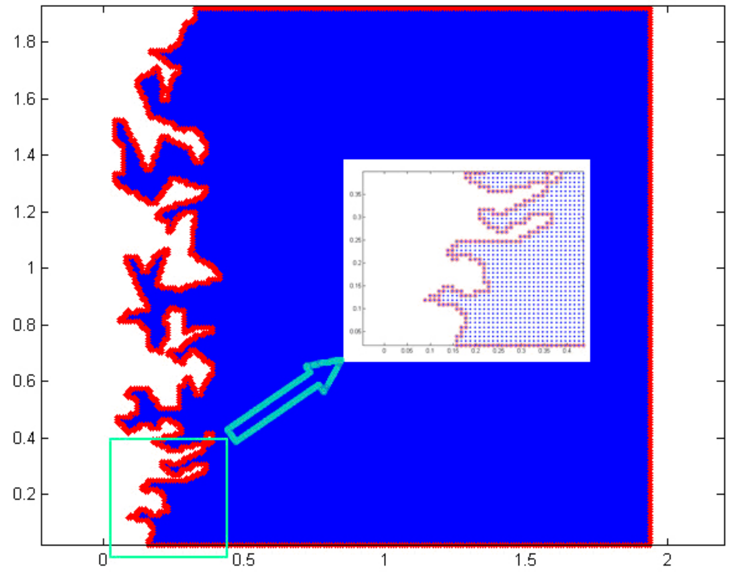
valeurs et fonctions propres



$\lambda_1=0,2592$ $\lambda_2=0,6403$ $\lambda_3=1,0752$

$\lambda_6=1,244$ $\lambda_7=1,3559$ $\lambda_8=1,7724$

Interface d’une surface catalytique



Approximation Diffuse
Approximation glissante à moindres carrés

$$\varphi_i^*(x_i,y_i)=\varphi+(x_i-x)\frac{\partial\varphi}{\partial x}+(y_i-y)\frac{\partial\varphi}{\partial y}+\frac{(x_i-x)^2}{2!}\frac{\partial^2\varphi}{\partial x^2}+(x_i-x)(y_i-y)\frac{\partial^2\varphi}{\partial x\partial y}+\frac{(y_i-y)^2}{2!}\frac{\partial^2\varphi}{\partial y^2}+O(h^2)$$

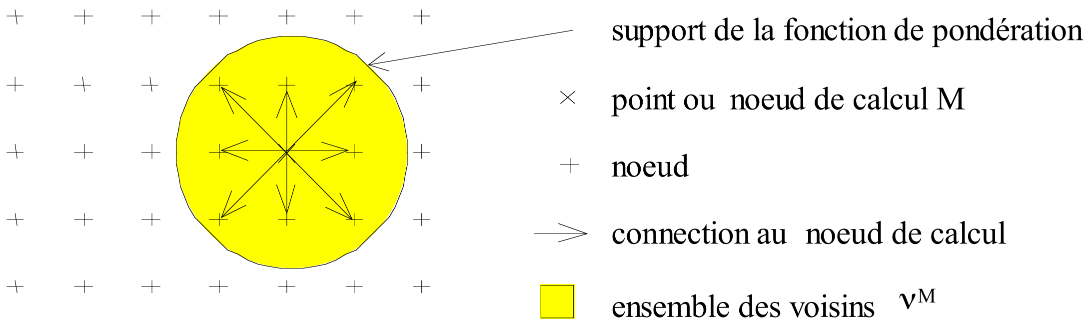
$$\boxed{\varphi_i^*(x_i,y_i)=\langle p(M_i,M)\rangle\cdot\langle\alpha_M\rangle^T}$$

$$\left\{\begin{array}{l}\langle p(M_i,M)\rangle=\langle 1,(x_i-x),(y_i-y),(x_i-x)^2,(x_i-x)(y_i-y),(y_i-y)^2\rangle\\[2ex] \langle\alpha_M\rangle^T=\left\langle \varphi(x,y)^*,\left(\frac{\partial\varphi}{\partial x}\right)^*,\left(\frac{\partial\varphi}{\partial y}\right)^*,\frac{1}{2!}\left(\frac{\partial^2\varphi}{\partial x^2}\right)^*,\left(\frac{\partial^2\varphi}{\partial x\partial y}\right)^*,\frac{1}{2!}\left(\frac{\partial^2\varphi}{\partial y^2}\right)^*\right\rangle^T\end{array}\right.$$

Erreur Quadratique – Fonction De Pondération

$$\boxed{I(\alpha_M) = \sum_{M_j \in \upsilon^M} \left\{ \omega(M_j,M) \left[\varphi_j - \langle p(M_j,M) \rangle \cdot \langle \alpha_M \rangle^T \right]^2 \right\}}$$

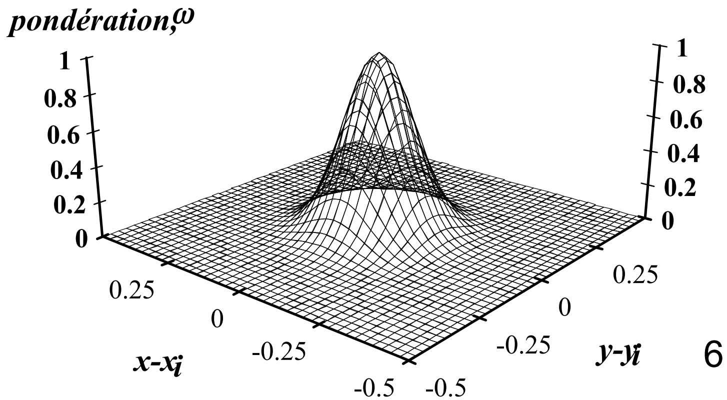
$\omega(M_j,M)$: fonction de pondération définie sur un support borné



Fonction de Gauss:

$$\omega(M_j,M) = \exp\left[-3\ln(10) \cdot \left(\frac{r}{\sigma}\right)^2 \right]$$

$$\omega(M_j,M) = 0 \quad \text{si } r > \sigma^2$$



Expression Des Dérivées Partielles

Minimisation de l'erreur quadratique : $\dfrac{\partial I(\alpha_M)}{\partial \alpha_i} = 0 \quad i = 0, \cdots, 5$

$$\left[A^M\right] = \sum_{j=1}^{n} \omega(M_j, M) \langle p(M_j, M) \rangle^T \langle p(M_j, M) \rangle \qquad \langle B^M \rangle^T = \sum_{j=1}^{n} \omega(M_j, M) \langle p(M_j, M) \rangle^T \cdot \varphi_j$$

$$\left[A^M\right] \cdot \begin{Bmatrix} \varphi \\ \dfrac{\partial \varphi}{\partial x} \\ \dfrac{\partial \varphi}{\partial y} \\ \dfrac{\partial^2 \varphi}{2! \partial x^2} \\ \dfrac{\partial^2 \varphi}{\partial x \partial y} \\ \dfrac{\partial^2 \varphi}{2! \partial y^2} \end{Bmatrix}^* = \langle B^M \rangle^T$$

$x_j = x_{Mj} - x_M$ et $y_j = y_{Mj} - y_M$

$$\xrightarrow{\text{inversion}} \quad \boxed{\begin{Bmatrix} \varphi \\ \dfrac{\partial \varphi}{\partial x} \\ \dfrac{\partial \varphi}{\partial y} \\ \dfrac{1}{2!}\dfrac{\partial^2 \varphi}{\partial x^2} \\ \dfrac{\partial^2 \varphi}{\partial x \partial y} \\ \dfrac{1}{2!}\dfrac{\partial^2 \varphi}{\partial y^2} \end{Bmatrix}^*} = \boxed{\left[A^M\right]^{-1}} \cdot \boxed{\sum_{M_j \in \nu^M} \omega(M_j, M) \begin{Bmatrix} 1 \\ x_j \\ y_j \\ x_j^2 \\ x_j \cdot y_j \\ y_j^2 \end{Bmatrix}} \boxed{\varphi_j}$$

Expression Des Dérivées Partielles

2D : Matrice 6×6

$$\left[A^M\right] = \sum_{M_j \in \nu^M} \omega(M_j,M) \begin{bmatrix} 1 & x_j & y_j & x_j^2 & x_j \cdot y_j & y_j^2 \\ x_j & x_j^2 & x_j \cdot y_j & x_j^3 & x_j^2 \cdot y_j & x_j \cdot y_j^2 \\ y_j & x_j \cdot y_j & y_j^2 & x_j^2 \cdot y_j & x_j \cdot y_j^2 & y_j^3 \\ x_j^2 & x_j^3 & x_j^2 \cdot y_j & x_j^4 & x_j^3 \cdot y_j & x_j^2 \cdot y_j^2 \\ x_j \cdot y_j & x_j^2 \cdot y_j & x_j \cdot y_j^2 & x_j^3 \cdot y_j & x_j^2 \cdot y_j^2 & x_j \cdot y_j^3 \\ y_j^2 & x_j \cdot y_j^2 & y_j^3 & x_j^2 \cdot y_j^2 & x_j \cdot y_j^3 & y_j^4 \end{bmatrix}$$

$\langle a_i \rangle$: $i^{ème}$ ligne de la matrice $[A^M]^{-1}$

$\langle p_j \rangle = \quad \langle p(M_j,M) \rangle$

$$\boxed{\frac{\partial \varphi}{\partial y} = \sum_{M_j \in \upsilon^M} \omega(M_j,M) \langle a_3 \rangle \langle p_j \rangle^T \varphi_j}$$

$$\boxed{\frac{\partial^2 \varphi}{\partial x^2} = 2! \sum_{M_j \in \upsilon^M} \omega(M_j,M) \langle a_4 \rangle \langle p_j \rangle^T \varphi_j}$$

3D : Matrice 10×10

$$
\left[A^M\right] = \sum_{M_j \in \nu^M} \omega(M_j, M)
\begin{bmatrix}
1 & x_j & y_j & x_j^2 & x_j \cdot y_j & y_j^2 & z_j & x_j \cdot z_j & y_j \cdot z_j & z_j^2 \\
* & x_j^2 & x_j \cdot y_j & x_j^3 & x_j^2 \cdot y_j & x_j \cdot y_j^2 & x_j \cdot z_j & x_j^2 \cdot z_j & x_j \cdot y_j \cdot z_j & x_j \cdot z_j^2 \\
* & * & y_j^2 & x_j^2 \cdot y_j & x_j \cdot y_j^2 & y_j^3 & y_j \cdot z_j & x_j \cdot y_j \cdot z_j & y_j^2 \cdot z_j & y_j \cdot z_j^2 \\
* & * & * & x_j^4 & x_j^3 \cdot y_j & x_j^2 \cdot y_j^2 & x_j^2 \cdot z_j & x_j^3 \cdot z_j & x_j^2 \cdot y_j \cdot z_j & x_j^2 \cdot z_j^2 \\
* & * & * & * & x_j^2 \cdot y_j^2 & x_j \cdot y_j^3 & x_j \cdot y_j \cdot z_j & x_j^2 \cdot y_j \cdot z_j & x_j \cdot y_j^2 \cdot z_j & x_j \cdot y_j \cdot z_j^2 \\
* & * & * & * & * & y_j^4 & y_j^4 \cdot z_j & x_j \cdot y_j^2 \cdot z_j & y_j^3 \cdot z_j & y_j^2 \cdot z_j^2 \\
* & * & * & * & * & * & z_j^2 & x_j \cdot z_j^3 & y_j \cdot z_j^3 & z_j^4 \\
* & * & * & * & * & * & * & x_j^2 \cdot z_j^2 & x_j \cdot y_j \cdot z_j^2 & x_j \cdot z_j^3 \\
* & * & * & * & * & * & * & * & y_j^2 \cdot z_j^2 & y_j \cdot z_j^3 \\
* & * & * & * & * & * & * & * & * & z_j^4
\end{bmatrix}
$$

$\langle a_i \rangle$: $i^{\text{ème}}$ ligne de la matrice $[A^M]^{-1}$

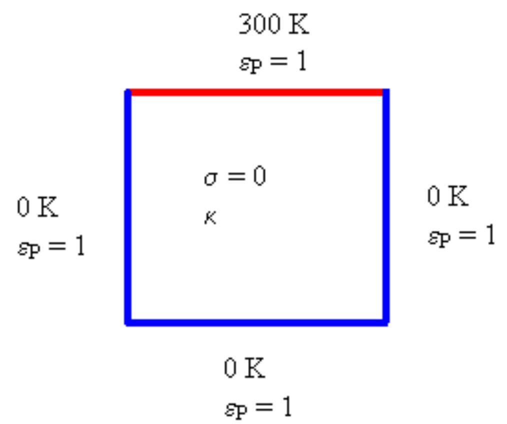
$\langle p_j \rangle = \langle p(M_j, M) \rangle$

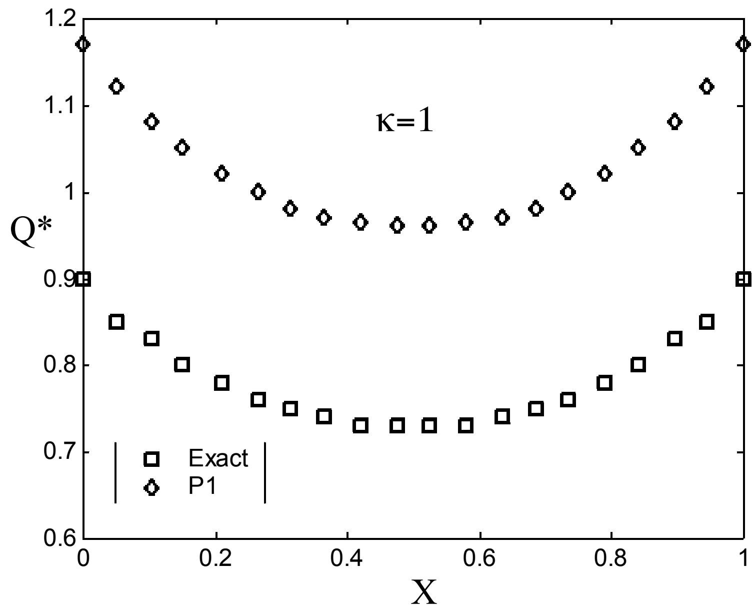
$$\boxed{\frac{\partial \varphi}{\partial y} = \sum_{M_j \in \upsilon^M} \omega(M_j, M) \langle a_3 \rangle \langle p_j \rangle^T \varphi_j}$$

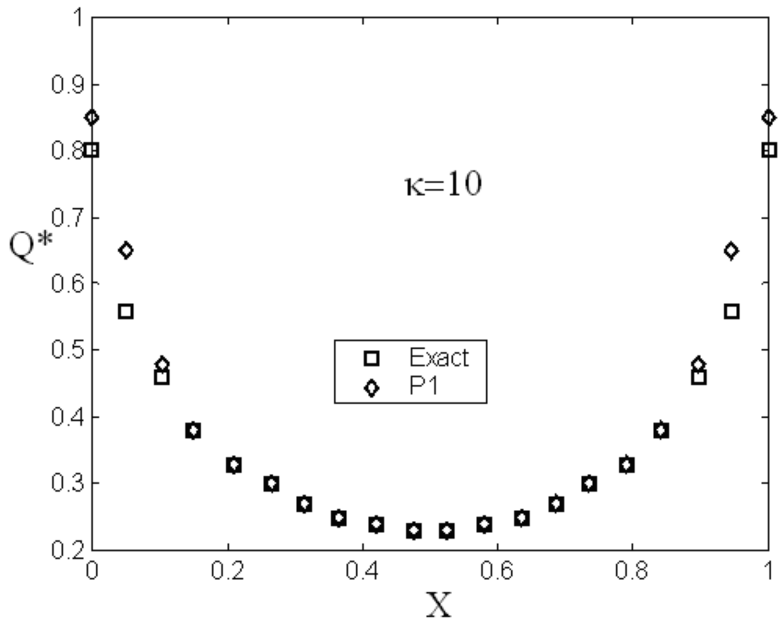
$$\boxed{\frac{\partial^2 \varphi}{\partial x^2} = 2! \sum_{M_j \in \upsilon^M} \omega(M_j, M) \langle a_4 \rangle \langle p_j \rangle^T \varphi_j}$$

Méthode P1

Equilibre Radiatif dans une cavité Carrée







Méthode S_N

ETR (Variables primaires)

$$\frac{dI(\Omega_i)}{ds} = -\beta I(\Omega_i) + \kappa I_b + \frac{\sigma}{4\pi}\sum_{j=1}^{J} I(\Omega'_j)\Phi(\Omega'_j,\Omega_i)W(\Omega'_j)$$

ETR (Variables Secondaires: Flux pairs)

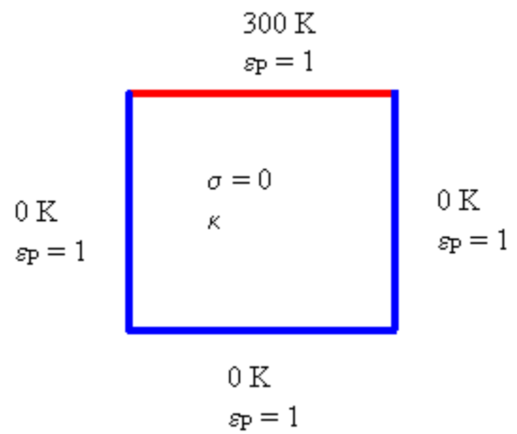
$$F(\Omega) = I^+(\Omega) + I^-(\Omega) \qquad G(\Omega) = I^+(\Omega) - I^-(\Omega)$$

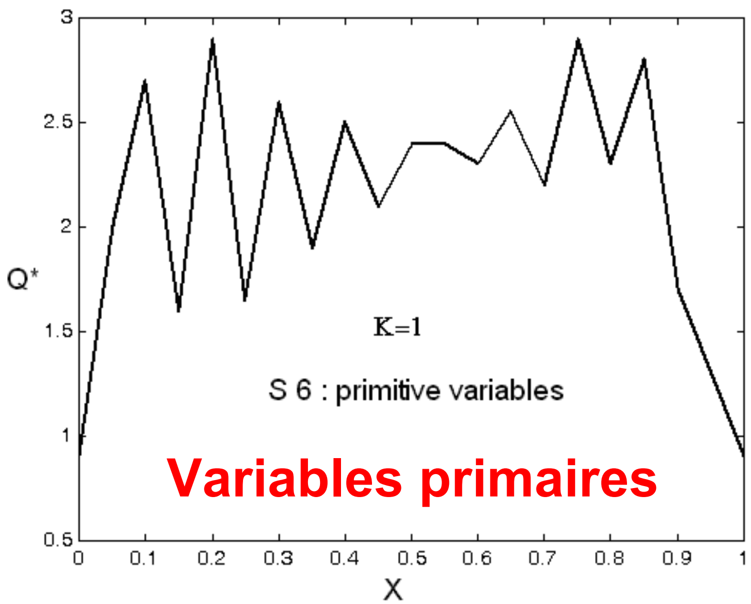
$$\left\{\begin{array}{l} \dfrac{1}{\beta}\dfrac{d^2F_i}{ds^2} - \beta F_i + \kappa I_b + \dfrac{\sigma}{4\pi}\displaystyle\sum_{j=1}^{J/2}(A_{ij}F_j + B_{ij}G_j) = 0 \\ \dfrac{\partial F_i(P,\vec{\Omega})}{\partial s_m} + \beta G_i(P,\vec{\Omega}) = 0 \end{array}\right.$$

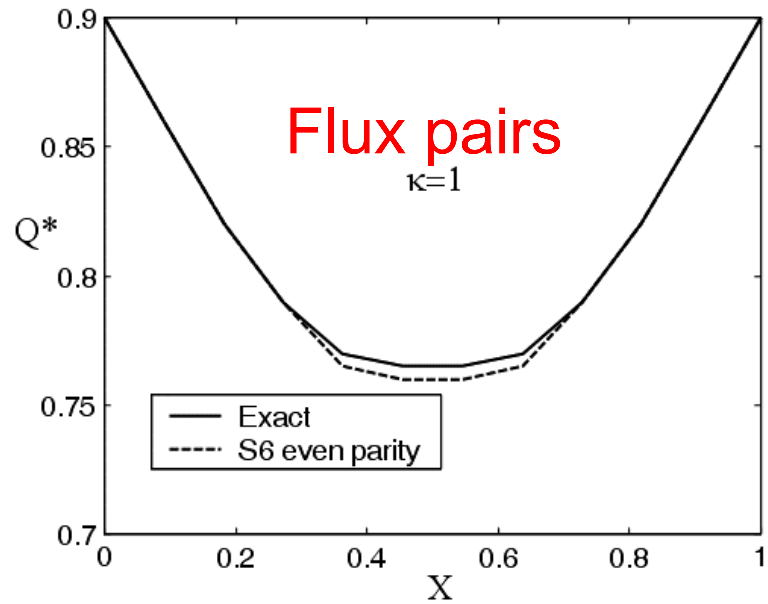
Remarque: Autre formulation du second ordre possible

Méthode SN

Equilibre Radiatif dans une cavité carrée

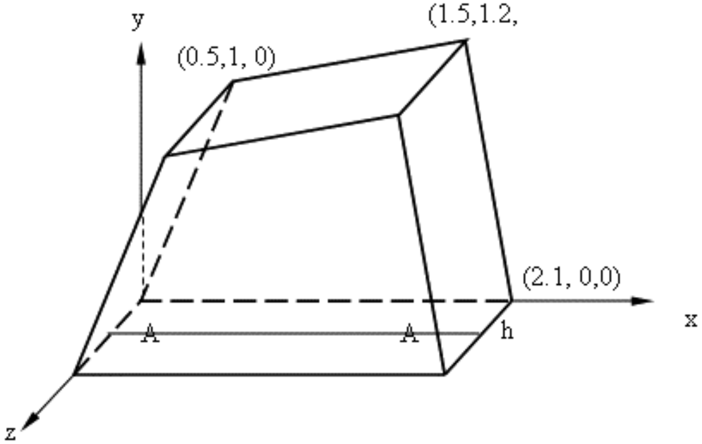






Flux pairs → Equilibre radiatif et problèmes couplés

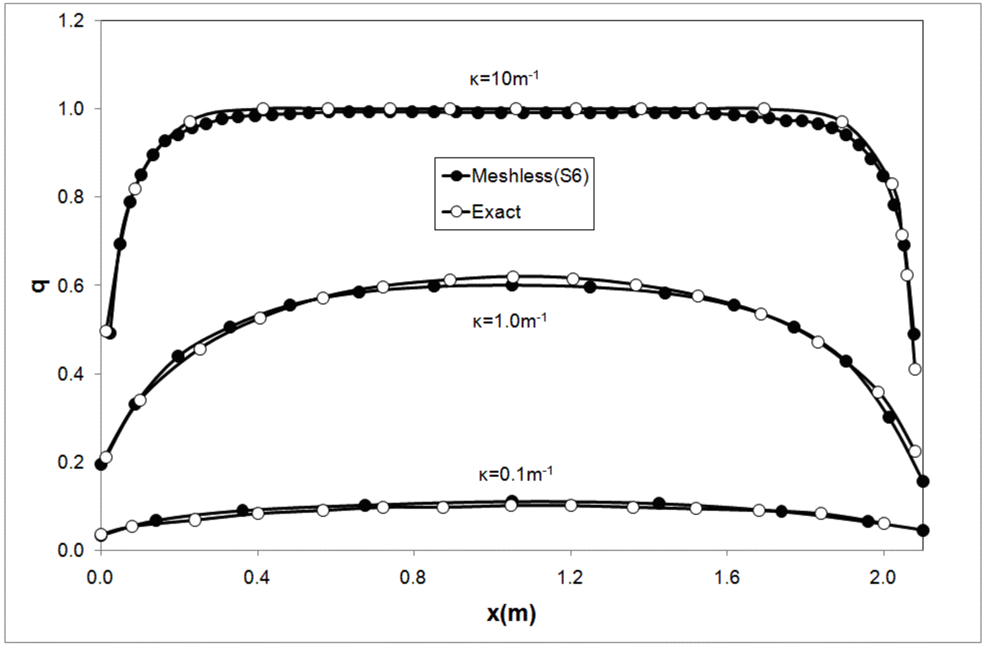
Equilibre radiatif dans une enceinte hexahédrique



h=1 m

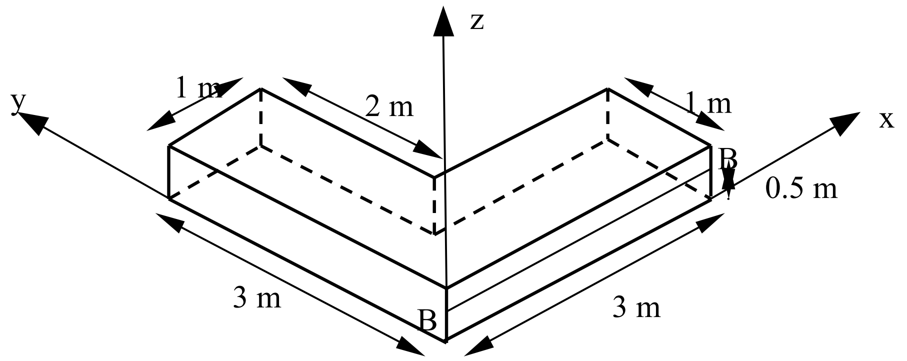
Milieu absorbant, émettant à Tm
Parois noires et froides à Tp

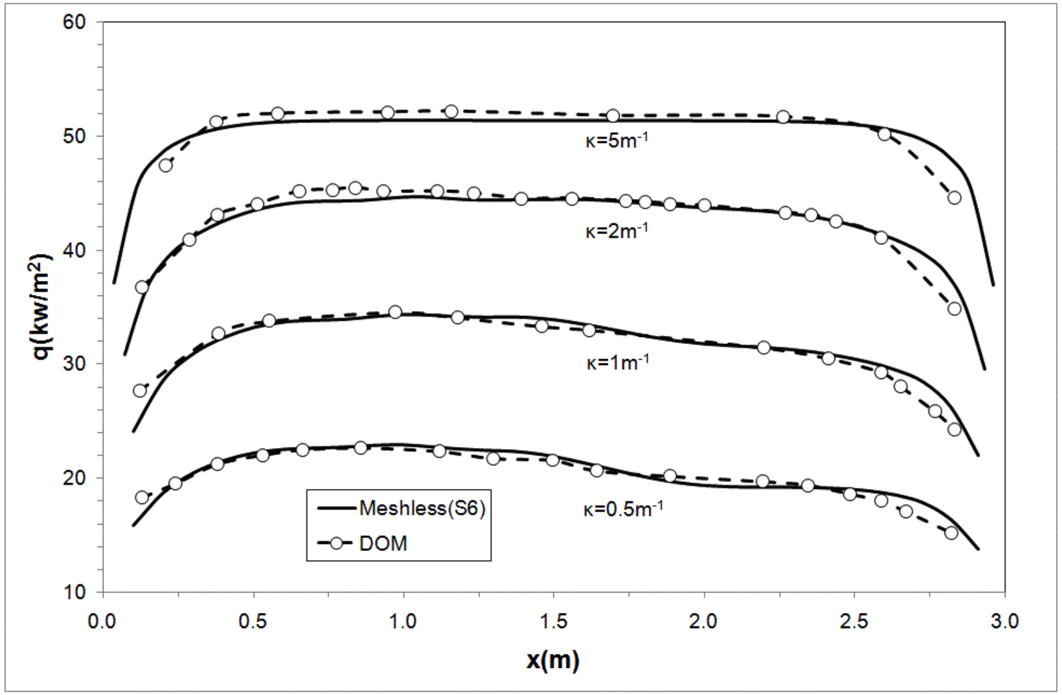
[Baek et al] NHT (Part B),1998
Nonorthogonal finite volume



Flux sur la ligne AA

Equilibre radiatif dans une enceinte en forme de "L"





Milieu absorbant, émettant à 1000K

Parois noires à 500K

15105 noeuds pour k=2 m^{-1} et $5m^{-1}$

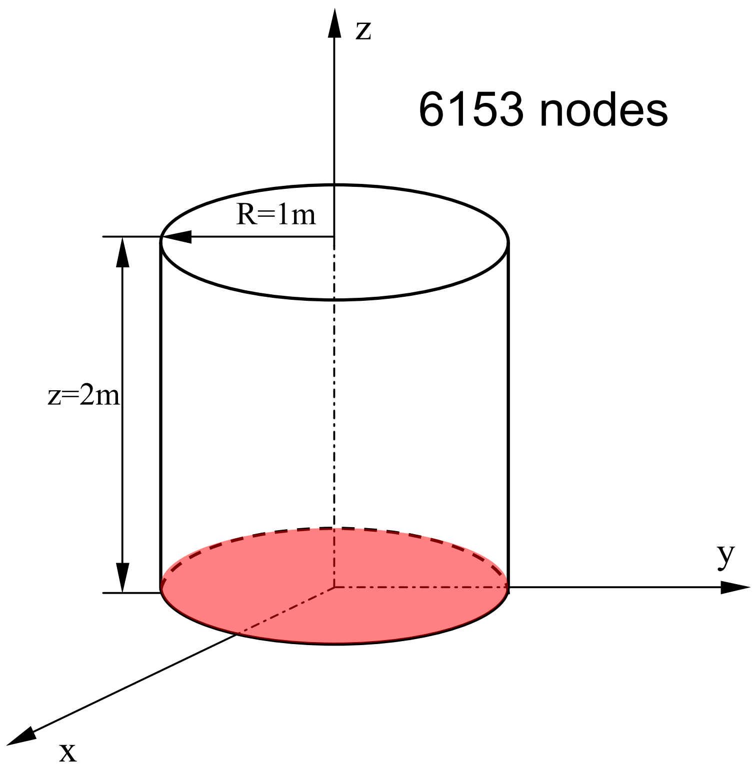
Flux sur la ligne BB

Journal Heat Transfer,1996

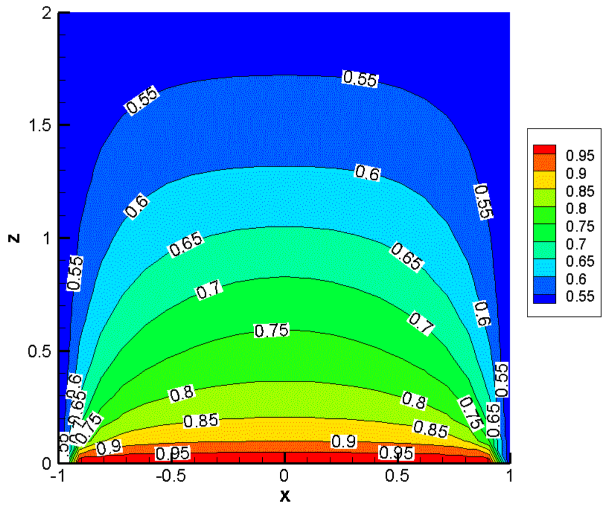
Couplage Conduction-Rayonnement

Milieu semi-transparent cylindrique

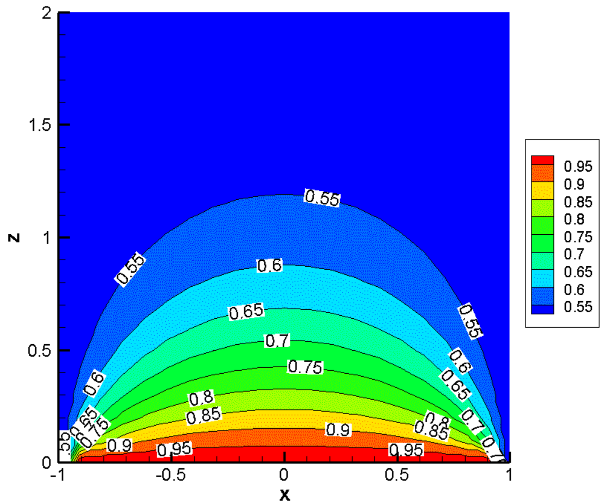
6153 nodes



Isothermes dans le plan y=0



Pr= 0.01



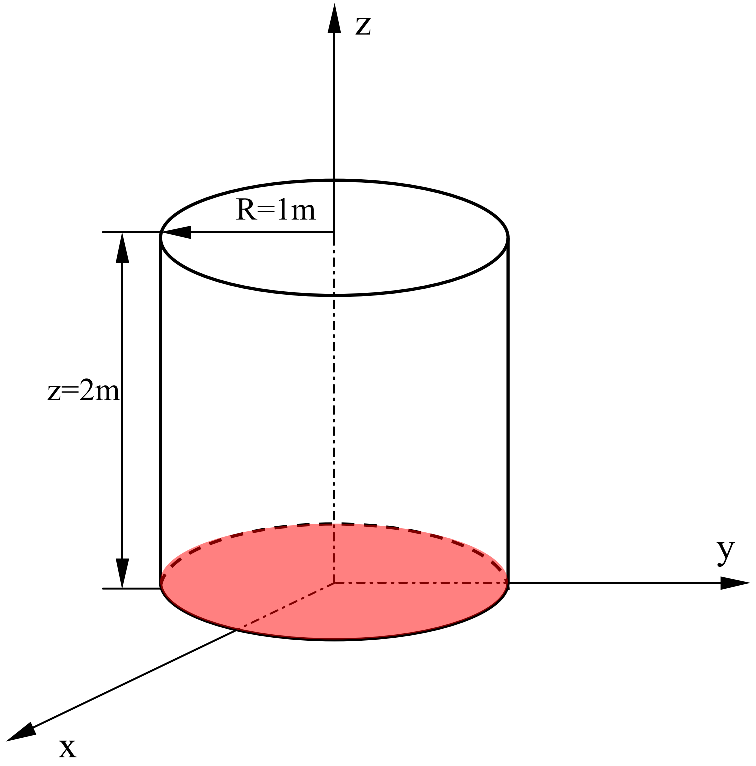
Pr = 1

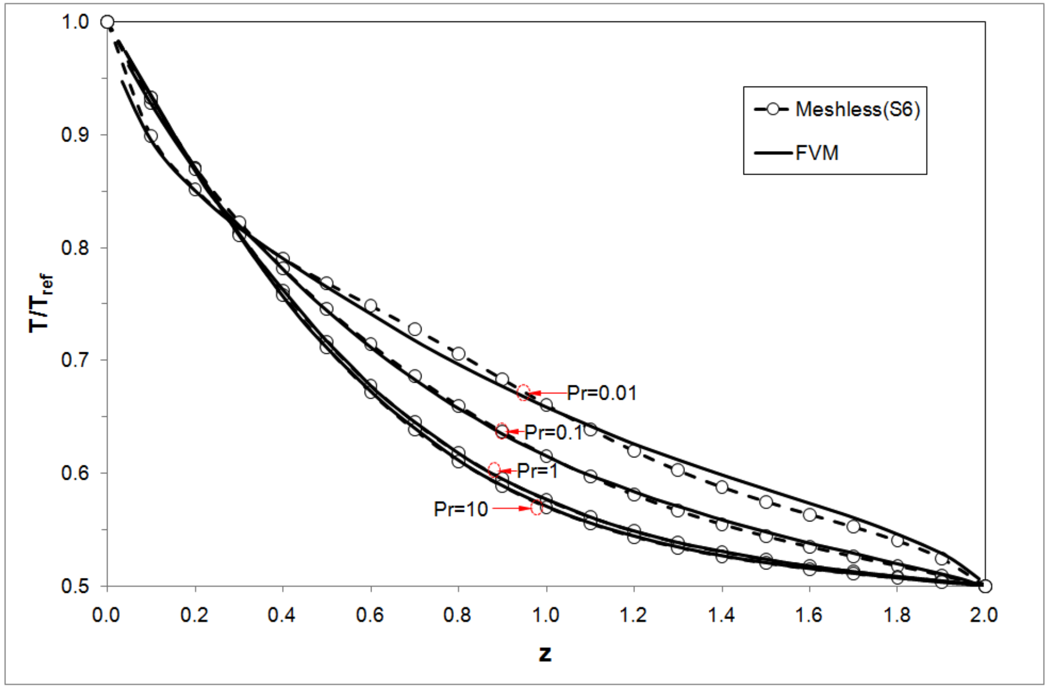
Base à Tc, les autres parois à T_f avec Tf/Tc=0.5

Effet du nombre de Planck

$Pr=(\lambda\beta)/(4\sigma T^3_{ref})$

Milieu semi-transparent cylindrique





temperature sur l'axe z

Effet du paramètre conducto-radiatif

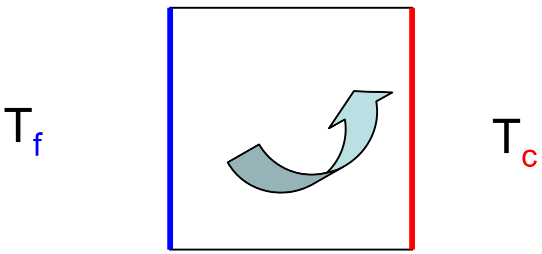
$$Pr=(\lambda\beta)/(4\sigma T^3_{ref})$$

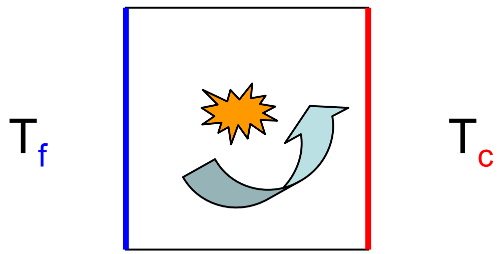
Couplage Convection-Rayonnement

Algorithme de Projection (P-V)
ou
Formulation Vitesse-Vorticité

+

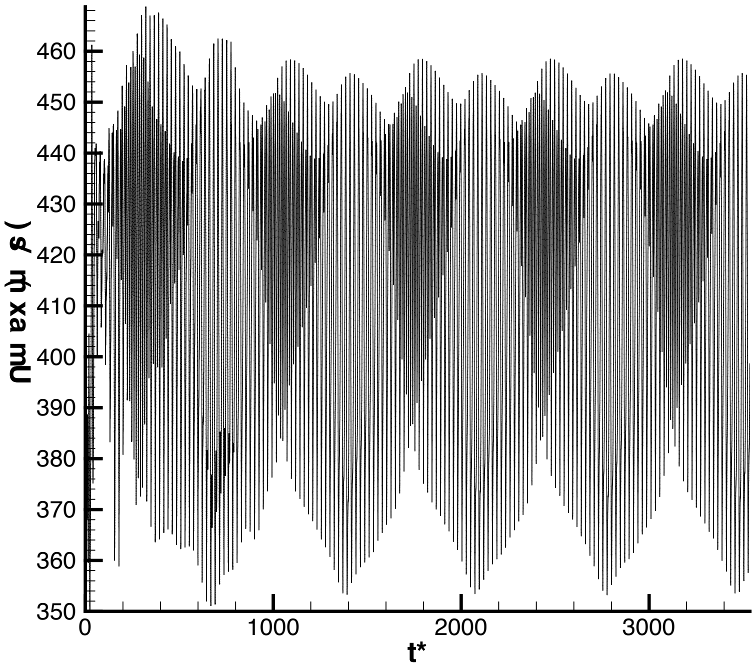
Méthode des Flux pairs

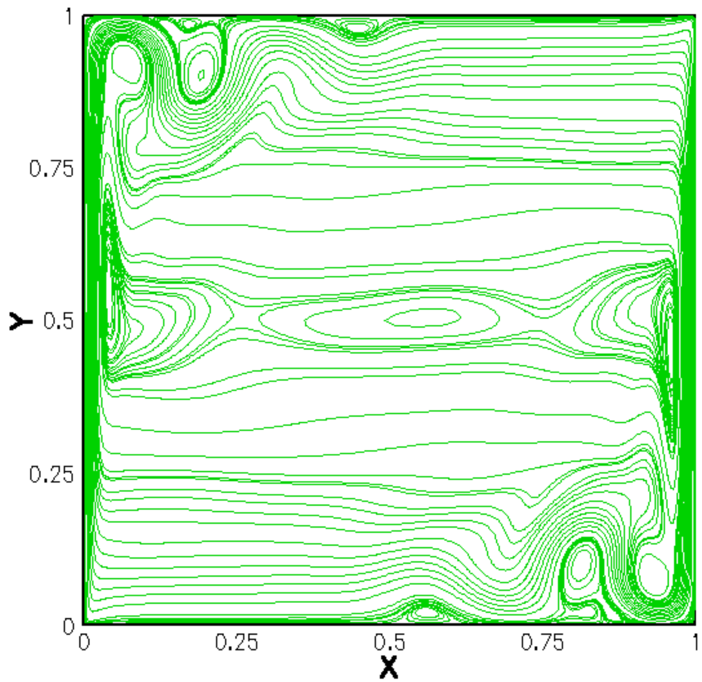




Cavité différentiellement chauffée à Ra=2. 10^8

Évolution temporelle de la vitesse

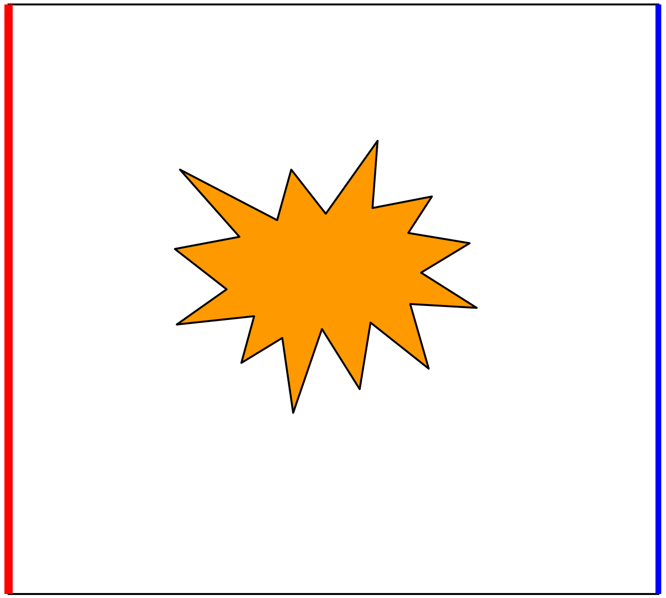




Bifurcation à un régime pseudo périodique

Couplage Convection-Rayonnement

Cavité avec generation interne de puissance



Ra=10^6

$$Ra^I = 4.10^4$$

$$Ra^I = \frac{g\beta\dot{q}L^5}{\nu_0\alpha_0 k}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \nabla^2 T - \frac{1}{Pl}\cdot\frac{1}{\Delta T}\cdot\frac{div\overrightarrow{q_r}}{4\sigma T_{ref}^3} + \frac{Ra^I}{Ra}$$

Equations de Poisson pour la vitesse

$$\nabla^2 u = -\frac{\partial \zeta}{\partial y} + \frac{\partial \eta}{\partial z} \qquad \nabla^2 v = -\frac{\partial \xi}{\partial z} + \frac{\partial \zeta}{\partial x} \qquad \nabla^2 w = -\frac{\partial \eta}{\partial x} + \frac{\partial \xi}{\partial y}$$

Equations de transport de la vorticité

$$\frac{\partial \xi}{\partial t} + u\frac{\partial \xi}{\partial x} + v\frac{\partial \xi}{\partial y} + w\frac{\partial \xi}{\partial z} = \xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z} + Pr\nabla^2\xi + RaPr\cos\phi(\frac{\partial T}{\partial y})$$

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + w\frac{\partial \eta}{\partial z} = \xi\frac{\partial v}{\partial x} + \eta\frac{\partial v}{\partial y} + \zeta\frac{\partial v}{\partial z} + Pr\nabla^2\eta + RaPr(\sin\phi\frac{\partial T}{\partial z} - \cos\phi\frac{\partial T}{\partial x})$$

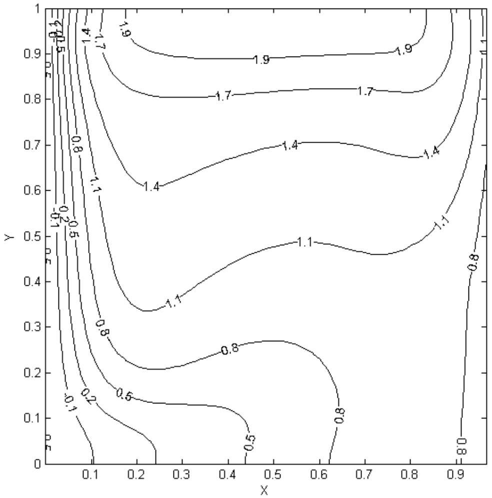
$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + w\frac{\partial \zeta}{\partial z} = \xi\frac{\partial w}{\partial x} + \eta\frac{\partial w}{\partial y} + \zeta\frac{\partial w}{\partial z} + Pr\nabla^2\zeta - RaPr\sin\phi(\frac{\partial T}{\partial y})$$

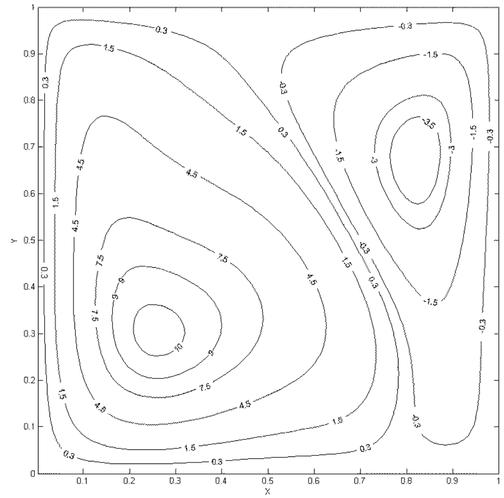
Equation de l' énergie

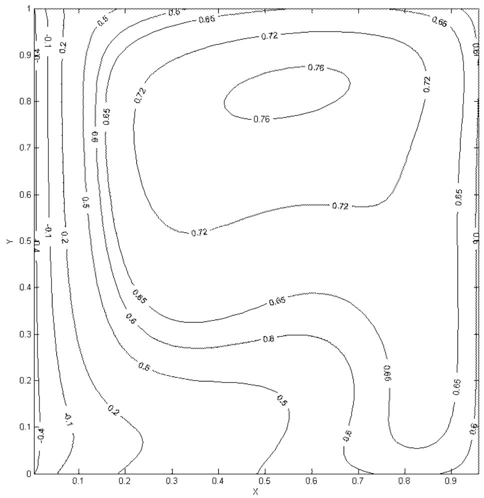
$$\boxed{\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \nabla^2 T - \frac{1}{Pl}\cdot\frac{1}{\Delta T}\cdot\frac{div\overrightarrow{q_r}}{4\sigma T_{ref}^3} + \frac{Ra^I}{Ra}}$$

Planck $Pl = \dfrac{k/L}{4\sigma T_{ref}^3}$ $\qquad Ra^I = \dfrac{g\beta\dot{q}L^5}{v_0\alpha_0 k}$ $\qquad$ **Rayleigh** $Ra = \dfrac{g\beta\Delta T L^3}{v_0\alpha_0}$

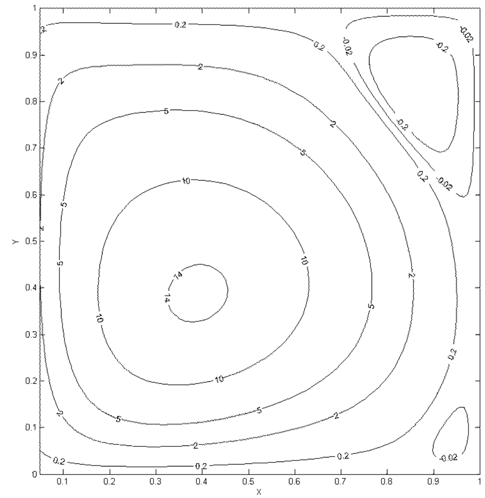
Convection







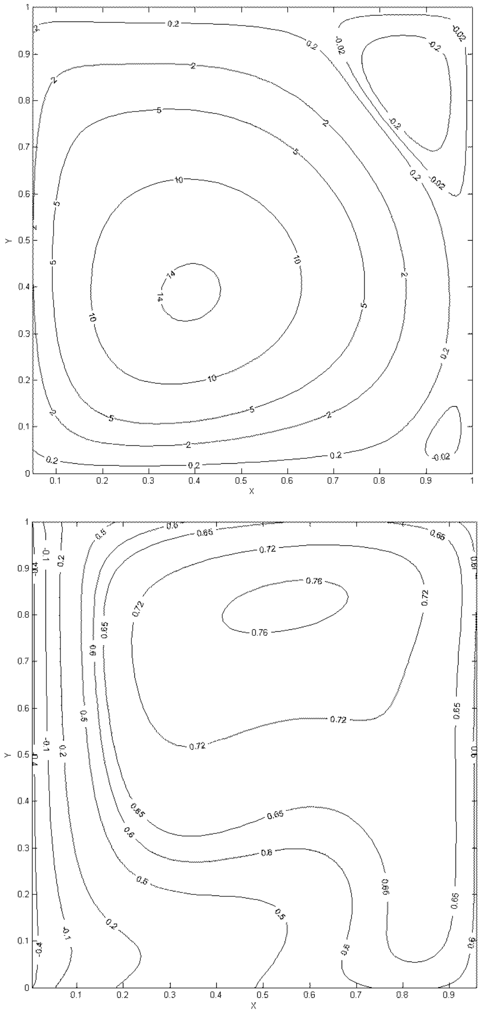
$\tau = 1.0$

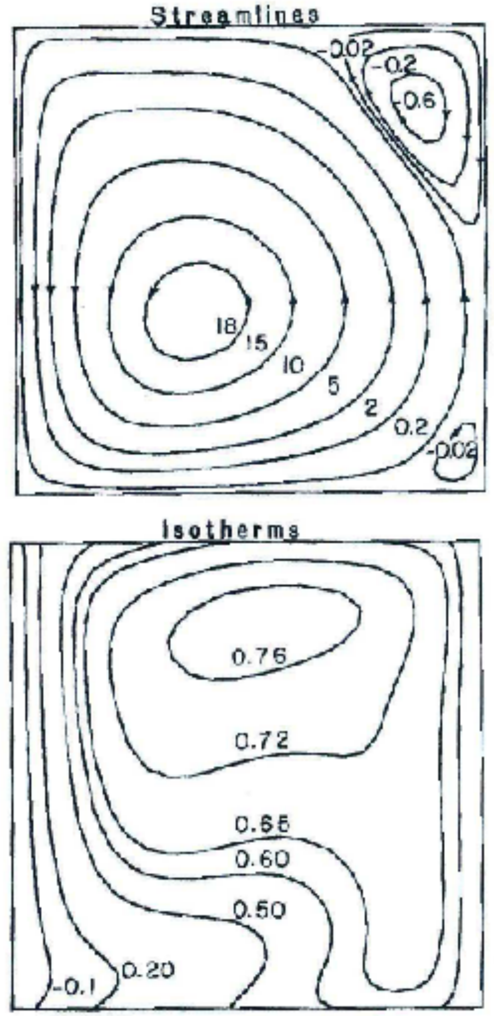


Convection+Rayonnement

Cavité avec generation interne de puissance

MESHLESS

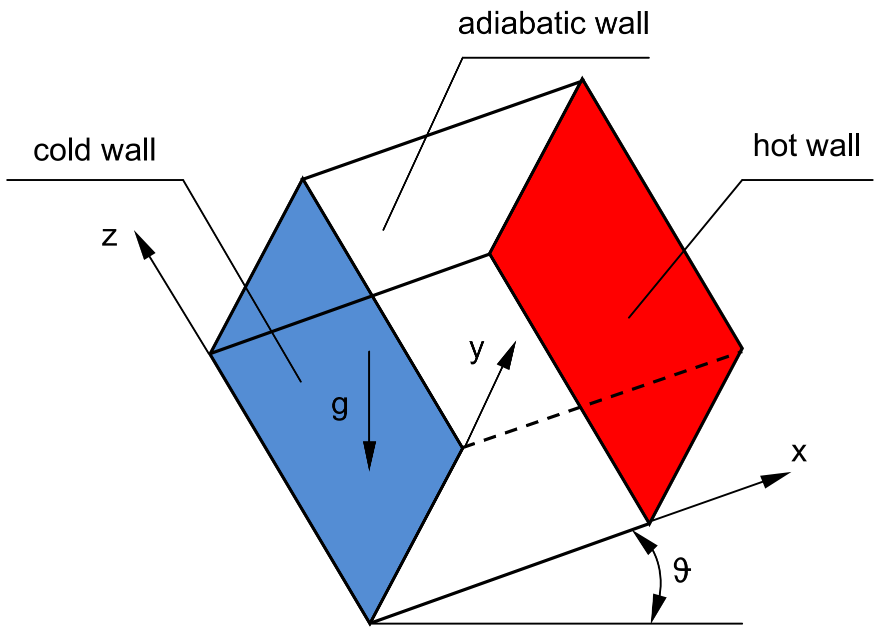




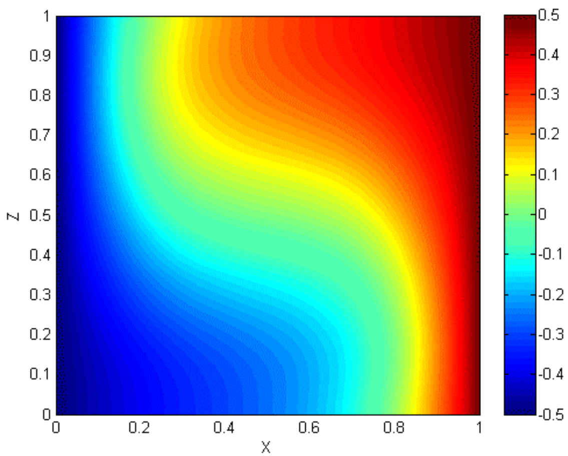
VOLUMES FINIS

[A. Yücel et al] NHT 2000
Volumes finis

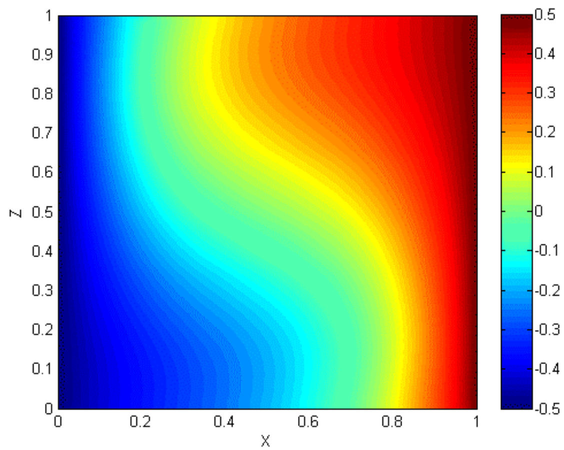
Cavité 3D différentiellement chauffée



Colomer et al,
IJHMT,2004



k=30



k=10

Merci