A New Look at the Modeling of Secondary Breakup

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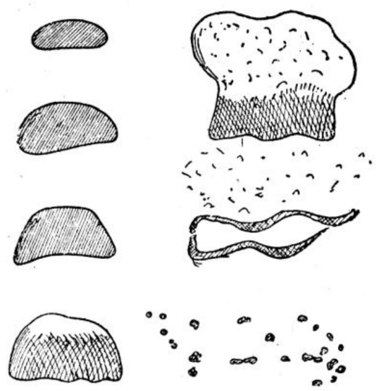
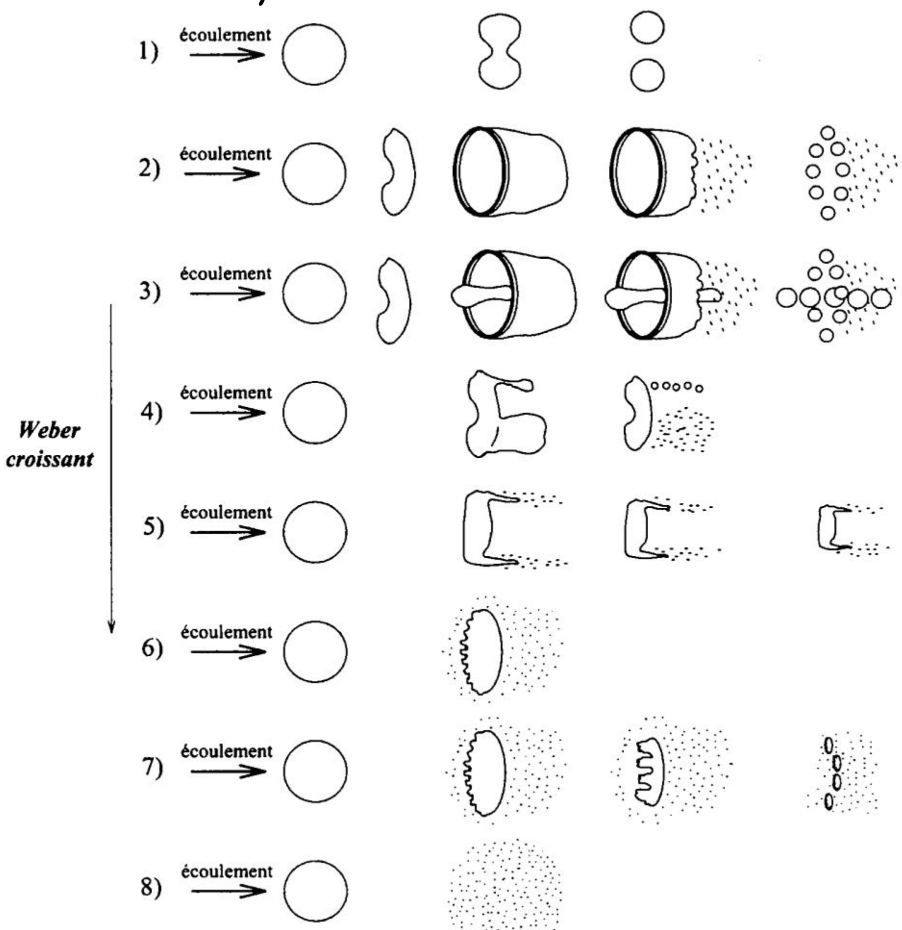
Experimental studies



- Since Eotvos
  - **Drop tower**
    - Hochwelder (1919)
  - **Shock tube**
    - Ranger & Nicholls (1968)
    - Faeth's group in Michigan (90[ies])
  - **Wind Tunnel**
    - Nowadays…
      - (Opfer et al. 2014, Kulkarni & Sojka 2014…etc.)
        - » Easier…
  - And many more…

First Classification (Pilch and Erdman, 1987)

$$We = \frac{\rho U^2 L}{\gamma} = \frac{\text{inertia}}{\text{surface tension}}$$



1. *Deformation We* < 12
2. *bag break-up:* 12 < *We* < 50
3. *umbrella break-up* for 50 < *We* < 100
4. *Multimode breakup*
5. *Shear break-up* for 100 < *We* < 350)
6. Wavy Shear Breakup: 350 < *We*
7. *Piercing break-up*
8. *Catastrophic break-up*

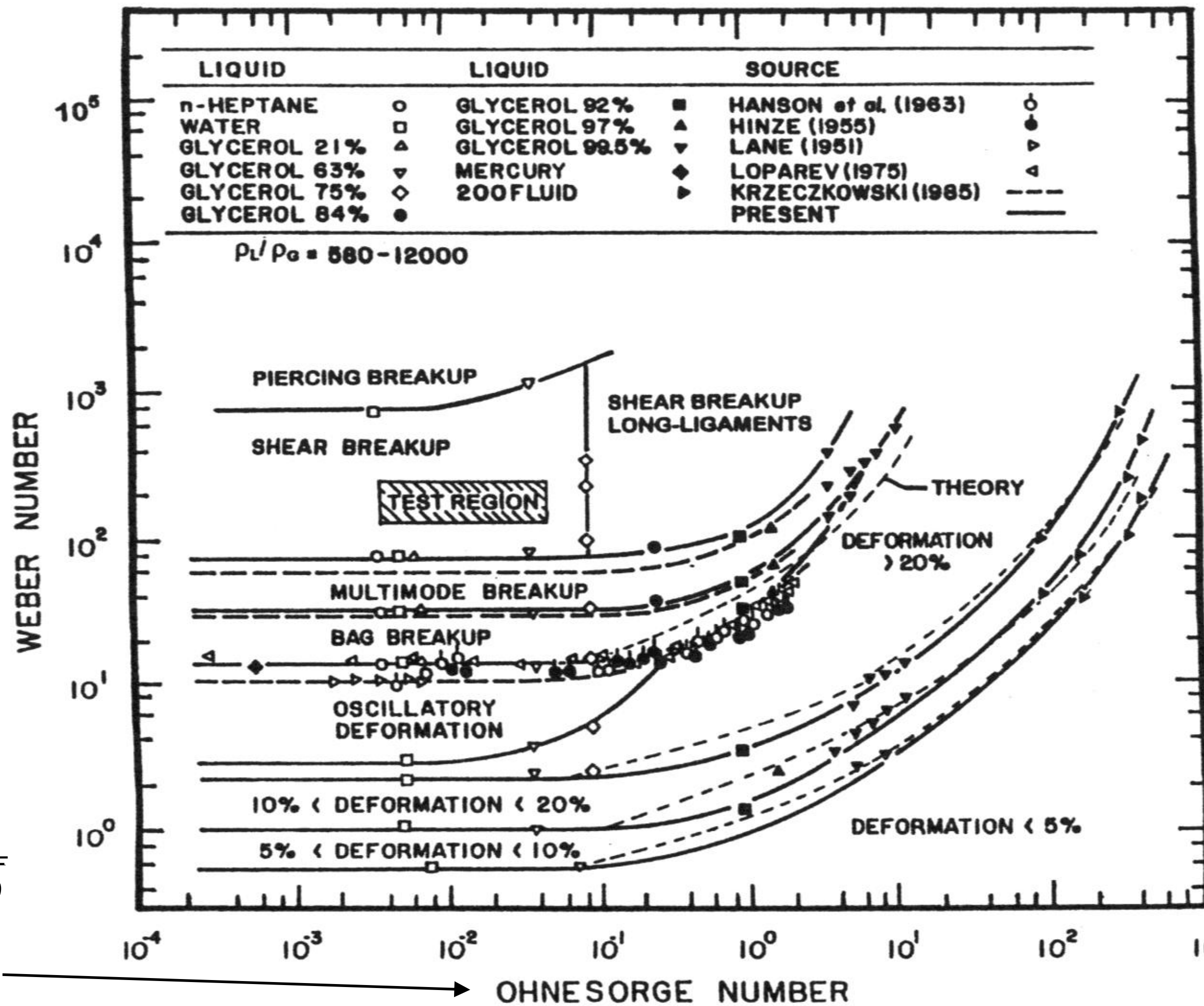
Second Classification (Faeth's Group)

Regime map for
shock wave
disturbance
Univ. Ann Arbor

Hsiang and Faeth *Drop Deformation and breakup due to shock wave and steady disturbance*
IJMF, 21, 545-560, 1995

$$Oh = Z = \frac{\sqrt{We}}{\mathrm{Re}} = \frac{\mu}{\sqrt{\gamma \rho D}}$$

$$= \frac{\text{Viscosity}}{\sqrt{\text{Inertia} \times \text{Surface Tension}}}$$



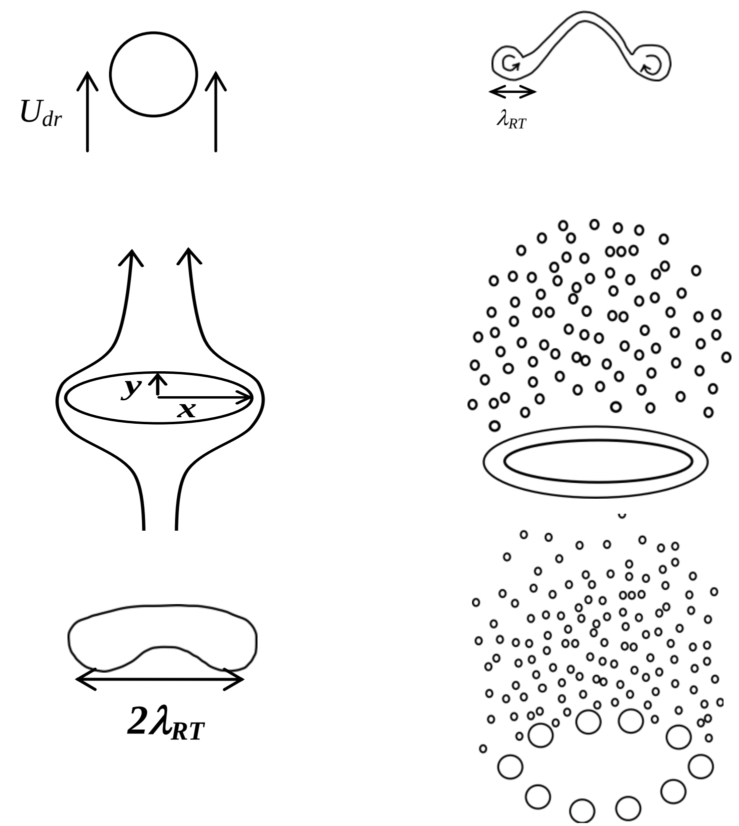
Droplet Deformation Models

- Droplet deformation and breakup model
  - Oscillator-like Model
    - TAB Taylor Analogy Breakup (*O'Rourke 1987*)
    - DDB Droplet Deformation and Breakup (*Ibrahim et al. 1993*)
  - More recent
    - Elongational deformation (*Villermaux & Bossa 2009, Kulkarni & Sojka 2014*)
    - Potential flow around a disc (*Opfer et al. 2014*)
- A new model
  - Without fitting parameters… Is it possible?

Bag-Breakup, Qualititavely

- In the literature

  12(23) < We < 50
- Six stages
  - Inception
  - Deformation
  - RT Wave Growth
  - Bag Growth
  - Bag breakup
  - Rim Breakup



Quantitatively

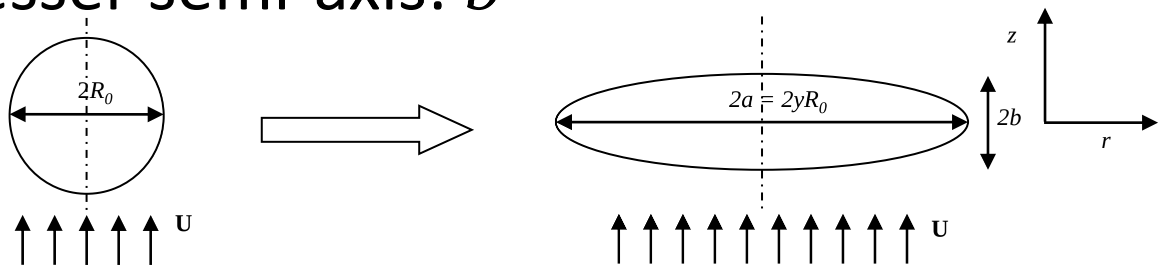
- Balance between:
  - Kinetic (deformation) energy: K
  - Surface energy: E_s
  - Viscous Dissipation: D
  - Air Pressure Work: W_p

$$\frac{dK}{dt} + \frac{dE_S}{dt} = W_P + D$$

- Simple modelling
  - Imposes a 1-parameter deformation path

Hypotheses

- Chosen deformation path: Oblate Ellipsoid
  - Greater semi-axis: a
  - Lesser semi-axis: b



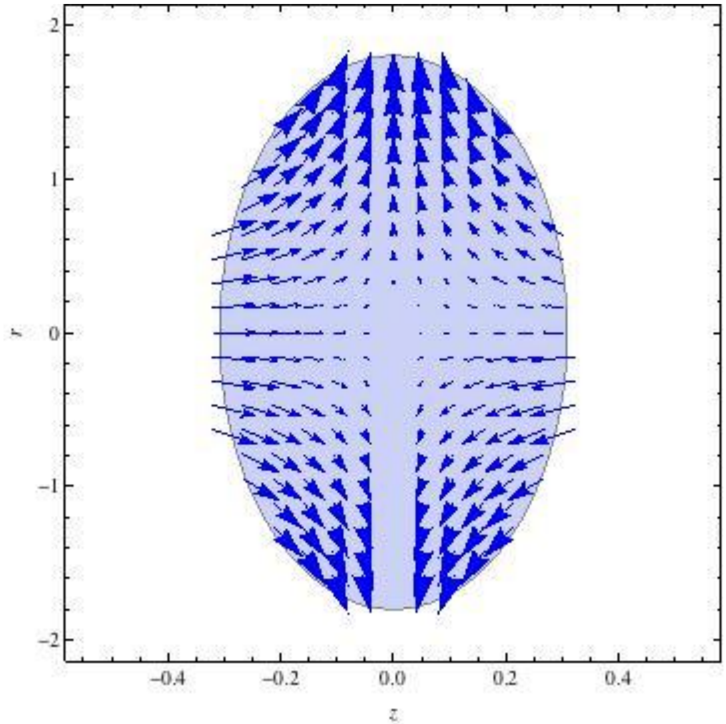
- Other hypotheses
  - Axisymetric potential outside flow
  - Viscous extensional flow inside
  - Volume conservation $a^2 b = R_0^3$

Flow inside the spheroid

- Viscous Extensional Flow

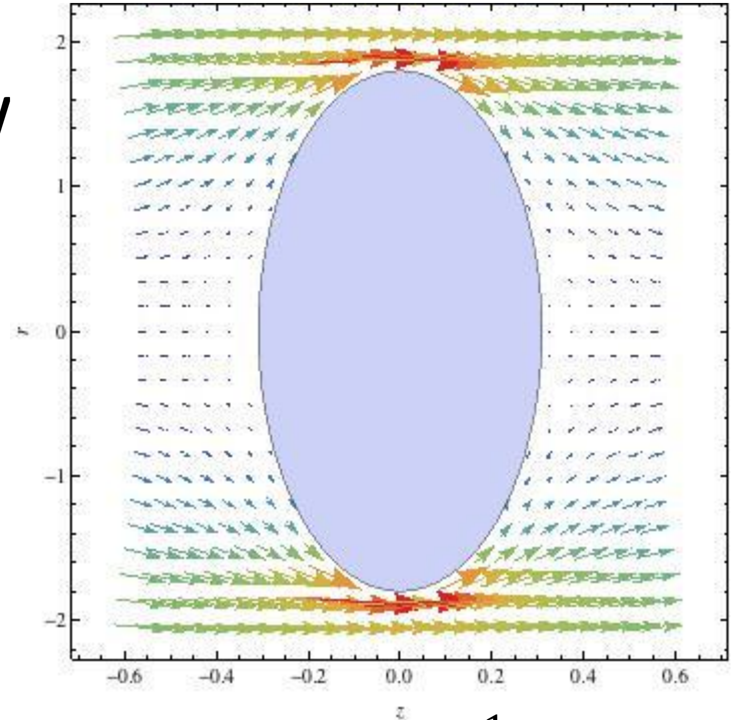
$$\begin{cases} v_r = \dfrac{\dot{y}}{y} r \\ v_z = -2\dfrac{\dot{y}}{y} z \end{cases}$$

$$y = \frac{a}{R_0}$$



Flow outside the spheroid

- Axisymmetric potential flow
  - Batchelor *An introduction to fluid dynamics*



$$U_z = \frac{-1}{r}\frac{\partial \psi}{\partial r}$$

$$U_r = \frac{1}{r}\frac{\partial \psi}{\partial z}$$

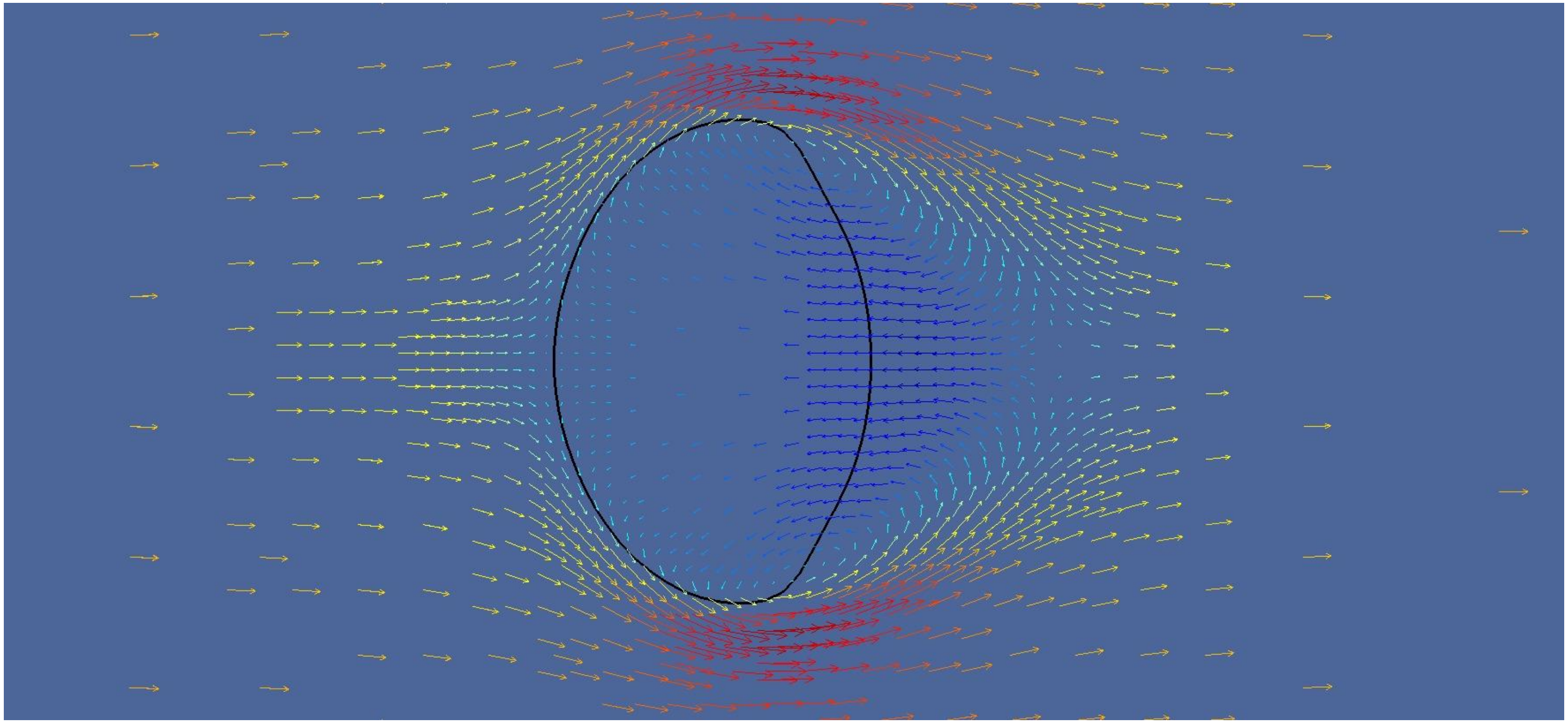
$$\psi = \frac{-\frac{1}{2}U_\infty\left(a^2-b^2\right)}{e\sqrt{1-e^2}-\mathrm{Sin}^{-1}(e)}\left(\mathrm{Sinh}(\xi)-\mathrm{Cosh}^2(\xi)\mathrm{Cot}^{-1}\left(\mathrm{Sinh}(\xi)\right)\right)\mathrm{Sin}^2(\eta)-\frac{1}{2}U_\infty r(\xi,\eta)^2$$

  - Conformal transformation

$$z+ir=\sqrt{a^2-b^2}\,\mathrm{Sinh}(\xi+i\eta)$$

Not that bad approximation

- Gerris Simulation (Azzara, 2014)



Kinetic Energy Term

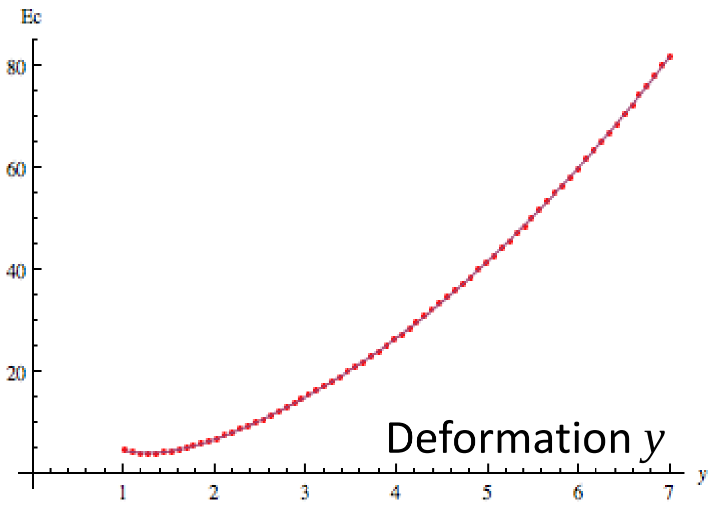
- Can be computed

$$K = \frac{1}{2}\rho_L \iiint_{V(t)} \mathbf{V}^2 d^3x = \frac{1}{2}\rho_L \left(\frac{\dot{y}}{y}\right)^2 \iint_{Ellipse(yR_0, R_0/y^2)} \left(r^2 + 2z^2\right) 2\pi r dr dz$$

  - Thanks to Mathematica!
  - And given the shape

$$K = \frac{1}{2}\rho_L K_C(y) \left(\frac{\dot{y}}{y}\right)^2$$

  - Where



- $K_c(y) \approx 22.1797 - 36.5808y + 25.9328y^2 - 8.2371y^3 + 1.5115y^4 - 0.1425y^5 + 0.0054y^6$

Surface Energy Term

- Exact formula (oblate)

$$S = 2\pi a^2 + \pi \frac{b^2}{e} \mathrm{Log}\left(\frac{1+e}{1-e}\right) \qquad e = \sqrt{1 - \frac{a^2}{b^2}}$$

- One gets

$$\frac{dE_S}{dt} = \sigma \frac{dE_S}{dy}\frac{dy}{dt} = \sigma K_S(y)\frac{1}{y}\frac{dy}{dt}$$

- Where K_S can be approximated by

$$K_S(y) \approx -39.682 + 41.523\, y - 5.242\, y^2 + 3.818\, y^3 - 0.404\, y^4 + 0.017\, y^5$$

Viscous Dissipation Term

- Cylindrical coordinates

$$D = \mu_L \int_V 2\left[\left(\frac{\partial V_r}{\partial r}\right)^2 + \left(\frac{\partial V_\theta}{r\partial\theta} + \frac{V_r}{r}\right)^2 + \left(\frac{\partial V_z}{\partial z}\right)^2\right] + \left[\left(r\frac{\partial V_\theta / r}{\partial r}\right) + \frac{\partial V_r}{r\partial\theta}\right]^2 + \left[\frac{\partial V_z}{r\partial\theta} + \frac{\partial V_\theta}{\partial z}\right]^2 + \left[\frac{\partial V_r}{\partial z} + \frac{\partial V_Z}{\partial r}\right]^2 d^3x$$

- Can be computed exactly

$$D = \mu_L \iiint_{S(y)} 12\left(\frac{\dot{y}}{y}\right)^2 d^3x = 16\pi R_0^3 \mu_L \left(\frac{\dot{y}}{y}\right)^2$$

  – *Schmehl et al. (ILASS 2002)*

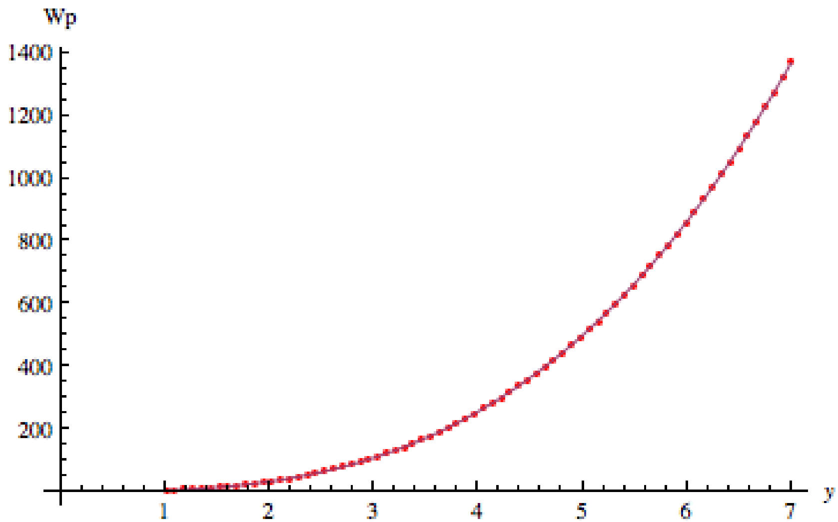
Air Pressure Work

- Air pressure work is given by

$$W_P = -\oint\!\!\oint_{\partial V} P(\mathbf{V.n})\,dS = 2\int_0^a \tfrac{1}{2}\mathbf{U}^2(\mathbf{V.n})\,2\pi r dr$$

- Which can be given by

$$W_P = \rho_G U_\infty^2 R_0^2 K_P(y)\frac{dy}{dt}$$



Deformation y

- Where

$$K_P(y) \approx -0.599 + 0.535y - 0.178y^2 + 4.026y^3 - 0.00137y^4$$

Final Equation

- Using non dimensional numbers:

$$K = \frac{\rho_L}{\rho_G} \qquad We_G = \frac{\rho_G U_\infty^2 2R_0}{\sigma} \qquad Re = \frac{\sqrt{\rho_G \rho_L} U_\infty 2R_0}{\mu_L}$$

$$Oh = \frac{\mu_L}{\sqrt{\sigma_L 2R_0}} \qquad \tau = \sqrt{\frac{\rho_G}{\rho_L}\frac{2R_0}{U_\infty}} \qquad t^* = \frac{t}{\tau}$$

- One gets:

$$\frac{d}{dt^*}\left(\left(\frac{\dot{y}}{y}\right)^2 K_C(y)\right) + \frac{128\pi}{Re}\left(\frac{\dot{y}}{y}\right)^2 + \frac{16}{We_G} K_S(y)\left(\frac{\dot{y}}{y}\right) = 8K_P(y)\left(\frac{\dot{y}}{y}\right)$$

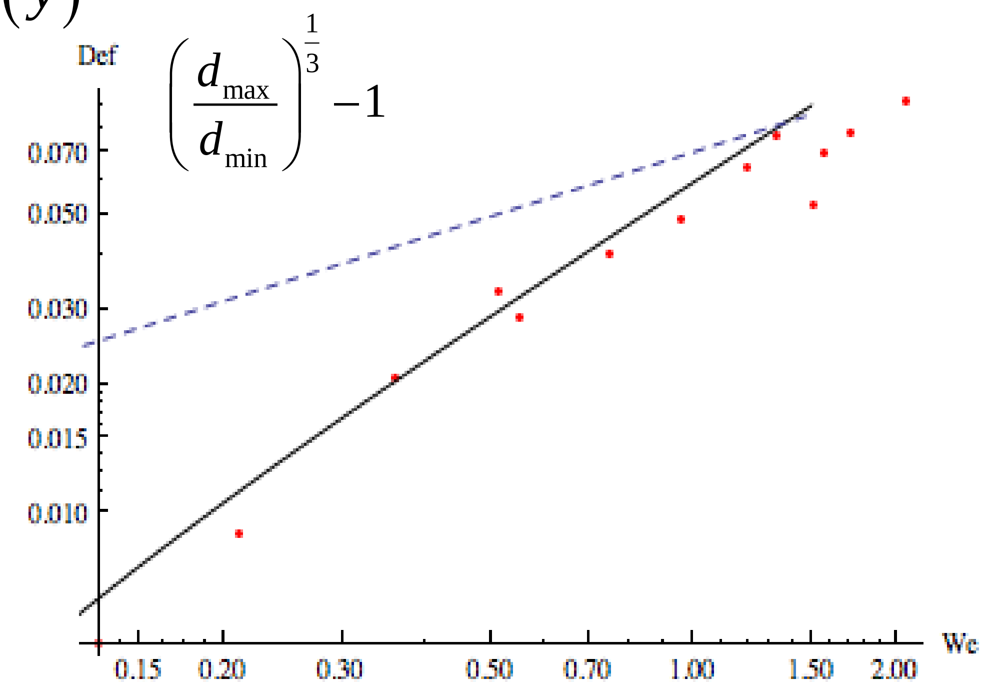
Steady State

- Equating Air Pressure and Surface Tension

$$We_G = \frac{2K_S(y)}{K_P(y)}$$

  - Compare with experimental data (Liquid/liquid)

    Hsiang & Faeth (1995)



Unsteady behavior

- Linearisation $\varepsilon = y - 1$

$$K_C(1)\ddot{\varepsilon} + \frac{64\pi}{\mathrm{Re}}\dot{\varepsilon} + \left(\frac{8}{We}\alpha_2 - 4\beta_2\right)\varepsilon = 0$$

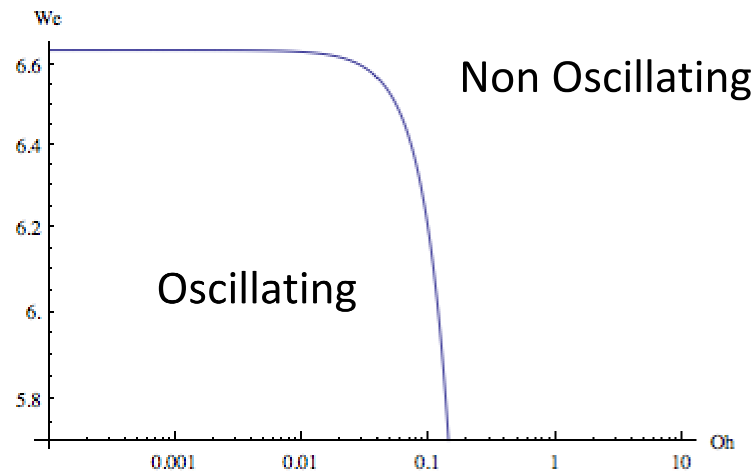
- Discriminant

$$\Delta = \left(\frac{64\pi}{Re}\right)^2 - 4K_C(1)\left(\frac{8\alpha_2}{We} - 4\beta_2\right)$$

- Oscillating condition

$$Oh^2 < \frac{4K_C(1)}{(64\pi)^2}(8\alpha_2 - 4\beta_2 We)$$

$$We_C = \frac{2\alpha_2}{\beta_2}$$



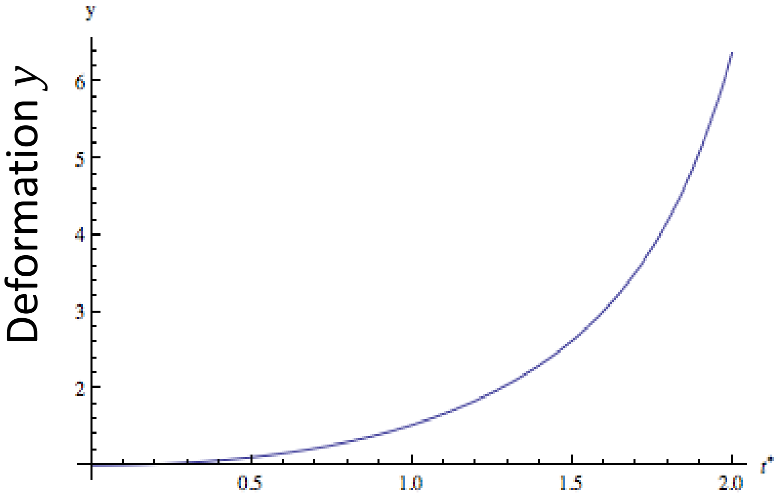
$$Oh_C = \frac{\sqrt{32K_C(1)\alpha_2}}{64\pi}$$

Decelerating (wind tunnel) drop

- Non dimensionalisation
  - U_0 initial velocity $U^* = \dfrac{U}{U_0}$
- Momentum equation

$$\frac{dU^*}{dt^*} = -\frac{3}{2\sqrt{K}} C_d y^{*2} U^{*2}$$

$$C_d(y, Re) = \frac{24}{Re}\left(1+0.1935 Re^{.6305}\right)\mathrm{Min}\left[\frac{3\,\mathrm{y}^3+4}{7}, 4\right]$$



U = 14 m/s We = 11.3, Oh=0.006, K= 769

- Deformation equation

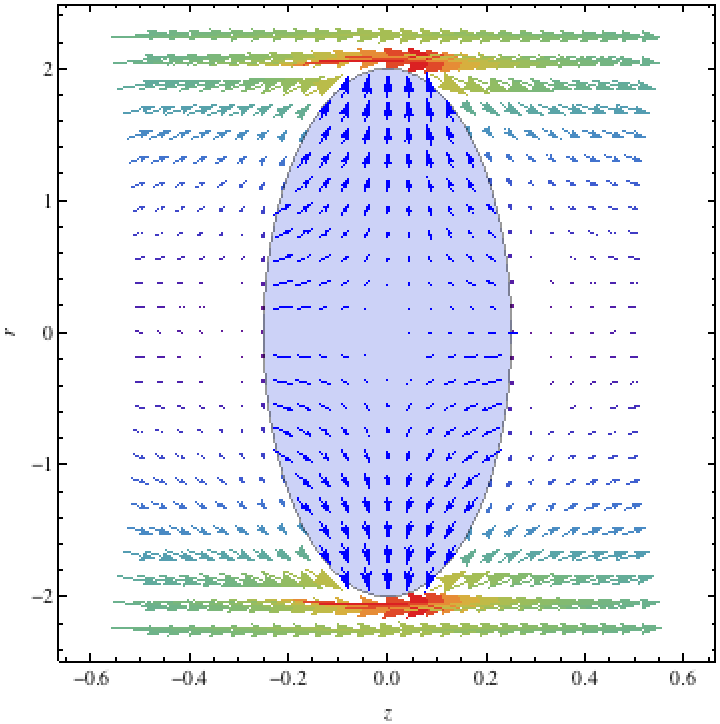
$$2\frac{d}{dt^*}\left(\left(\frac{\dot y}{y}\right)^2 K_C(y)\right) + \frac{128\pi}{Re}\frac{1}{U^*}\left(\frac{\dot y}{y}\right)^2 + \frac{16}{We_G}K_S(y)\frac{1}{U^{*2}}\left(\frac{\dot y}{y}\right) = \frac{8}{2}K_P(y)\left(\frac{\dot y}{y}\right)$$

Vortices inside

Vortices outside

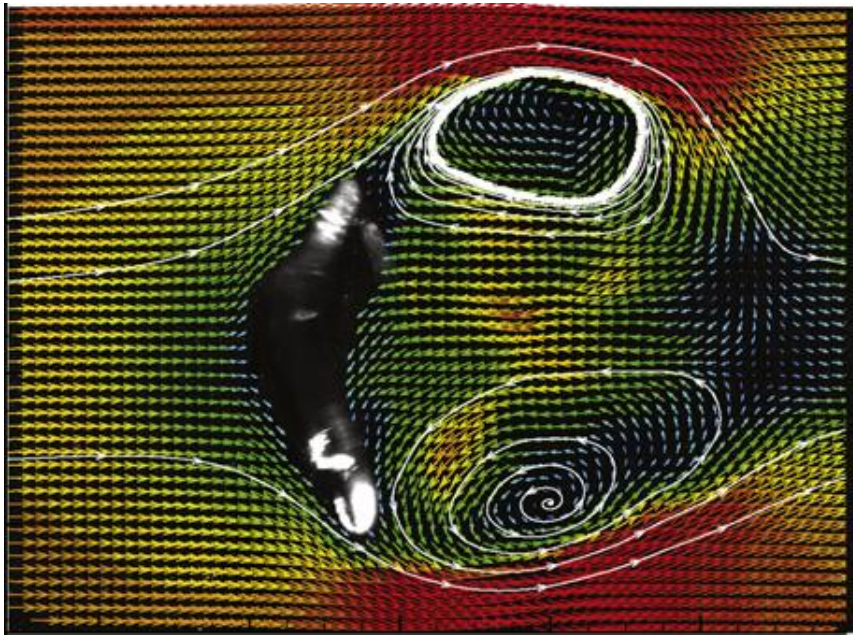
Why Fitting parameters?

« Model »

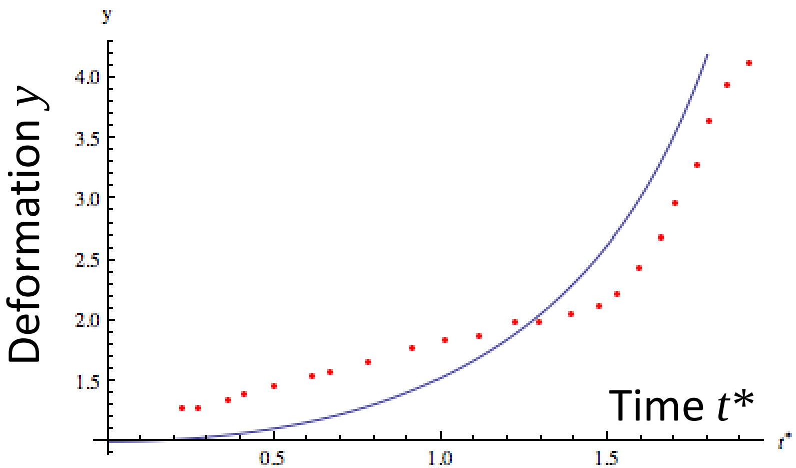


« Reality »

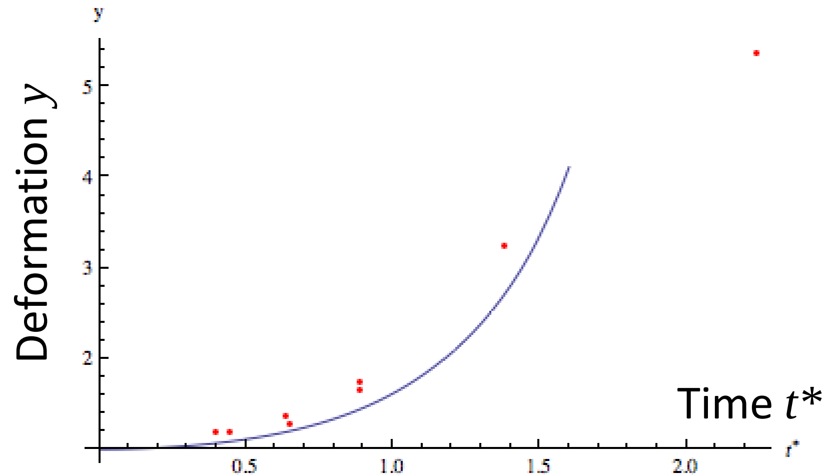
Flock et al. (2012)



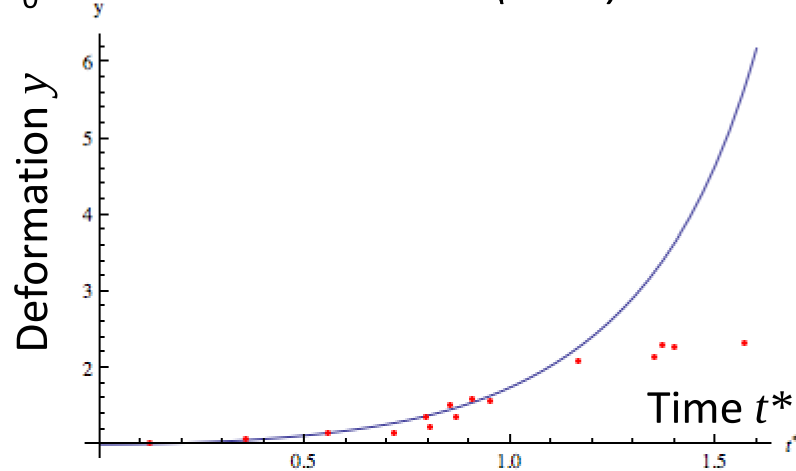
Comparison with experiments



U = 14 m/s We = 11.3, Oh=0.002, K= 769.
R_0= 1.6 mm *Opfer et al. (2014)*



U= 13.5 m/s We = 18.4, Oh=0.0014, K= 770.
R_0= 2.8 mm *Krzeczkowski (1980)*



U= 32 m/s We = 103.5, Oh=0.0014, K= 770.
R_0= 2.8 mm *Krzeczkowski (1980)*

Accelerating (freefall) drop

- Non dimensionalisation

$$Eo = \frac{(\rho_L - \rho_G) g (2R_0)}{\sigma}$$

$$U_l = \sqrt{\frac{4(K-1)R_0}{3gC_d}}$$

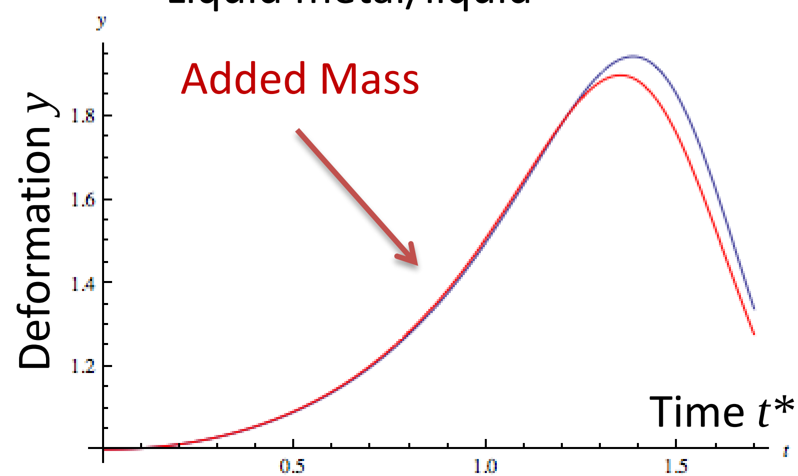
- Momentum equation

$$\frac{dU^*}{dt^*} = -\frac{3}{2\sqrt{K}} C_d y^{*2} U^{*2} + \frac{K^{3/2}}{K-1} \frac{Eo}{We_G}$$

  – Added mass effect

$$\left(1 + y^3 \frac{1}{K} \frac{e - \left(\mathrm{Sin}^{-1} e\right)\sqrt{1-e^2}}{\mathrm{Sin}^{-1} e - e\sqrt{1-e^2}}\right) \frac{dU^*}{dt^*} = -\frac{3}{2\sqrt{K}} C_d y^{*2} U^{*2} + \frac{K^{3/2}}{K-1} \frac{Eo}{We_G}$$

Liquid metal/liquid



U_l=1.05 m/s, We = 26.88, Oh=0.00014, K= 8, R_0 = 6mm

WHAT HAPPENS WHEN SIZE DOUBLE?

Rayleigh-Taylor Growth

- Most amplified wavelength is given by

$$\lambda_{RT,\max} = 2\pi\sqrt{\frac{3\gamma}{f\,\Delta\rho}}$$

- Droplet deceleration is given by

$$f = \frac{3}{8}\frac{\rho_G}{\rho_L}\frac{x^2}{r^3}C_d U^2$$

- Which turns to

$$\frac{\lambda_{RT,\max}}{r} = 2\pi\left(\frac{r}{x}\right)\sqrt{\frac{8\gamma}{C_d\rho_G U^2 r}}$$

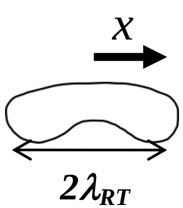
- C_d is equal to (Pilch et Erdman, 1987)
  - 1.7 (falling droplet)
  - 3.0 (shock tube)

Critical Weber Number

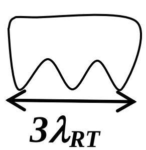
- By assuming the following two-waves breakup condition: $x=\lambda=2r$, one gets:

$$\left(\frac{x}{r}\right)^4 = 2^4 = 16 = 64\pi^2 \frac{1}{C_d We}$$

- Which turns to
  - Free Fall $\quad We_{\min} = \frac{4\pi^2}{C_d} \approx 23.2$
  - Shock tube $\quad We_{\min} = \frac{4\pi^2}{C_d} \approx 13$

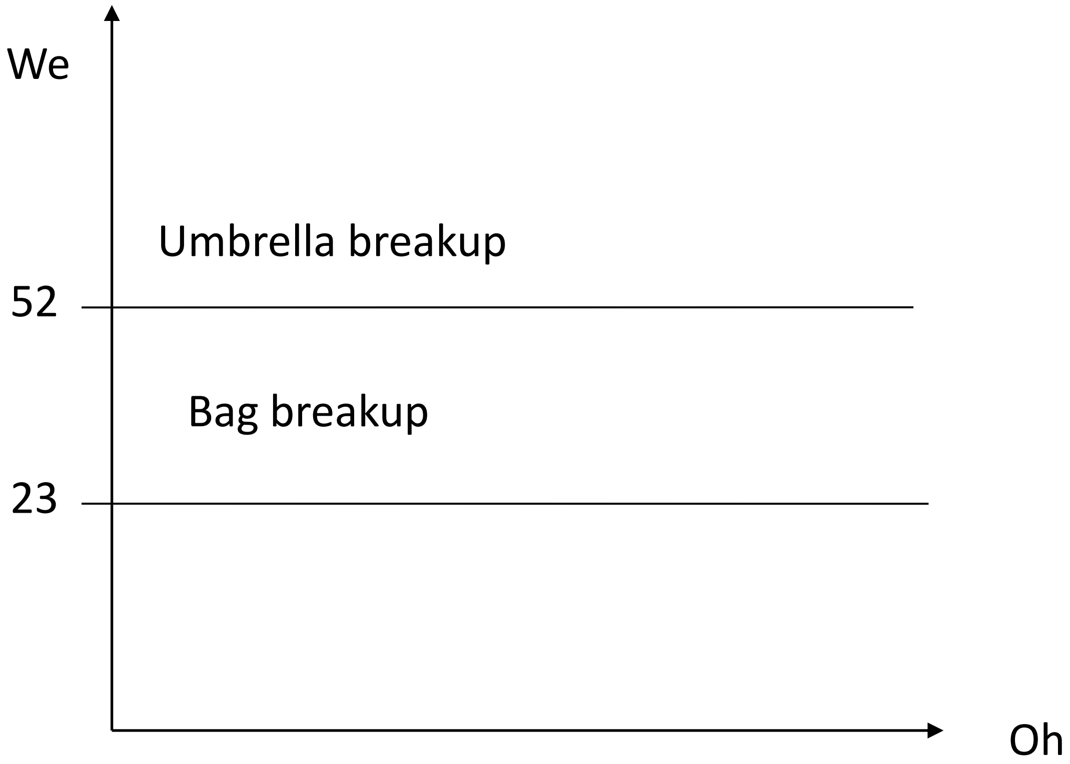


- With three waves
  - Umbrella breakup $\quad 2x = 3\lambda_{RT}$



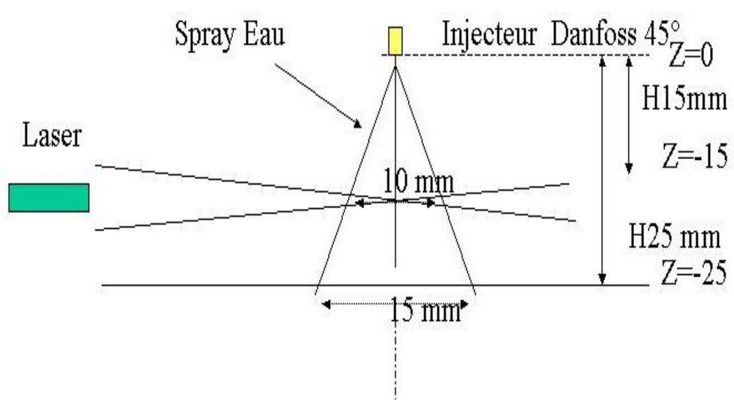
$$\left(\frac{x}{r}\right)^4 = 12^2\pi^2 \frac{1}{C_d We} \qquad We_{\min} = \frac{144\pi^2}{16 C_d} = 52 \text{ (chute libre)}$$

Theoretical Diagram



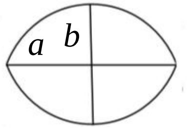
DAUGHTER DROPS PDF

Bag Breakup
Dispositif Experimental



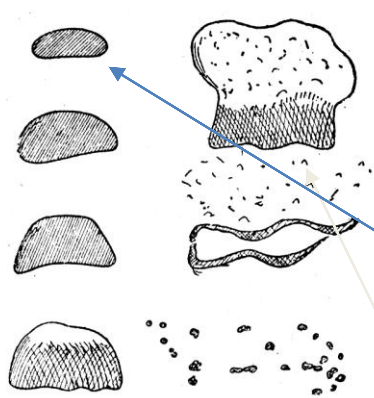
N. Rimbert and G. Castanet *Evidences of turbulent cascading atomization in the bag-breakup regime* Phys. Rev. E (2011)

- **Tuyère Lechler ref. 665-042, 8 bars**
  - Montée verticalement
  - Oeil de chat, "Fan spray", 80 L/minute

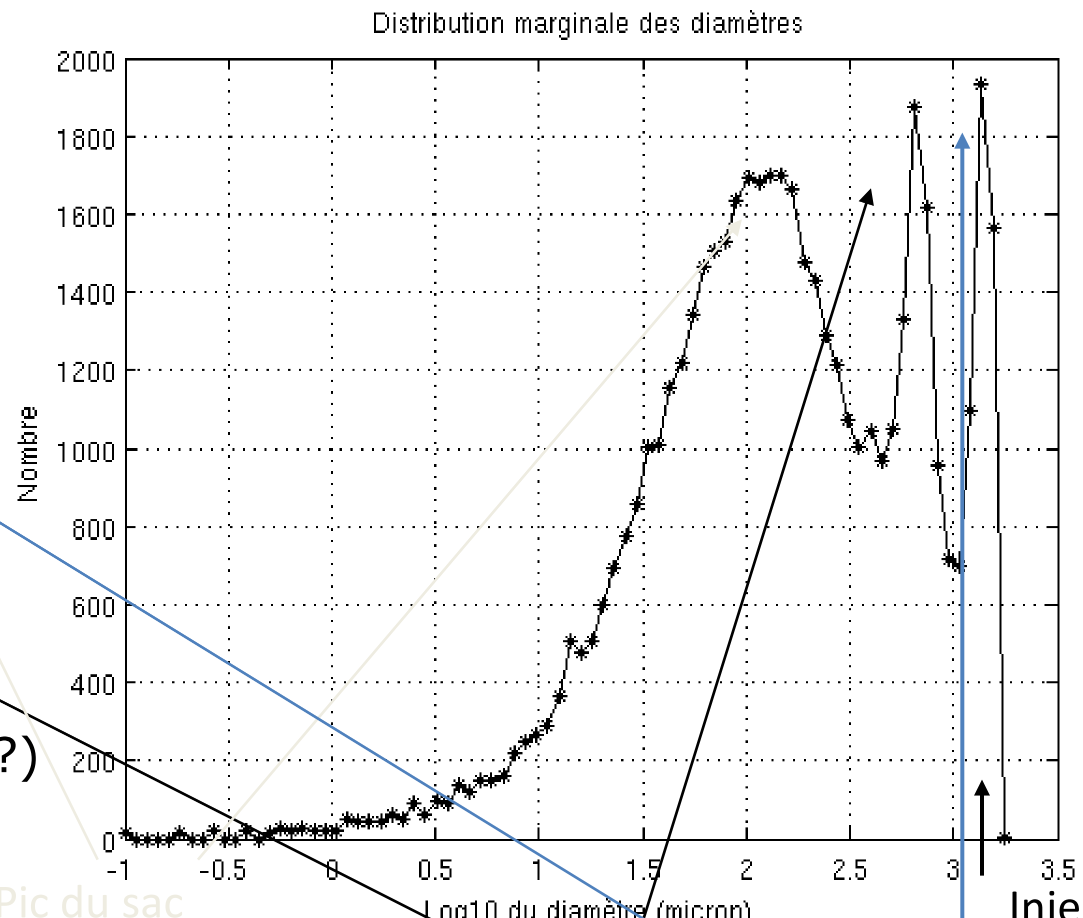


- La lumière Laser est guidée optiquement dans une fibre
- PDPA: mesure la taille et la vitesse des gouttes

PDF à trois pics



(Hochschwender, 1919?)



Pic du sac

Pic de l'anneau

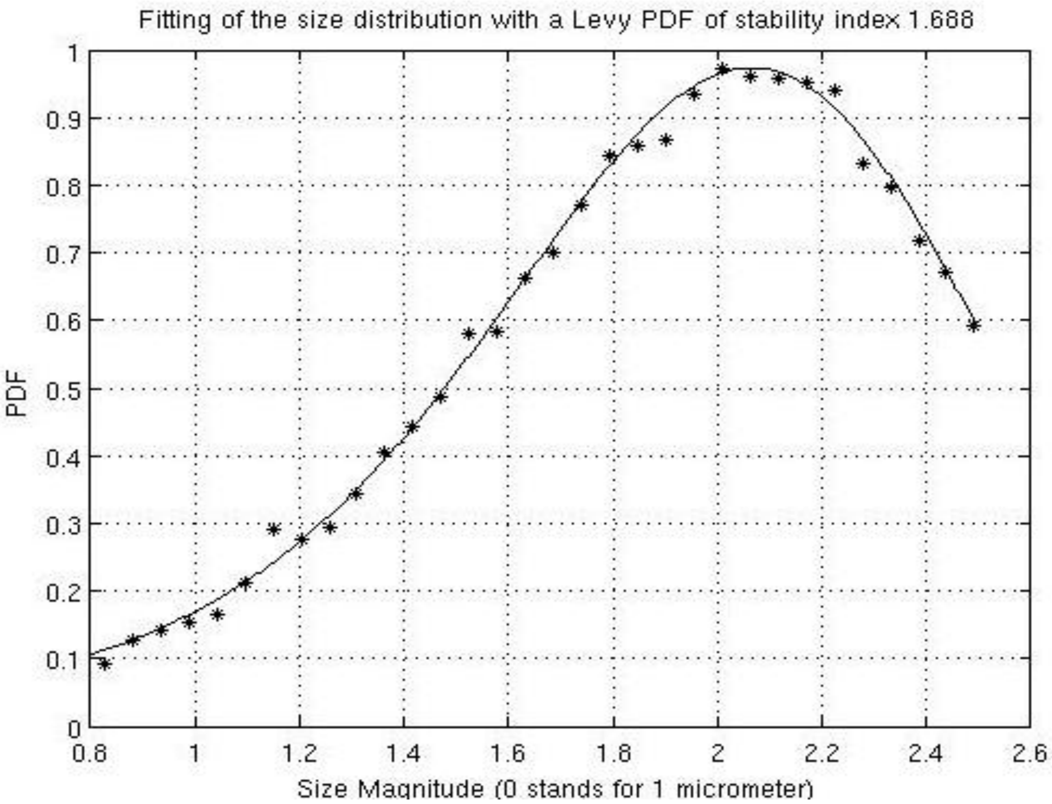
Pic initial

Injecteur: 3.23

$$\frac{SMD_{bag}}{d_{init}} = \frac{200}{1350} \approx 0.148$$

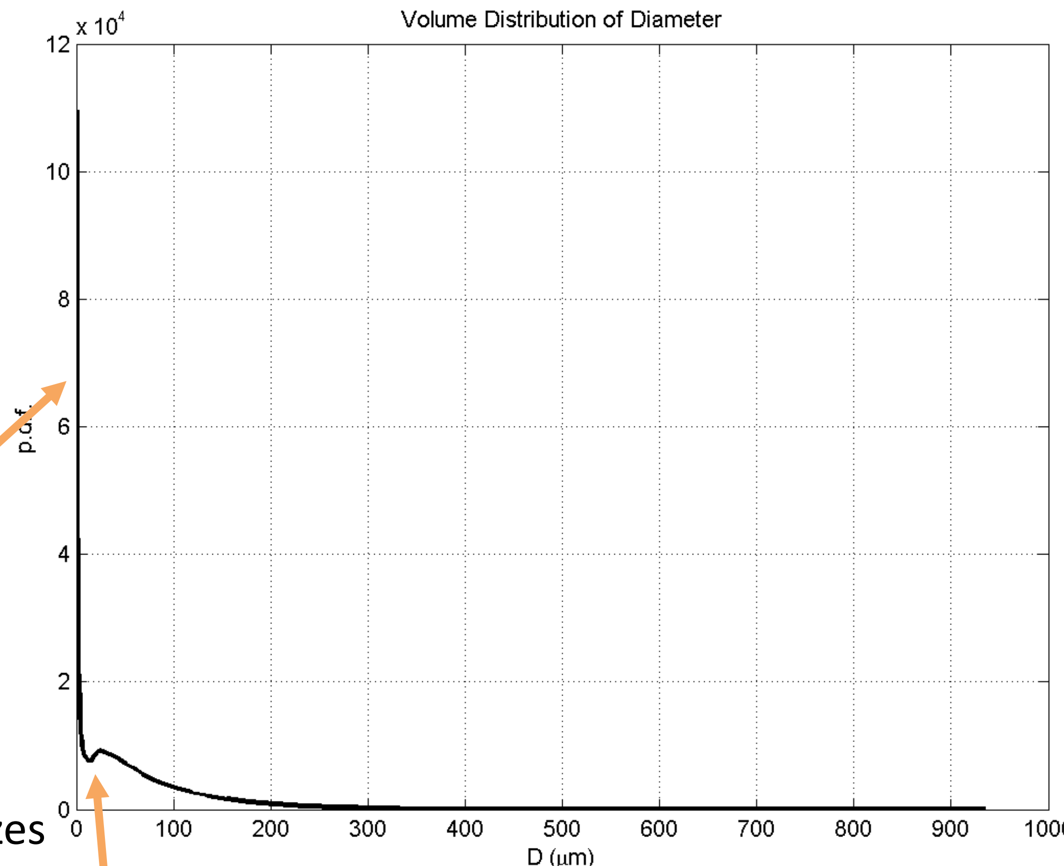
Log Lévy Stable PDF

- *Novikov and Dommermuth, Phys. Rev. E, 1997*
- *Rimbert and Séro-Guillaume, Phys. Rev. E, 2004*
  - Extension de résultats de Kolmogorov
  - Distribution volumique
  - Données exp. communiquées par Simmons and Hanratty
- Ici, nos propres données
  - 50,000 gouttelettes
  - Distribution numérique
  - α = 1.69



$$\sigma_{\ln d} \approx 1.1$$

Other representation of Log-Lévy PDF



$$\frac{1}{x}\frac{1}{(\log x)^{\alpha+1}}$$

Divergence toward small sizes

Apparently two modes