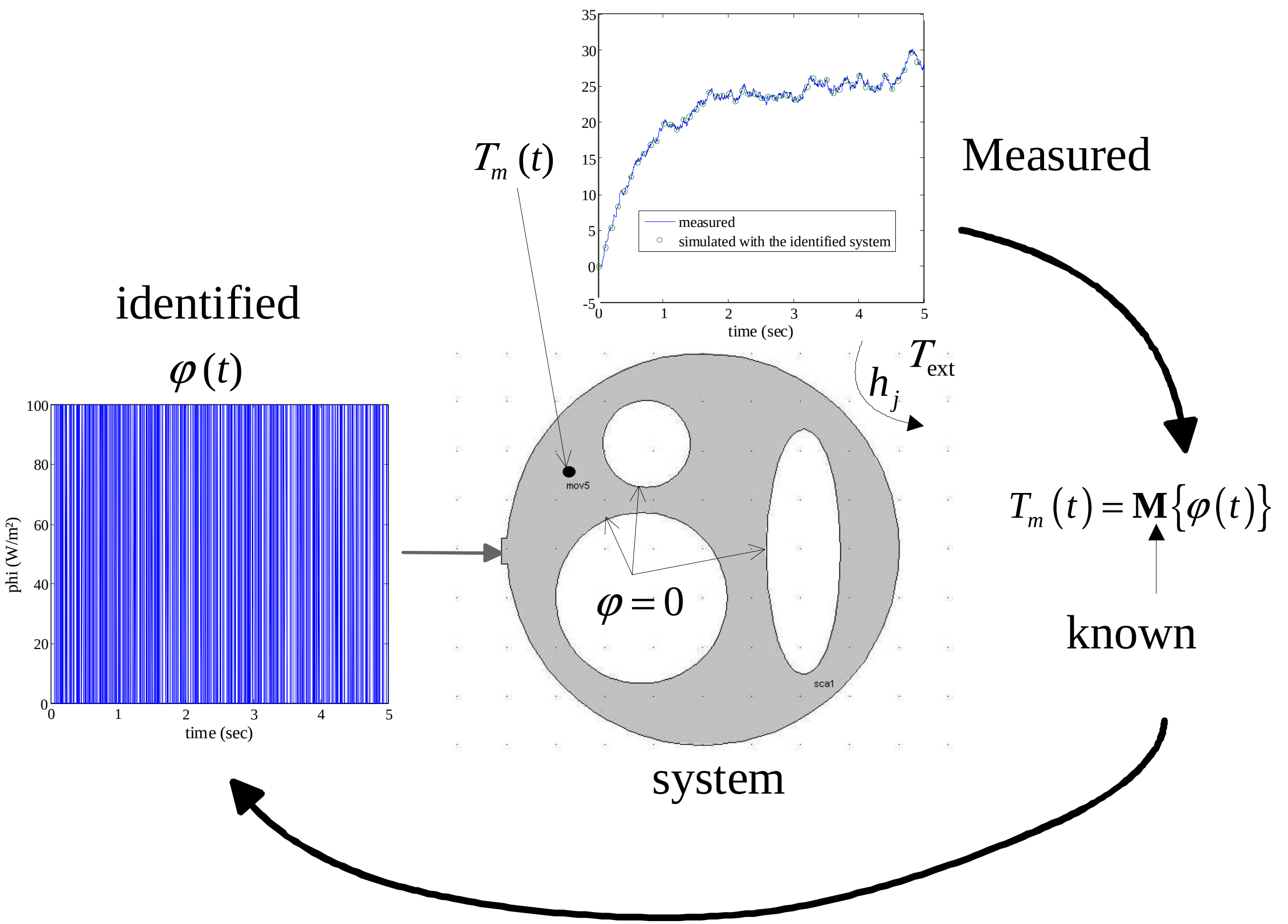
Lecture 8 & Tutorial 1:

Experimental idention of low order model

Jean-Luc Battaglia, Laboratory I2M,

Department TREFLE, I2M Bordeaux

Measurement inversion



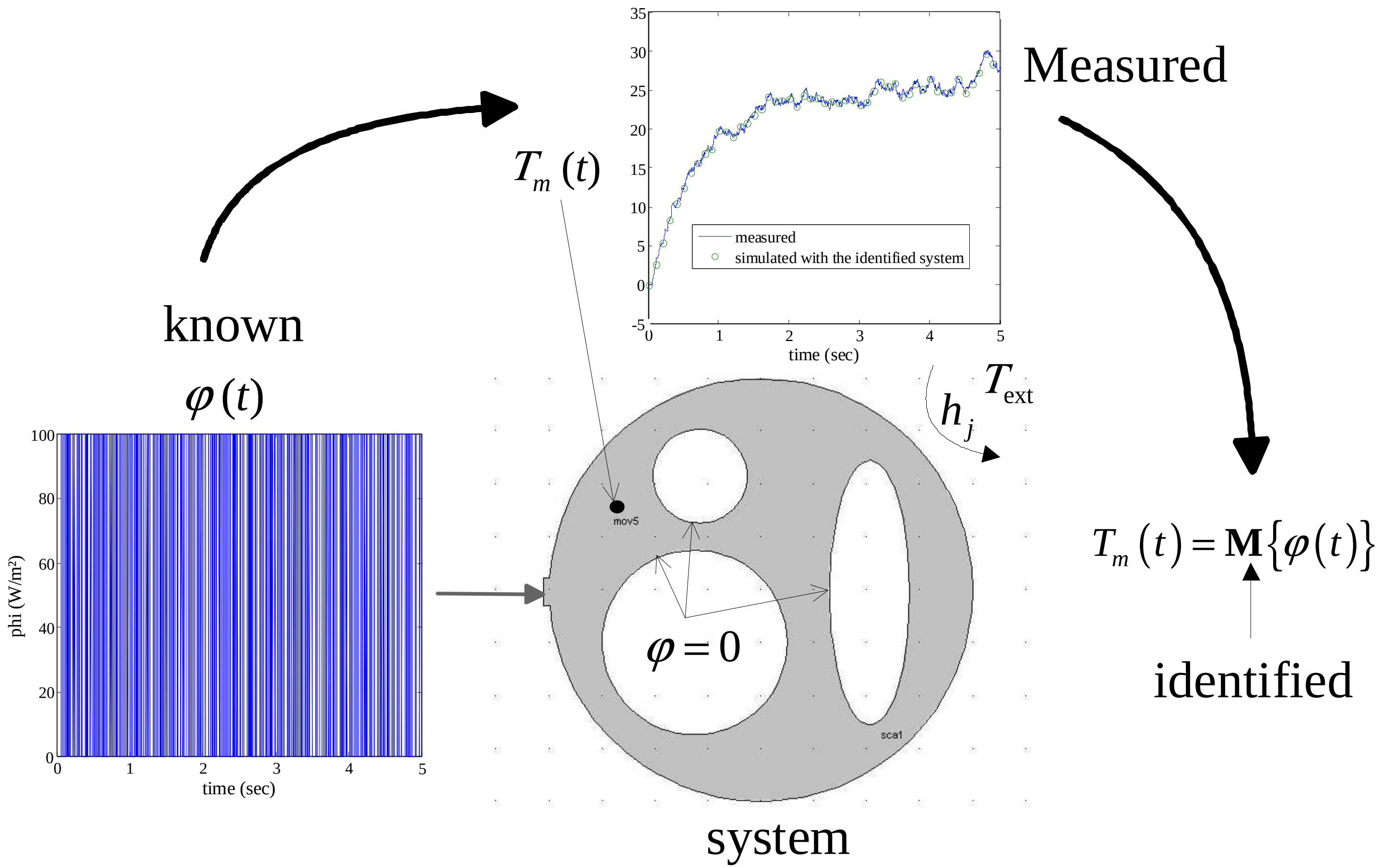
Classical requirements for a good resolution of the inverse problem

- Implementing thermal sensors in the system at strategic locations and making measurements

- Having a reliable and accurate model that describes well the experiment. The reliability of the direct model rests on the accuracy on two sets of data:
  - the thermal properties
  - the location of the sensor.

  Uncertainties on those data will lead to a very low confidence domain for the estimated heat flux

System identification



Linear monovariable systems

- The impulse response

$$T_m(t) = h_m(t) * \varphi(t) = \int_0^{\infty} h_m(t-\tau)\varphi(\tau)\,\mathrm{d}\tau$$

- For monovariable linear systems, the impulse response fully characterizes the thermal behaviour. Therefore, any kind of inverse strategy can be based on the direct model expressed as the impulse response of the system

System identification approach

- **Advantages**
- The system identification approach will be first interesting to obtain a reliable and accurate *low order model* that will require less computational time for simulation.
- There is no need to know the thermal properties of the system (thermal conductivity, density, specific heat, heat exchange coefficients, thermal resistances at the interfaces, parameters related to thermal radiation…).
- It is not required to know the sensor location inside the system.
- It is not required calibrating the sensor.
- The identification procedure is fast (this will be viewed later with the description of the different techniques).
- **Drawbacks**
- The model identification must be achieved in the exactly same conditions as those encountered during the inversion (heat exchanges between the surrounding and the system must remain the same for the two configurations).
- The prediction of the identified model rests on strong assumptions (in particular, it is better reaching the stationary behaviour during the system identification process). In general, the identified system is only valid for the time duration of the system identification process.

The deconvolution technique 1/2

$$T_m(k\,\Delta t) = \sum_{i=0}^{k} h_m\big((k-i)\,\Delta t\big)\varphi(k\,\Delta t) = \sum_{i=0}^{k} h_m(k\,\Delta t)\varphi\big((k-i)\,\Delta t\big)$$

$$\begin{bmatrix} T_0 \\ T_1 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} \varphi_0 & & & \\ \varphi_1 & \varphi_0 & & \\ \vdots & \ddots & \ddots & \\ \varphi_N & \cdots & \varphi_1 & \varphi_0 \end{bmatrix} \begin{bmatrix} h_{m0} \\ h_{m1} \\ \vdots \\ h_{mN} \end{bmatrix}$$

Assuming an additive measurement error of normal distribution (zero mean and constant standard deviation)

$$y_m(k\,\Delta t) = T_m(k\,\Delta t) + e(k\,\Delta t) = \sum_{i=0}^{k} h_m(k\,\Delta t)\varphi\big((k-i)\,\Delta t\big) + e(k\,\Delta t)$$

The deconvolution technique 2/2

$$\lim_{k\to\infty} h_k = 0$$

$$\underbrace{\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_Q \\ \vdots \\ y_N \end{bmatrix}}_{Y_N} = \underbrace{\begin{bmatrix} \varphi_0 & & & \\ \varphi_1 & \varphi_0 & & \\ \vdots & \ddots & \ddots & \\ \varphi_Q & \cdots & \varphi_1 & \varphi_0 \\ \vdots & \cdots & \vdots & \vdots \\ \varphi_N & \cdots & \varphi_{N-Q+1} & \varphi_{N-Q} \end{bmatrix}}_{\Phi_N} \underbrace{\begin{bmatrix} h_{m0} \\ h_{m1} \\ \vdots \\ h_{mQ} \end{bmatrix}}_{H_Q} + \underbrace{\begin{bmatrix} e_0 \\ e_1 \\ \vdots \\ e_Q \\ \vdots \\ e_N \end{bmatrix}}_{E_N}$$

$$\left(E_N \, E_N{}^T\right) \qquad\qquad H_Q = \left(\Phi_N \, \Phi_N{}^T\right)^{-1} \Phi_N{}^T \, Y_N$$

The correlation technique

$$y_m(t) = \int_0^{\infty} h_m(t-\tau)\varphi(\tau)\,\mathrm{d}\tau + e(t)$$

$$\int_0^{\infty} y_m(t)\varphi(t-\tau)\,\mathrm{d}t = \int_0^{\infty}\int_0^{\infty} h_m(t-\tau)\varphi(\tau)\varphi(t-\tau)\,\mathrm{d}t\,\mathrm{d}\tau + \int_0^{\infty} \varphi(t-\tau)e(t)\,\mathrm{d}t$$

$$C_{y_m\varphi}(\tau) = \int_0^{\infty} h_m(t-\tau)C_{y_m\varphi}(\tau)\,\mathrm{d}\tau + h_m(t-\tau)C_{e\varphi}(\tau)$$

$$C_{\varphi\varphi}(\tau) = \delta(\tau)$$

$$C_{y_m\varphi}(\tau) = h(\tau)$$

Spectral technique 1/2

$$\mathrm{FFT}\left[C_{y_m\varphi}(\tau)\right]=\mathrm{FFT}\left[\int_0^\infty h_m(t-\tau)C_{\varphi\varphi}(\tau)\,\mathrm{d}\tau\right]=Y_m(f)\Phi(f)=S_{y_m\varphi}(f)$$

$$\mathrm{FFT}\left[C_{\varphi\varphi}(\tau)\right]=\mathrm{FFT}\left[\int_0^\infty \varphi(t-\tau)\varphi(\tau)\,\mathrm{d}\tau\right]=\Phi(f)^2=S_{\varphi\varphi}(f)$$

$$S_{y_m\varphi}(f)=H(f)S_{\varphi\varphi}(f)+S_{\varphi e}(f)$$

$$H(f)=\frac{S_{y_m\varphi}(f)}{S_{\varphi\varphi}(f)}$$

$$\varphi_\Pi(t)=\varphi(t)\Pi_\tau(t)$$

Spectral technique 2/2

$$\Phi_{\Pi}(f) = \Phi(f) * \left(\tau \frac{\sin(\pi \tau f)}{\pi \tau f} \right)$$

$$\varphi_{\Pi}(t) = \varphi(t)\, g_{\tau}(t)$$

$$g_{\tau}(t) = 0.5\left(1 - \cos\left(\frac{2\pi t}{\tau} \right) \right)$$

The parametric approach

$$T_m(t) + \alpha_1 \frac{\mathrm{d}T_m(t)}{\mathrm{d}t} + \alpha_2 \frac{\mathrm{d}^2 T_m(t)}{\mathrm{d}t^2} + \cdots = \beta_0 \varphi(t) + \beta_1 \frac{\mathrm{d}\varphi(t)}{\mathrm{d}t} + \beta_2 \frac{\mathrm{d}^2\varphi(t)}{\mathrm{d}t^2} + \cdots$$

$$\frac{\partial T(x,t)}{\partial t} = a \frac{\partial^2 T(x,t)}{\partial x^2} \qquad 0 < x < e, t > 0$$

$$-k \frac{\partial T(x,t)}{\partial x} = \varphi(t) \qquad x = 0, t > 0$$

$$\frac{\partial T(x,t)}{\partial x} = 0 \qquad x = e, t > 0 \qquad T(x,t) = 0$$

The parametric approach

$$L\{T_m(t)\} = \theta_m(s) = \frac{1}{k\,\beta\sinh(\beta e)} L\{\varphi(t)\} = \frac{1}{k\,\beta\sinh(\beta e)}\Phi(s) \qquad \beta = \sqrt{s/a}$$

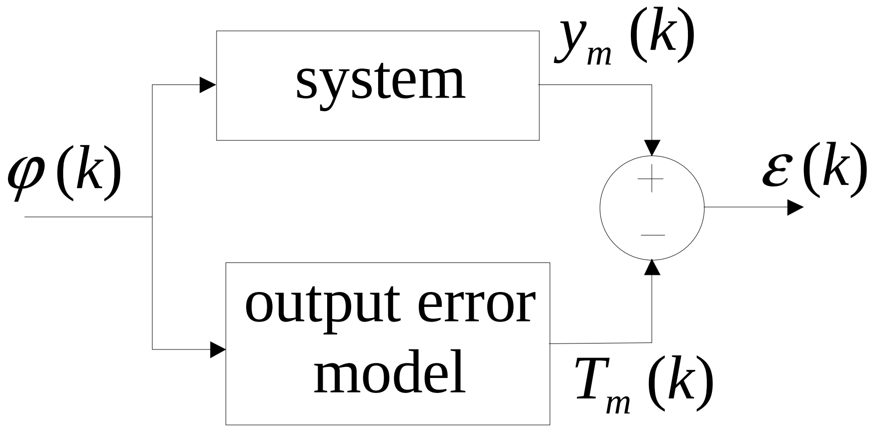
$$\sinh(z) = \sum_{n=0}^{\infty}\frac{z^{2n+1}}{(2n+1)!},\quad \forall z \geq 0 \qquad \theta_m(s) = \frac{1}{k\,\beta\displaystyle\sum_{n=0}^{\infty}\frac{(\beta e)^{2n+1}}{(2n+1)!}}\Phi(s) = \frac{1}{k\displaystyle\sum_{n=0}^{\infty}\frac{e^{2n+1}s^{n+1}}{a^{n+1}(2n+1)!}}\Phi(s)$$

$$L\left(\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n}\right) = s^n F(s) - \sum_{k=0}^{n-1} s^{n-k-1}\frac{\mathrm{d}^k f(0)}{\mathrm{d}t^k}$$

$$\sum_{n=0}^{\infty}\alpha_n \frac{\mathrm{d}^{n+1}T_m(t)}{\mathrm{d}t} = \varphi(t)$$

Output error method

$$T_m(k) = b_0\,\varphi(k) + b_1\,\varphi(k-1) + b_2\,\varphi(k-2) + \cdots - a_1\,T_m(k-1) - a_2\,T_m(k-2) - \cdots$$



$$S_{a_i}(k) = \frac{\partial T_m(k)}{\partial a_i},\ i = 1,\ldots,n \qquad S_{b_i}(k) = \frac{\partial T_m(k)}{\partial b_i},\ i = 0,\ldots,n$$

$$S_{a_i}(k) + a_1\,S_{a_i}(k-1) + \cdots + a_n\,S_{a_i}(k-n) = -T_m(k-i),\ i = 1,\ldots,n$$

Output error method

$$S_{a_i}(k)+a_1 S_{a_i}(k-1)+\cdots+a_n S_{a_i}(k-n)=-T_m(k-i),\ i=1,\ldots,n$$

$$S_{a_i}(0)=S_{a_i}(1)=\cdots=S_{a_i}(n-1)=0$$

$$S_{b_i}(0)=S_{b_i}(1)=\cdots=S_{b_i}(n-1)=0$$

$$b_0 S_{b_i}(k)+b_1 S_{b_i}(k-1)+\cdots+b_n S_{b_i}(k-n)=\varphi(k-i),\ i=0,\ldots,n$$

$$\varepsilon(k)=y_m(k)-T_m(k)=\sum_{i=1}^{n} S_{a_i}(k)\Delta a_i+\sum_{i=0}^{n} S_{b_i}(k)\Delta b_i$$

$$\mathbf{E}=\begin{bmatrix}\varepsilon(n)\\ \varepsilon(n+1)\\ \vdots\\ \varepsilon(N)\end{bmatrix}=\mathbf{S}\begin{bmatrix}\Delta a_1\\ \vdots\\ \Delta a_n\\ \Delta b_0\\ \vdots\\ \Delta b_n\end{bmatrix}=\mathbf{S}\Delta\Theta$$

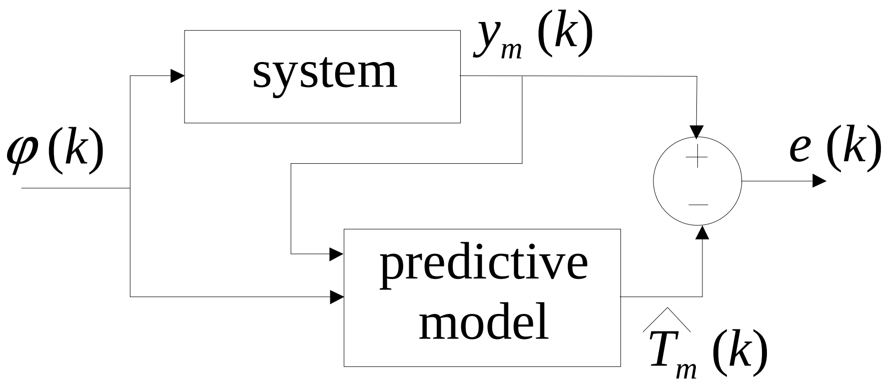
$$\mathbf{S}=\begin{bmatrix}S_{a_1}(n) & \cdots & S_{a_n}(n) & S_{b_0}(n) & \cdots & S_{b_n}(n)\\ \vdots & & \vdots & \vdots & & \vdots\\ S_{a_1}(N) & \cdots & S_{a_n}(N) & S_{b_0}(N) & \cdots & S_{b_n}(N)\end{bmatrix}$$

$$\Theta_\nu=\Theta_{\nu-1}+\Delta\Theta_{\nu-1}$$

$$\Delta\Theta=\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T\mathbf{E}$$

Predicton error method

$$\hat{T}_m(k) = b_0\varphi(k) + b_1\varphi(k-1) + b_2\varphi(k-2) + \cdots - a_1 y_m(k-1) - a_2 y_m(k-2) - \cdots$$



$$y_m(k) = \mathbf{H}(k)\Theta + e(k)$$

$$\Theta^T = [a_1 \quad \cdots \quad a_n \quad b_0 \quad \cdots \quad b_n]$$

$$\mathbf{H}(k) = [-y_m(k-1) \quad \cdots \quad -y_m(k-n) \quad \varphi(k) \quad \cdots \quad \varphi(k-n)]$$

Predicton error method

$$\mathbf{Y}_N = \Psi_N \; \Theta + \mathbf{E}_N$$

$$\mathbf{Y}_N{}^T = \left[y_m(n) \quad \cdots \quad y_m(N+n) \right]$$

$$\Psi_N{}^T = \left[\mathbf{H}(n) \quad \cdots \quad \mathbf{H}(N+n) \right]$$

$$\mathbf{E}_N{}^T = \left[e(n) \quad \cdots \quad e(N+n) \right]$$

$$\hat{\Theta} = \left(\Psi_N \, \Psi_N{}^T \right)^{-1} \Psi_N{}^T \, \mathbf{Y}_N$$

$$\hat{\Theta} = \Theta + \left(\Psi_N \, \Psi_N{}^T \right)^{-1} \Psi_N{}^T \, \mathbf{E}_N$$

$$E\left\{ \hat{\Theta} \right\} = \Theta + \left(E\left\{ \mathbf{H}(k) \mathbf{H}(k)^T \right\} \right)^{-1} E\left\{ \mathbf{H}(k)^T e(k) \right\}$$

It thus appears that if $e(k)$ is correlated with $\mathbf{H}(k)$ or if $E\left\{ e(k) \right\}$ in not zero, the estimation is biased and $E\left\{ \hat{\Theta} \right\} \neq \Theta$.

The use of recursivity

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \mathbf{L}(k)\left[y_m(k) - \mathbf{H}(k)\hat{\Theta}(k-1) \right]$$

$$\mathbf{L}(k) = \frac{\mathbf{P}(k-1)\mathbf{H}(k)^T}{\lambda(k) + \mathbf{H}(k)\mathbf{P}(k-1)\mathbf{H}(k)^T}$$

$$\mathbf{P}(k) = \mathbf{P}(k-1) - \frac{\mathbf{P}(k-1)\mathbf{H}(k)^T \mathbf{H}(k)\mathbf{P}(k-1)}{\lambda(k) + \mathbf{H}(k)\mathbf{P}(k-1)\mathbf{H}(k)^T}$$

$$\hat{\Theta}(0) = \mathbf{0}_D \qquad \mathbf{P}(0) = 10^6 \mathbf{I}_D$$

Limit of the d^n/dt^n models

$$L\{T_m(t)\} = \theta_m(s) = \frac{\cosh(\beta e)}{k\beta\sinh(\beta e)} L\{\varphi(t)\} = \frac{\cosh(\beta e)}{k\beta\sinh(\beta e)} \Phi(s)$$

$$\beta = \sqrt{s/a}$$

$$\cosh(z) = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} \quad \text{and} \quad \sinh(z) = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}, \quad \forall z$$

$$\theta_m(s) = \frac{\displaystyle\sum_{n=0}^{\infty} \frac{(\beta e)^{2n}}{(2n)!}}{\displaystyle k\beta \sum_{n=0}^{\infty} \frac{(\beta e)^{2n+1}}{(2n+1)!}} \Phi(s) = \frac{\displaystyle\sum_{n=0}^{\infty} \frac{e^{2n} s^n}{a^n (2n)!}}{\displaystyle k \sum_{n=0}^{\infty} \frac{e^{2n+1} s^{n+1}}{a^{n+1} (2n+1)!}} \Phi(s)$$

$$\alpha_n = k \frac{e^{2n+1}}{a^{n+1}(2n+1)!}$$

$$\sum_{n=0}^{\infty} \alpha_n s^{n+1} \theta_m(s) = \sum_{n=0}^{\infty} \beta_n s^n \Phi(s)$$

$$\beta_n = \frac{e^{2n}}{a^n (2n)!}$$

$$\sum_{n=0}^{\infty} \alpha_n \frac{\mathrm{d}^{n+1} T_m(t)}{\mathrm{d}t} = \sum_{n=0}^{\infty} \beta_n \frac{\mathrm{d}^n \varphi(t)}{\mathrm{d}t}$$

Asymptotic behaviours

$$\lim_{s\to\infty} \frac{\cosh\left(e\sqrt{s}/\sqrt{a}\right)}{k\,e\sqrt{s}/\sqrt{a}\,\sinh\left(e\sqrt{s}/\sqrt{a}\right)} = \frac{1}{k/\sqrt{a}\,\sqrt{s}}$$

$$\lim_{s\to\infty} \frac{\sum_{n=0}^{\infty}\beta_n\, s^n}{\sum_{n=0}^{\infty}\alpha_n\, s^{n+1}} = \frac{\beta_n\, s^n}{\alpha_n\, s^{n+1}} = \frac{2n+1}{ek/a}\frac{1}{s}$$

The use of dα/dtα

$$L\left(\frac{\mathrm{d}^{\nu} f(t)}{\mathrm{d}t^{\nu}}\right) = s^{\nu} F(s) - \sum_{k=0}^{n-1} s^{n-k-1} \frac{\mathrm{d}^{\nu} f(0)}{\mathrm{d}t^{\nu}}$$

$$\mathrm{D}^{\nu}\{f(t)\} = \mathrm{D}^{n} \quad \left\{\mathrm{I}^{n-\nu}\{f(t)\}\right\} \qquad n \in \mathrm{N},\ \mathrm{Re}(\nu) > 0, \quad n-1 \leq \mathrm{Re}(\nu) < n$$

$$\mathrm{I}^{\nu}\{f(t)\} = \frac{1}{\Gamma(\nu)} \int_{0}^{t} (t-u)^{\nu-1} f(u)\,\mathrm{d}u$$

$$\Gamma(\nu) = \int_{0}^{\infty} u^{\nu-1} \exp(-u)\,du$$

$$L^{-1}\left\{\frac{1}{k/\sqrt{a}} \frac{1}{\sqrt{s}} \Phi(s)\right\} = \frac{1}{k/\sqrt{a}} \mathrm{I}^{1/2}\{\varphi(t)\}$$

$$\sum_{n=0}^{\infty} \alpha_n \mathrm{D}^{n/2}\{T_m(t)\} = \sum_{n=0}^{\infty} \beta_n \mathrm{D}^{n/2}\{\varphi(t)\}$$

explanation

$$\cosh(z) = \frac{e^{z} + e^{-z}}{2} = \frac{e^{-z}\left(1 + e^{2z}\right)}{2}$$

$$\sinh(z) = \frac{e^{z} - e^{-z}}{2} = \frac{e^{-z}\left(-1 + e^{2z}\right)}{2}$$

$$\theta_m(s) = \frac{e^{2\beta e}}{k\,\beta\left(e^{2\beta e} - 1\right)}\Phi(s)$$

$$e^{z} = \sum_{n=0}^{\infty} \frac{z^n}{n!}, \quad \forall z$$

$$\theta_m(s) = \frac{\sum_{n=0}^{\infty} \beta'_n\, s^{n/2}}{\sum_{n=0}^{\infty} \alpha'_n\, s^{(n+1)/2}}\Phi(s)$$

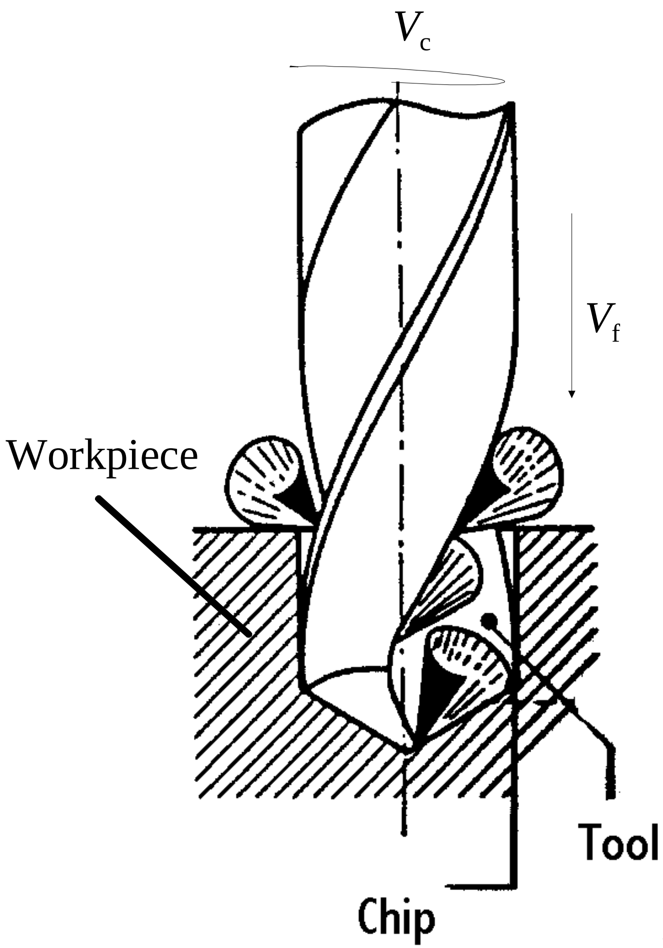
$$\alpha'_n = \frac{k\, e^{n-1}}{a^{(n-1)/2}\, n!} \qquad \textit{and} \qquad \beta'_n = \frac{e^{n-1}}{a^{(n-2)/2}\, n!}$$

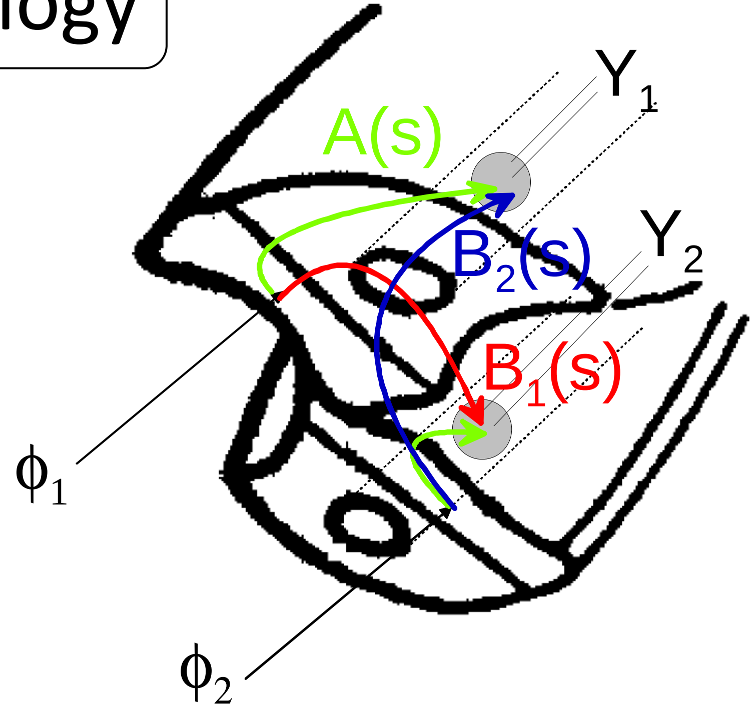
$$\sum_{n=0}^{\infty} \alpha'_n\, \mathrm{D}^{(n+1)/2}\left\{T_m(t)\right\} = \sum_{n=0}^{\infty} \beta'_n\, \mathrm{D}^{n/2}\left\{\varphi(t)\right\}$$

Heat flux estimation in machining

Heat flux estimation in fast drilling process,
Tool coating influence

Drilling: the phenomenology

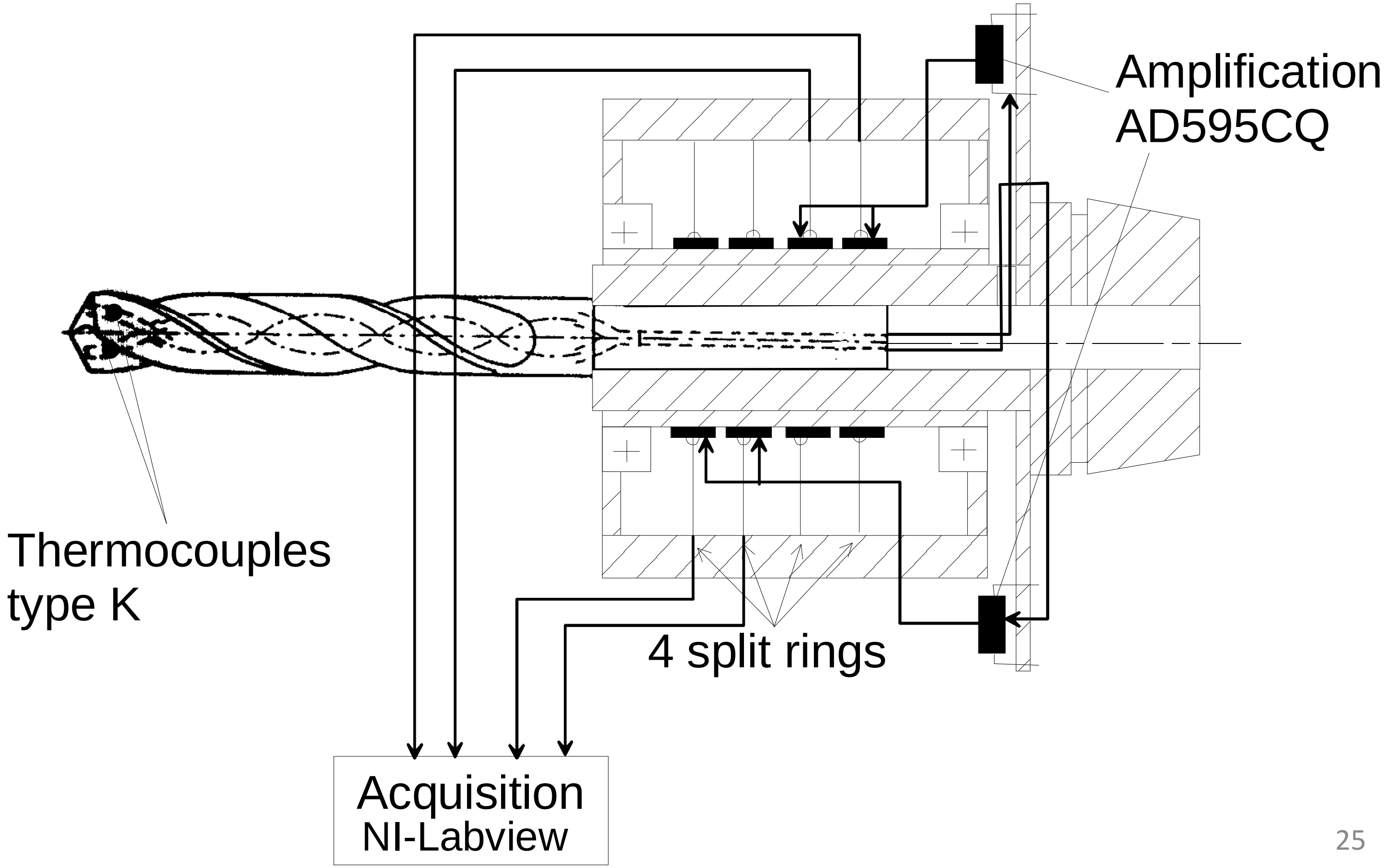




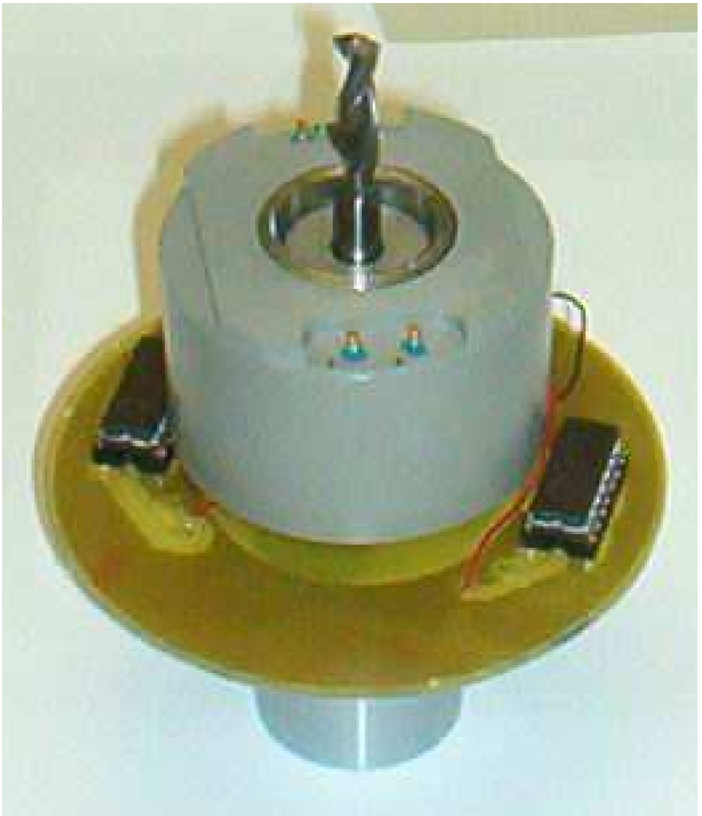
$$\begin{bmatrix} \overline{T}_1(s) \\ \overline{T}_1(s) \end{bmatrix} = \underbrace{\begin{bmatrix} F_{1,1}(s) & F_{1,2}(s) \\ F_{2,1}(s) & F_{2,2}(s) \end{bmatrix}}_{\mathbf{F}(s)} \begin{bmatrix} \overline{\phi}_1(s) \\ \overline{\phi}_2(s) \end{bmatrix}$$

Multivariable system : two sensors and two heat fluxes.

Temperature measurement in the drill



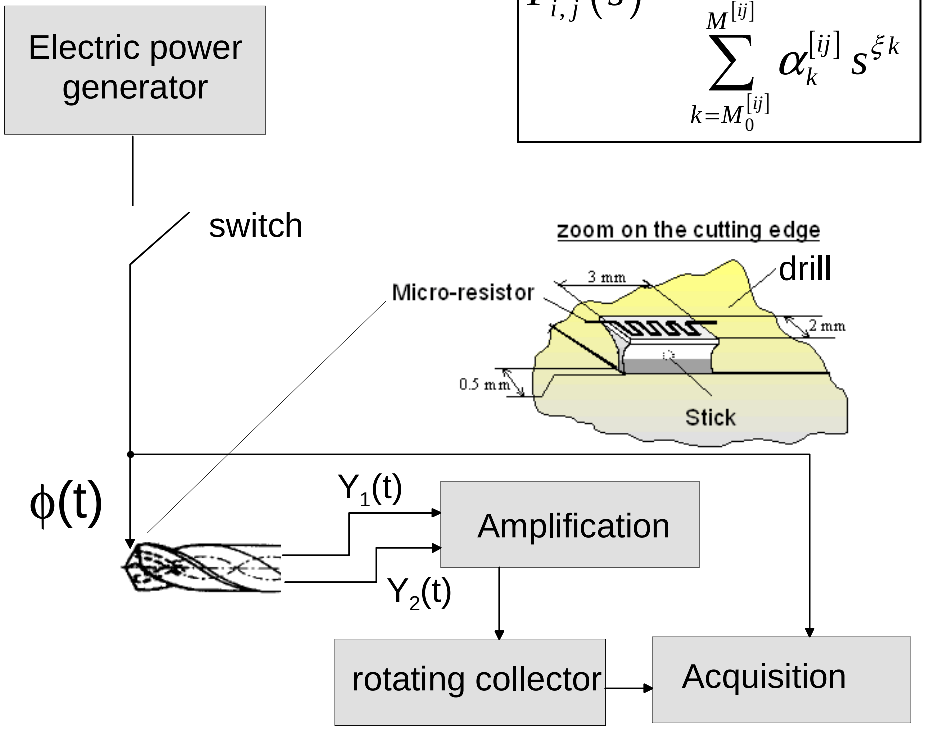
The complete device

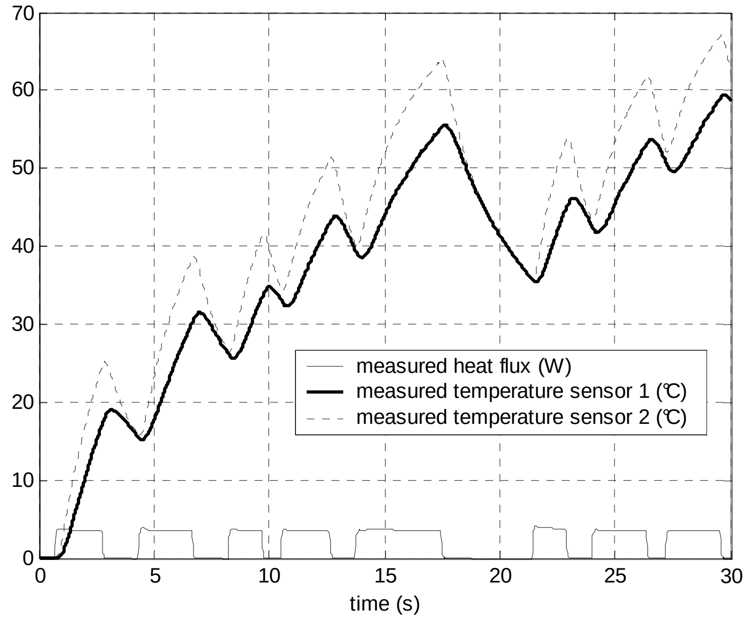


Non integer system identification

$$F_{i,j}(s) = \frac{\displaystyle\sum_{k=L_0^{[ij]}}^{L^{[ij]}} \beta_k^{[ij]}\, s^{\xi k}}{\displaystyle\sum_{k=M_0^{[ij]}}^{M^{[ij]}} \alpha_k^{[ij]}\, s^{\xi k}}$$

$$\alpha_{M_0}^{[ij]} = 1 \qquad \xi = \frac{1}{2}$$

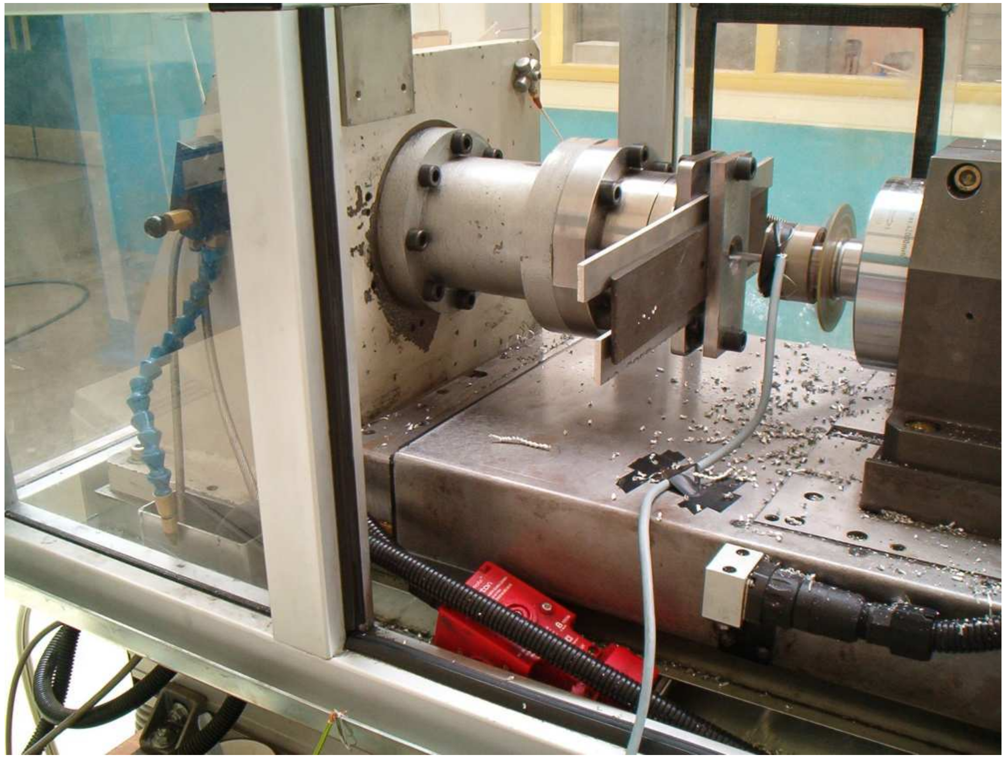




Application: Aluminum machining

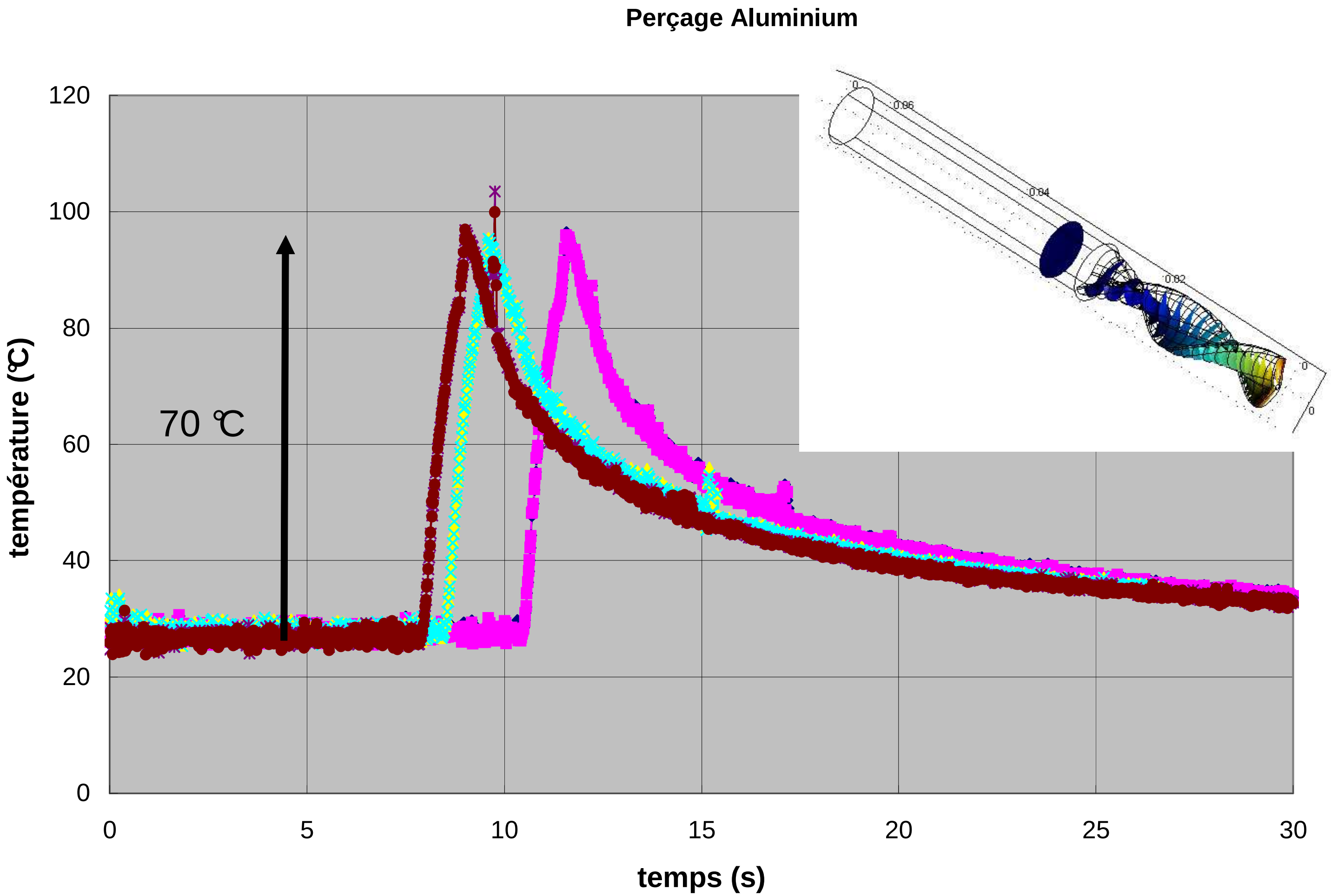
Industrial partners:
DASSAULT, CETIM, SLCA

Project MEDOC
ANR CONTROLTHER

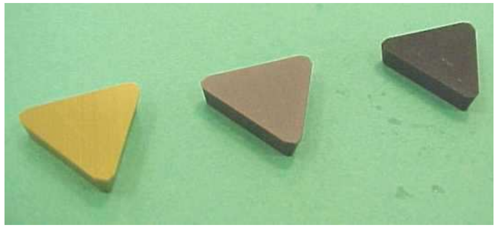


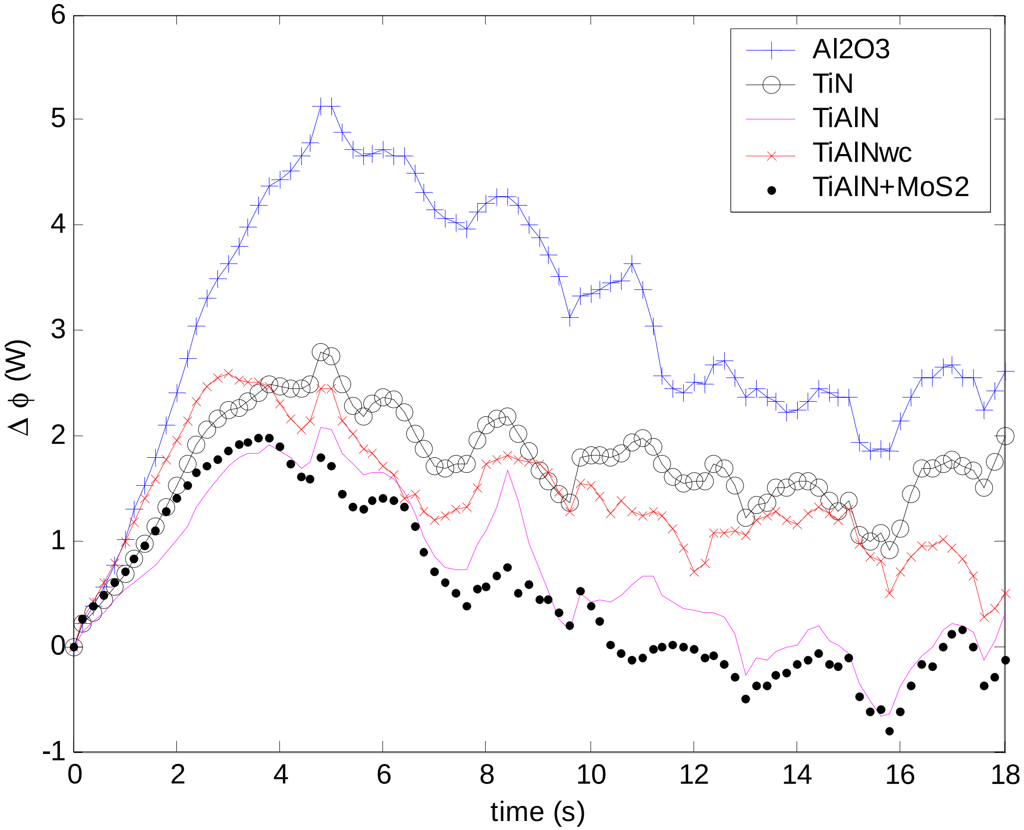
10000 tr mn^{-1}

Aluminium drilling



Comparison of the performances of the various coatings





Raising operation of the surface of a disc from the periphery towards the center.
One represents the difference between the flux estimated for the tool without coating and that for the covered tool.

$$\Delta\phi = \phi_{nr} - \phi_{rev}$$

Heat flux estimation in severe thermal conditions

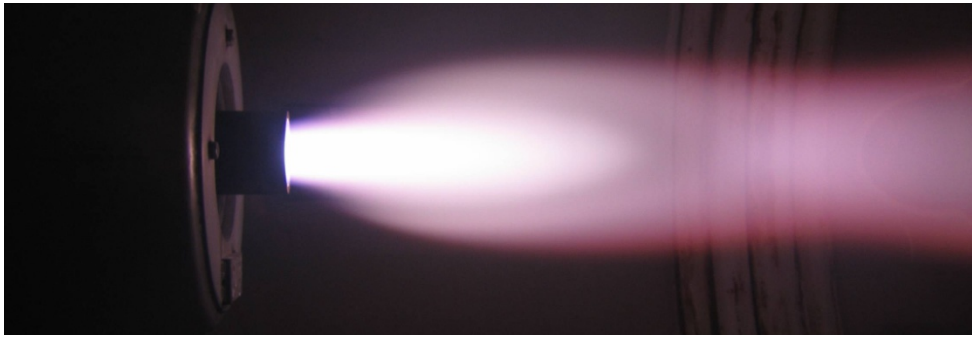
Heat flux estimation in a plasma tunnel

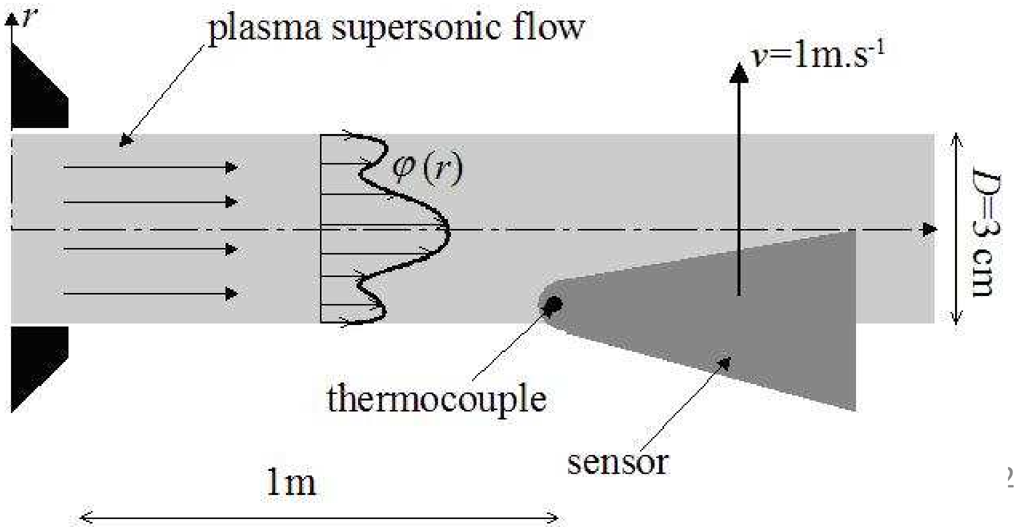
Estimation de flux dans un jet supersonique de plasma

Industrial partners: EADS-ASTRIUM, CEA-CESTA

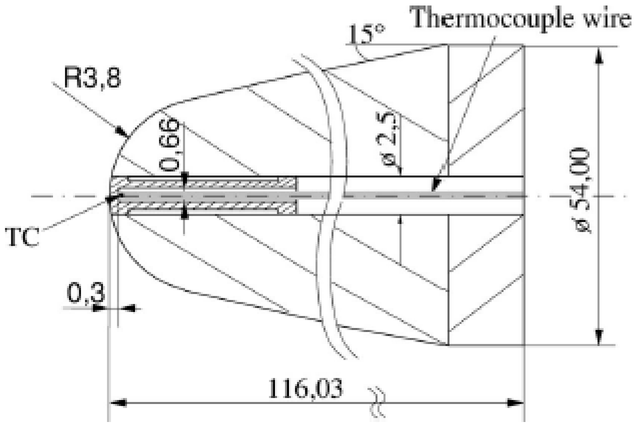
Applications :

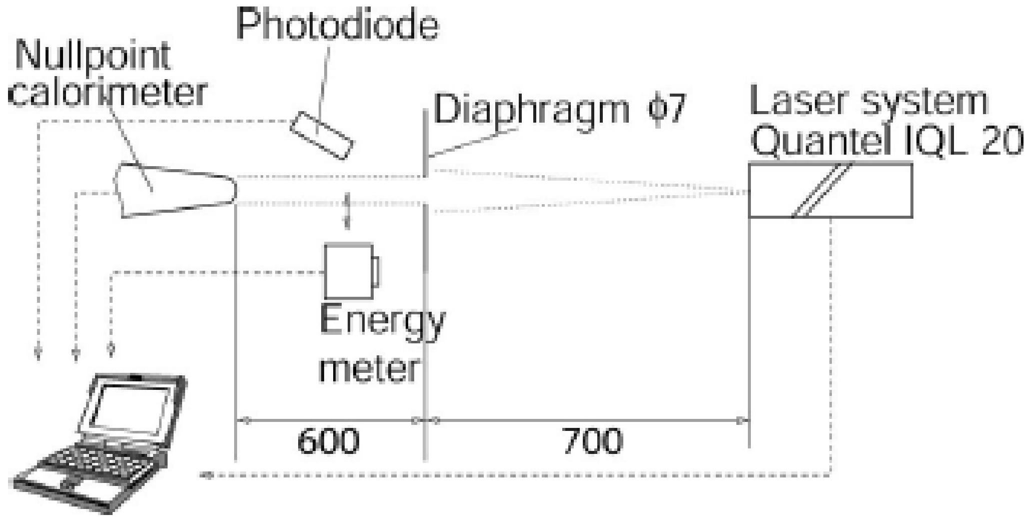
- re-entry simulation
- Degradation of materials under high heat flux density and high temperature



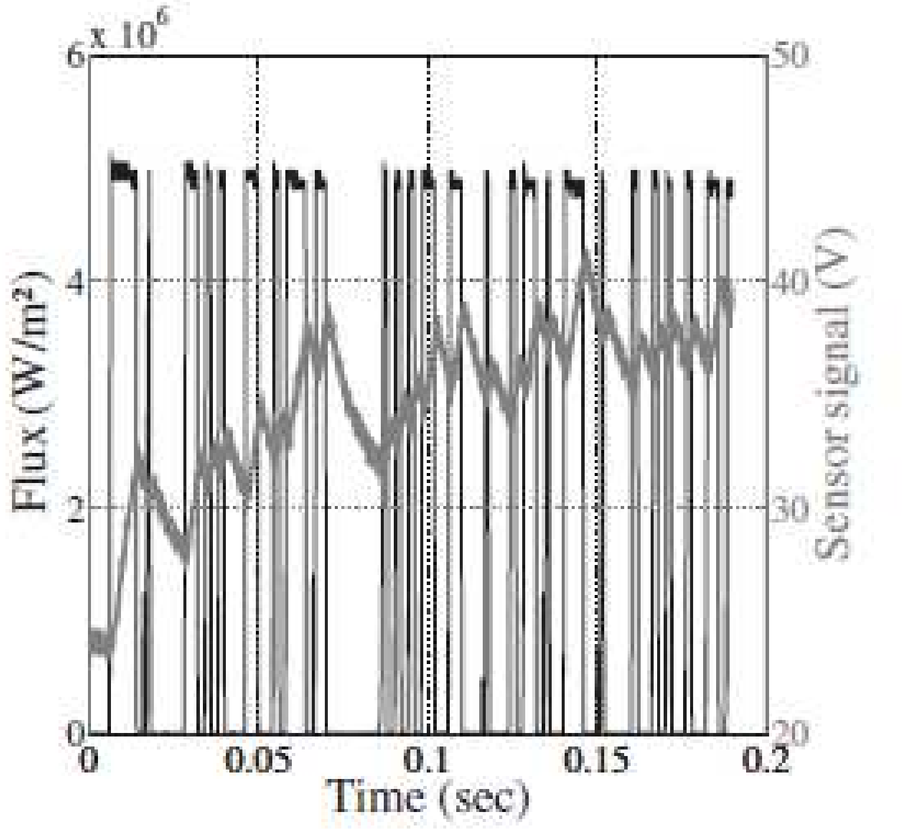


System identification

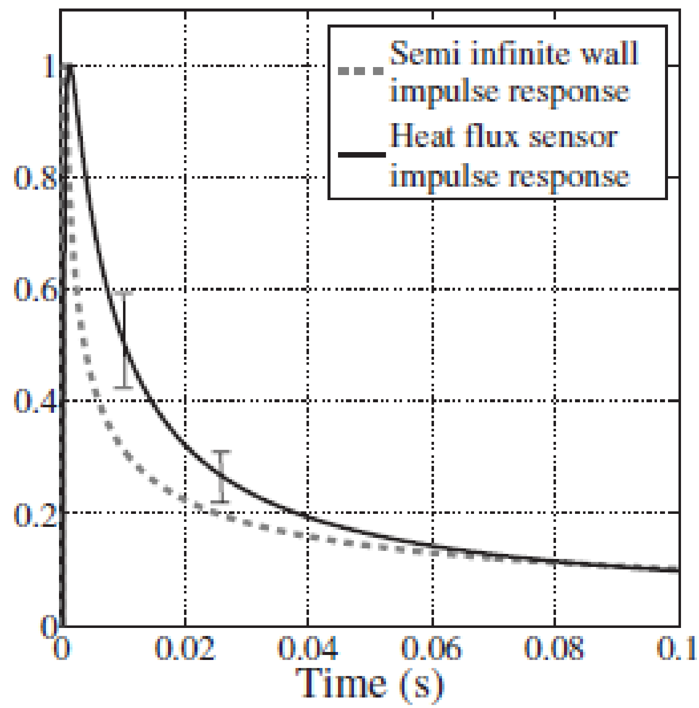




System identification



$$[\alpha_0 D^{-1} + \alpha_1 D^{-0.5} + \alpha_2]V(t) = [\beta_0 D^{-1.5} + \beta_1 D^{-1} + \beta_2 D^{-0.5}]\varphi(t).$$



The « pen » sensor