Time/Space noise and « thermal » processing of temperature signal

J.C. Batsale, C. Pradere

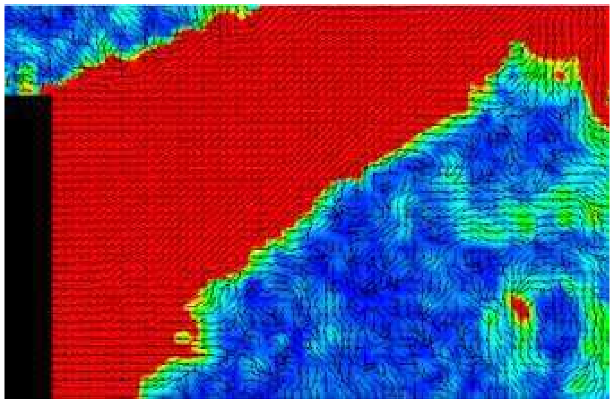
Lecture 6

Introduction

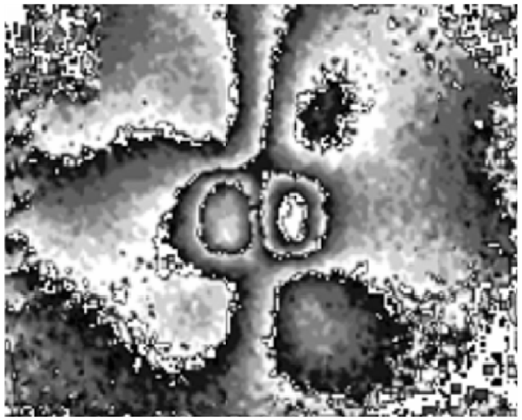
- **The processing of space and time temperature fields is more and more necessary** (not only in heat transfer but also in all domains related to continuous media such as solid or fluid mechanics…)
- **Simple devices are now currently available** in order to quickly measure, store and process thermal information (Infrared thermography , optical or mechanical scans, …)
- **"how to process" and "how to estimate" thermophysical properties from a great amount of thermal data, such as temperature fields?**
  - Difficulties occurring with such instruments (noise and signal perturbation, systematic errors…)
  - Difficulties related to the manipulation of a great amount of data and the suitable processing of such data

The processing of space and time temperature fields is more and more necessary

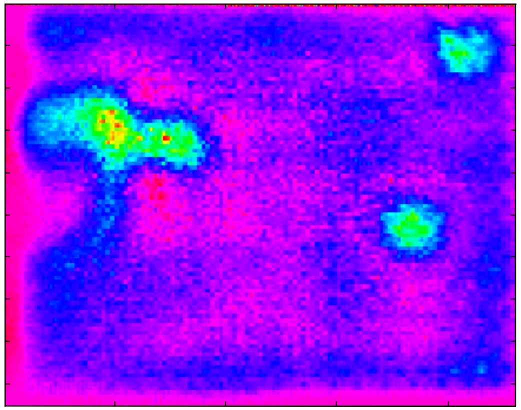
- Velocity , strain or temperature fields, are any more experimentally accessible for **Continuum Mechanics**…
- In a near future, several different fileds will be simultaneously recorded and processed.



PIV velocity fields



Shearography: strain fields



Thermography: temperature fields

For thermal analysis, simple devices are now currently available!

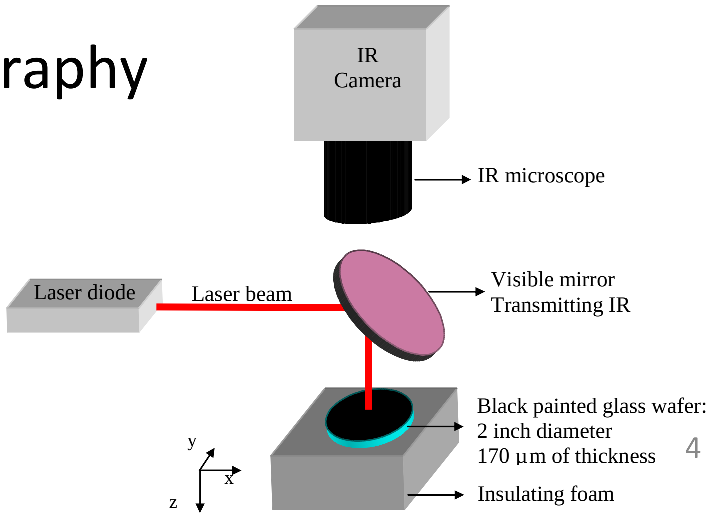
Irisys

FLIR-Indigo-A10

FLIR-CEDIP Orion, Titanium…

With a lot of active heating possibilities (Laser, flash lamps, acoustic or electromagnetic sources….)

- Scanners and in the future …tomography

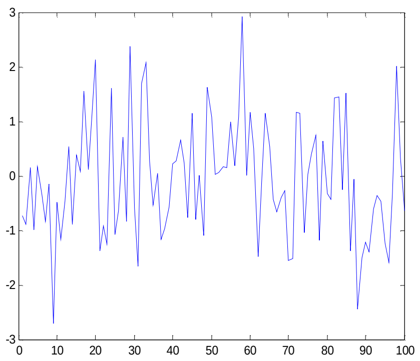


“How to process” and “how to estimate” thermophysical properties from a great amount of thermal data, such as temperature fields?

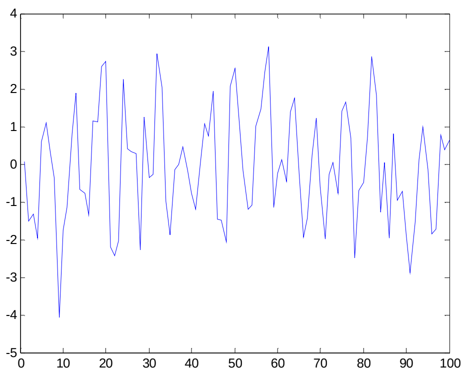
- **Part1**-Difficulties occurring with **the instruments** (noise and signal perturbation, systematic errors, filters, resolution…)
- **Part2-**Difficulties related to the manipulation of a **great amount of data** and the suitable processing of such data, by **considering a heat transfer model.**

1-1 Noise characterization

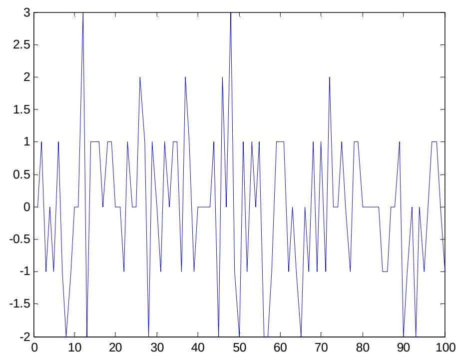
1-1-1Monosensor stationnary signal-simple observation



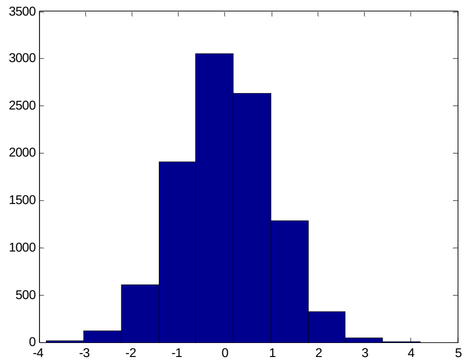
Perfectly gaussian noise



Correlated signal (parasitic periodic noise superposed to the signal)



Digitized noise



histogram

All of them have the same rough statistical characteristics (zero mean value, standard deviation), oher ways to study such signal?

Studies with Linear least squares theorem

$$T = X\beta$$

<u>Hypothesis :</u>

-zero mean and additive errors

$-\beta$ constant and unknown before the estimation and X_{ij} known without error

-constant variance (σ known) and uncorrelated errors

$\beta_{optimum}$ minimize the sum squares function S between theory and experiment

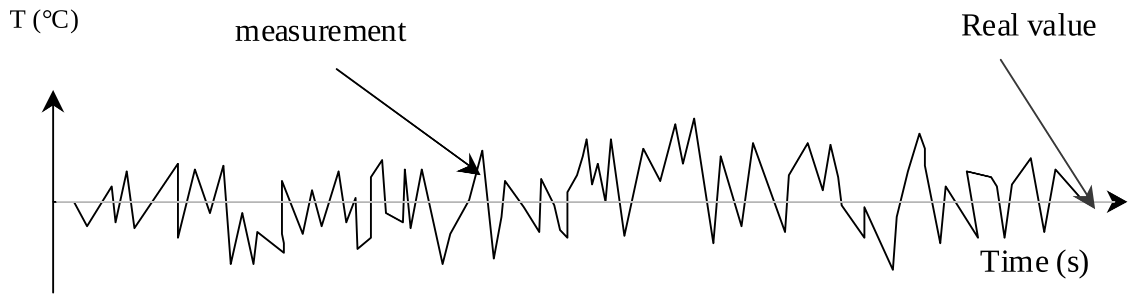
Estimator

$$\hat{\beta} = (X^t X)^{-1} X^t \hat{T}$$

Estimation error

$$\mathrm{cov}(e_{\beta}) = (X^t X)^{-1} \sigma^2$$

Example : Stationary Signal -Estimation of the mean value



$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ 1 \end{bmatrix} \beta \qquad \hat{\beta} = \frac{\sum_{i=1}^{N} \hat{T}_i}{N} \qquad \sigma_{\beta} = \sigma / \sqrt{N}$$

The processing of a great amount *N* of noisy and stationary data improves the accuracy of the estimation.

Example :Estimation of several parameters from the previous signal

Case where *f* and *g* are orthogonal

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} f_1 & g_1 \\ f_2 & g_2 \\ \vdots & \vdots \\ \vdots & \vdots \\ f_N & g_N \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

$$\hat{\beta}_1 = \frac{\left(\boldsymbol{f}^t \cdot \hat{\boldsymbol{T}}\right)}{\left(\boldsymbol{f}^t \cdot \boldsymbol{f}\right)} \qquad \hat{\beta}_2 = \frac{\left(\boldsymbol{g}^t \cdot \hat{\boldsymbol{T}}\right)}{\left(\boldsymbol{g}^t \cdot \boldsymbol{g}\right)}$$

$$\operatorname{cov}\left(\hat{\boldsymbol{B}}\right) = \sigma^2 \begin{pmatrix} \left(\boldsymbol{f}^t \boldsymbol{f}\right)^{-1} & 0 \\ 0 & \left(\boldsymbol{g}^t \boldsymbol{g}\right)^{-1} \end{pmatrix}$$

$$\operatorname{cov}\left(\hat{\boldsymbol{B}}\right) \approx \sigma^2 \left(\frac{N}{t_{\max}}\right)^{-1} \begin{pmatrix} \|f\|^{-2} & 0 \\ 0 & \|g\|^{-2} \end{pmatrix} \qquad \operatorname{cond}\left(\operatorname{cov}\left(\hat{\boldsymbol{B}}\right)\right) \approx \frac{\|f\|^2}{\|g\|^2}$$

- T_i regularly spaced, *N* must be chosen <u>as great as possible!</u>

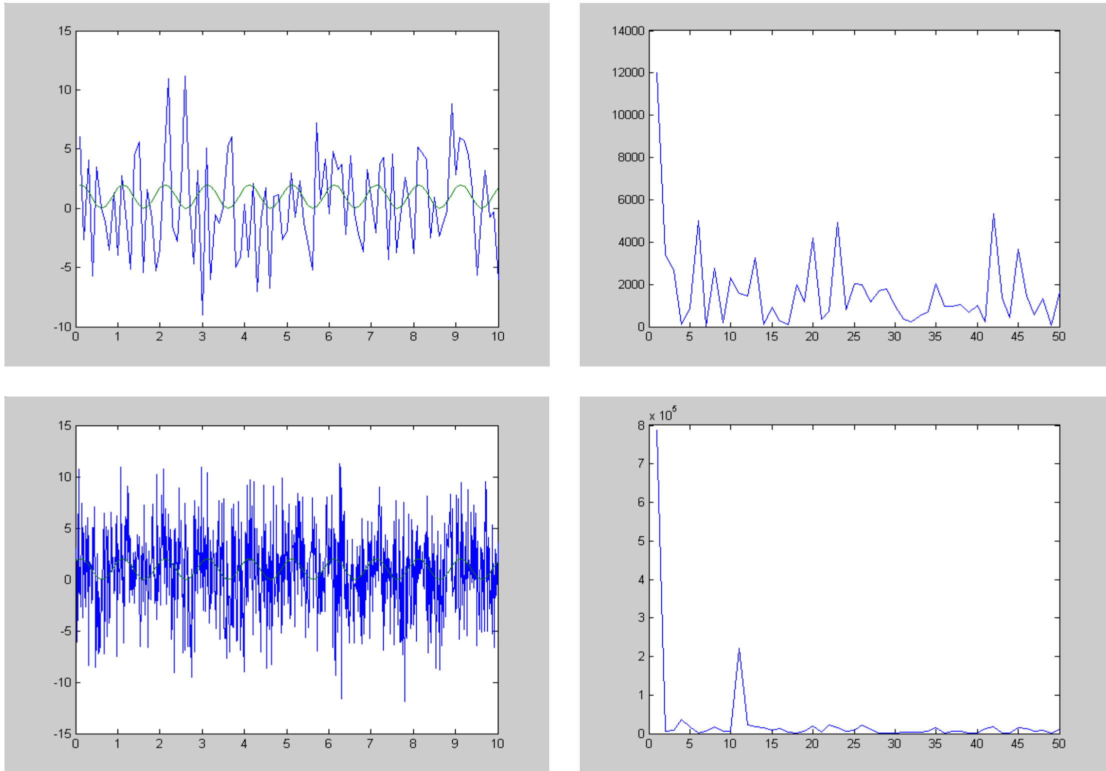
-The conditioning number is <u>non-dependant on *N*</u> *!*

Example: T=B1+B2*sin(wt+phi)+NOISE

Signal

Power Spectral density

fft(T).*conj(fft(T))



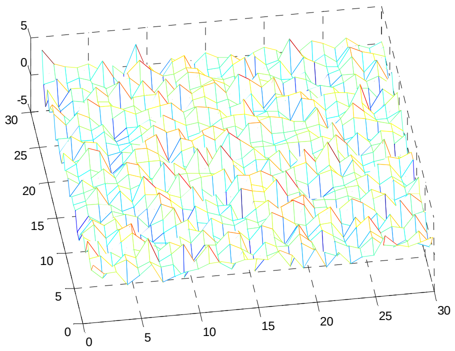
N=100

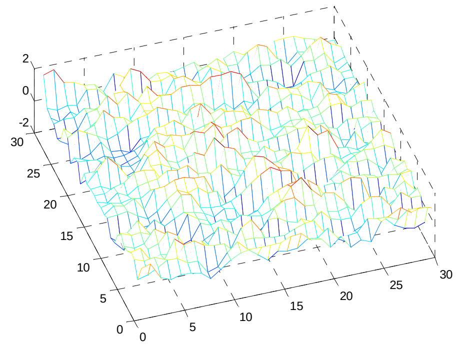
N=800

-Even if the signal is noisy, it is advantageous to process a great amount of data

- The function: *sin(wt)* is « orthogonal » to the function: *f(t)=1*

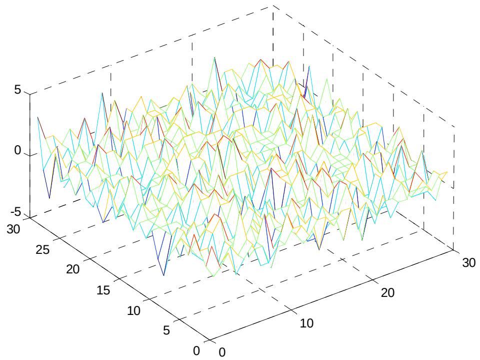
1-1-2 Sensor array- stationnary observation

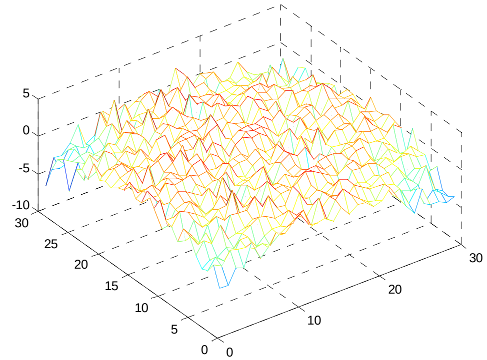




Random

Space correlation





Digitized noise

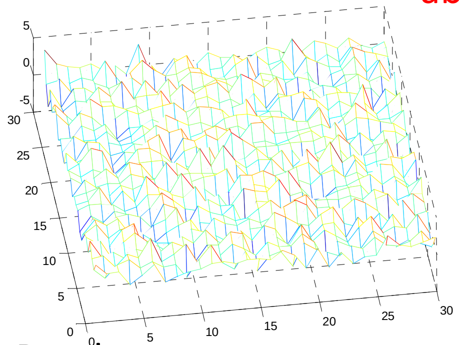
Non-uniformity distortion

When a signal is multidimensional, instead of projecting on an orthogonal basis, it is possible to set out a singular value decomposition (SVD).

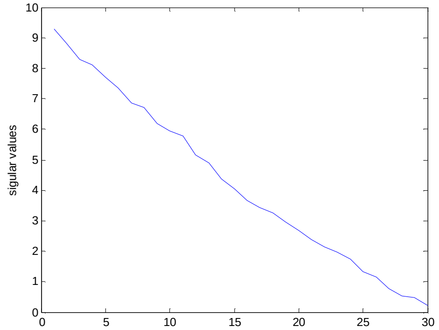
SVD decomposition of the previous images

The Singular values are giving an idea about the « complexity » of the images

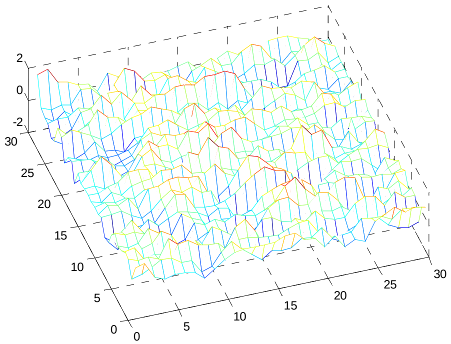
$$\hat{\mathbf{T}} = \mathbf{U}_{nxn}\Sigma_{nxn}\mathbf{V}_{nxm}{}^{T}$$



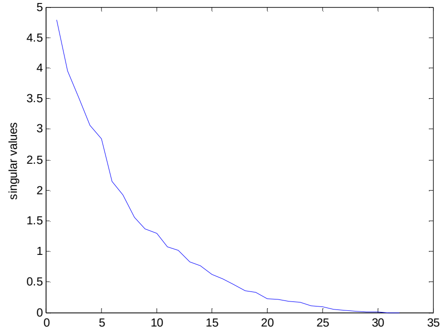
Random



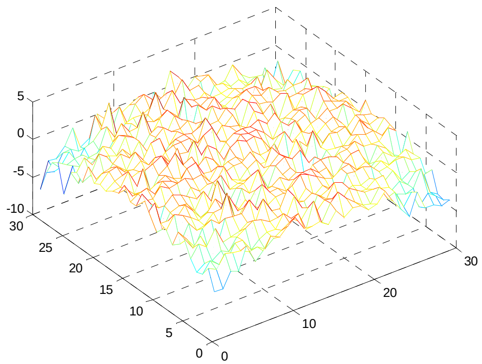
Uniform singular values distribution (non-compressible signal)



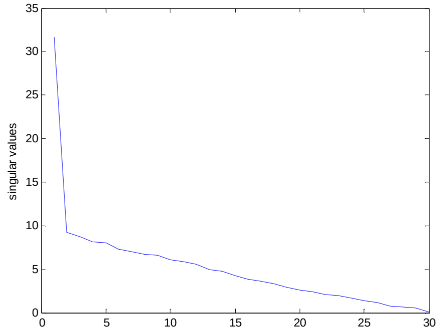
Space correlation



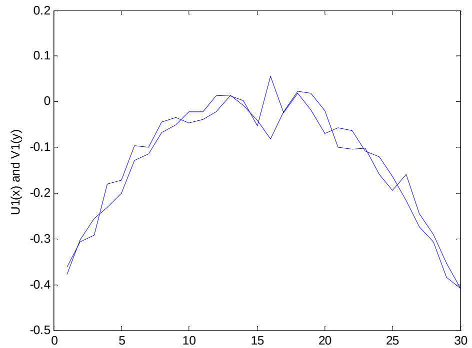
High order zero singular values (compressible signal)



Non-uniformity distortion



Σ values

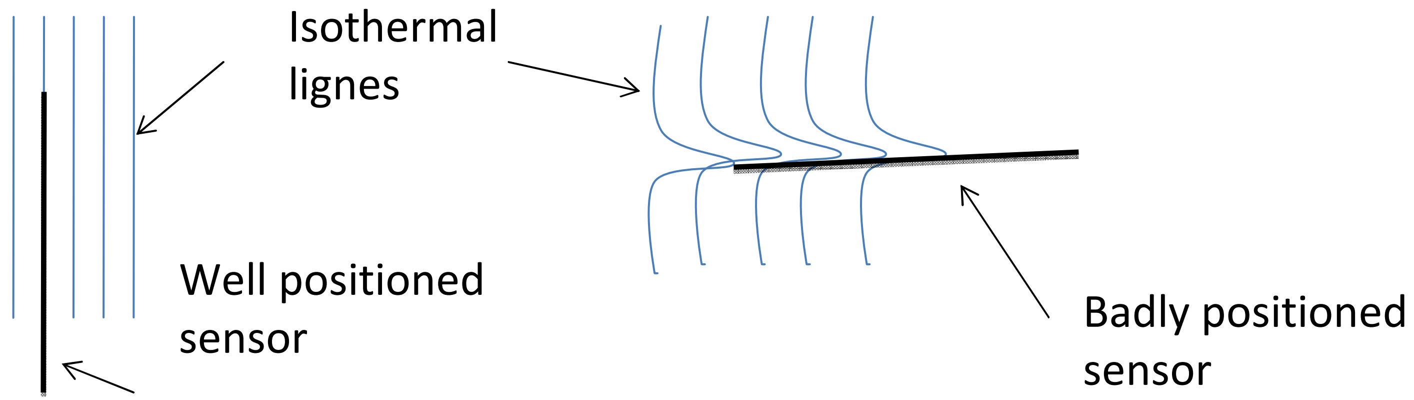


U1(x) and V1(y)

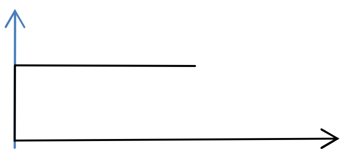
1-2 Systematic errors

1-2-1 Monosensor (thermocouple, resistor, pyrometer…)

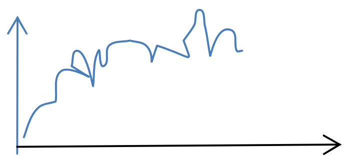
- In the best case, the sensor is measuring the « temperature of the sensor »! (see Bourouga and Bardon, 2000), several illustrations:
  - **Perturbation of the Isothermal lines and Position error**



  - **Inertia of the sensor**



Real temperature evolution



Observed signal
(noisy but also delayed)

B. Bourouga, V. Goizet, J.-P. Bardon, Les aspects théoriques régissant l'instrumentation d'un capteur thermique pariétal à faible inertie, International Journal of Thermal Sciences 39 (1) (January 2000) 96–109.

1-2-1 Systematic error: inertia of a thermocouple

$$Y(t) = \frac{K}{\rho c L} \int_0^t \exp\left(-\frac{h}{\rho c L} \tau \right) U(t-\tau) d\tau$$

h: exchange coefficient
ρcL: heat capacity

$$Y_i = \sum_{j=1}^{i} H_{i-j} U_j \Delta t$$

U(t): real temperature behaviour
Y(t): observed behaviour

$$\begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_m \end{bmatrix} = \Delta t \begin{bmatrix} H_1 & 0 & & & 0 \\ H_2 & H_1 & 0 & & \\ H_3 & H_2 & H_1 & & \\ . & . & & . & 0 \\ H_m & H_{m-1} & & H_2 & H_1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ . \\ . \\ U_m \end{bmatrix}$$

The observable signal *Y(t)* is a correlated signal!
Even if *N* observations are made, only *less than N informations* are really available.

1-2-2 Systematic errors with thermographic sensor arrays

Calibration emission and reflexion, with infrared thermography

-Radiative balance between the sensor and the environment (proper emission, reflexion and influence of the environment) (see [3], [4]).

- Luminance function of the temperature of the surface (calibration with Planck's law)

Non uniformity correction (NUC)

A distribution of gain and offset for each pixel must be regularly re-estimated (Non Uniformity Correction).

Bad or dead Pixels

Generally, these pixels are recognized initially by the device provider and corrected by a signal averaged from the neighbouring pixels (Bad Pixel Replacement).

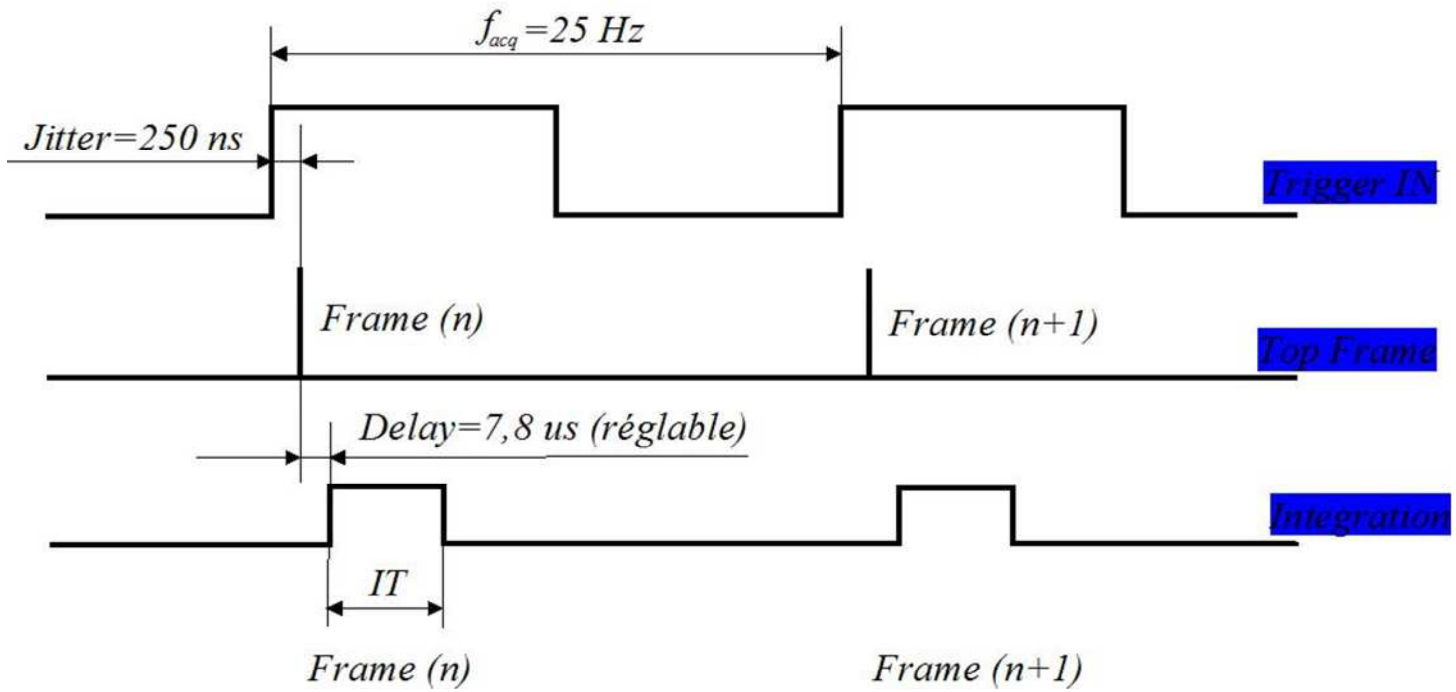
Thermal stability of the instrument

The freezing of the detector array and the thermal regulation is not always stable (1 to 5 mK).

Time recording, dead time step

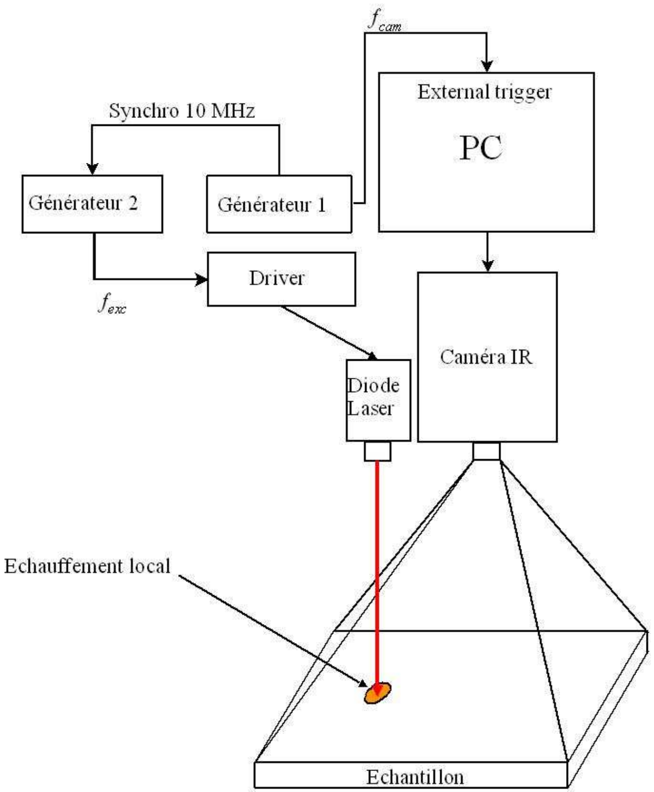
Space resolution

Time recording, dead time step



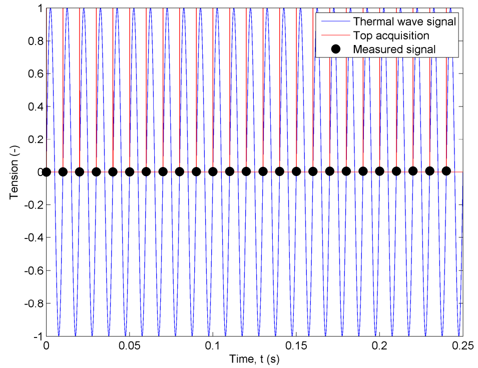
The integration time of FPA cameras with quantum detectors is generally about **100µs**. The time for electronic recording and storage is greater (about **40 ms** at 25 Hz). If an accurate triggering is possible, the heterodyne methods can be implemented.

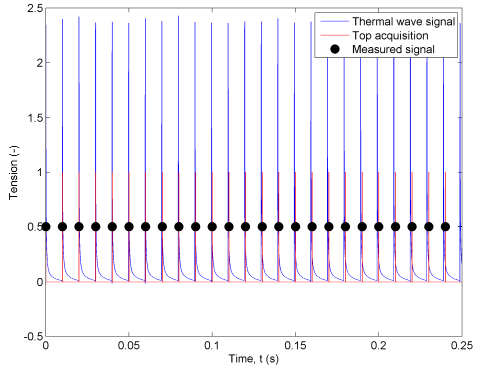
Principle of the stroboscopic effect or heterodyne technics



$$f_{exc} = k.f_{cam}$$

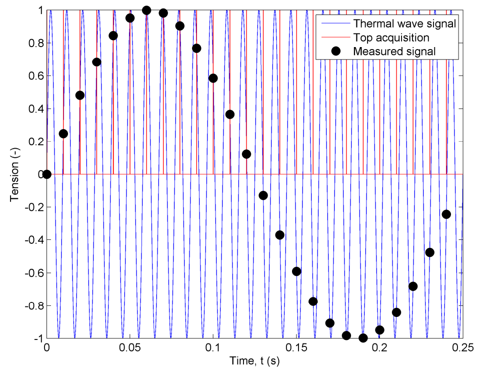
(*k integer*)

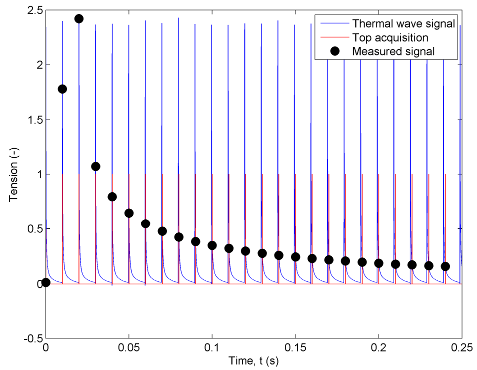




$$f_{acq} = f_{exc} / \left(k + 1/N\right)$$

(*k* and *N* integers)

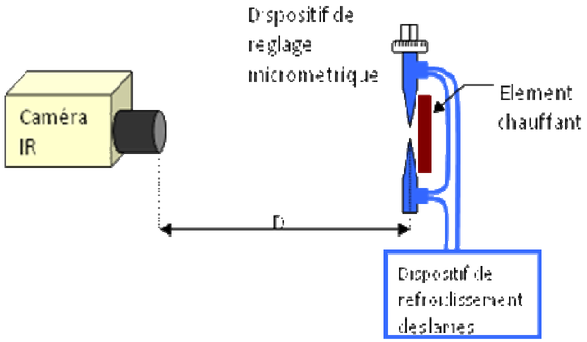


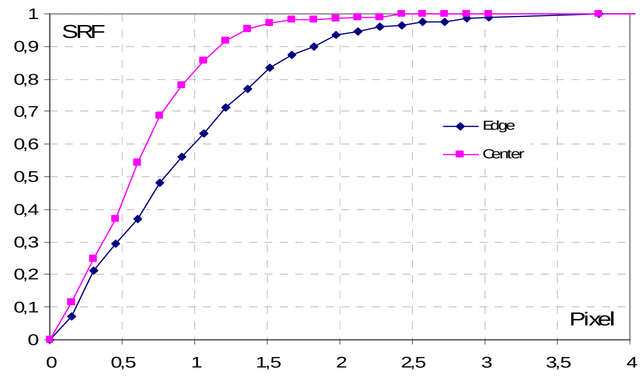


It is possible to reconstruct very fast transient periodic phenomena with a 25 Hz camera!

Space resolution

- Slit response function

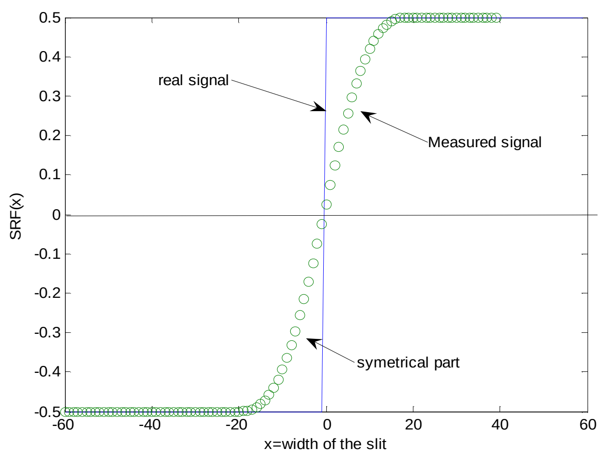




$$SRF(x) = \frac{T(x) - Tslit}{Tplate - Tslit}$$

$$Y(x) = \int_0^L p(\chi) U(x-\chi) d\chi$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_m \end{bmatrix} = \Delta x \begin{bmatrix} p_1 & p_2 & p_3 & .. & 0 \\ p_2 & p_1 & p_2 & & .. \\ p_3 & p_2 & p_1 & & \\ . & . & & . & p_2 \\ 0 & 0 & & p_2 & p_1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ . \\ . \\ U_m \end{bmatrix}$$



The pixels of an infrared camera are generally correlated (**N pixels, but less than N informations**)!

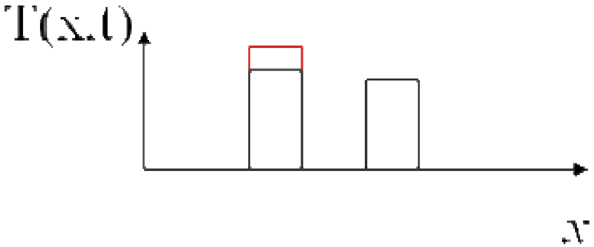
Conclusions of part 1

- **The experimental space/time temperature signal is coming from an experiment (with unknowns and non perfect characteristics)**

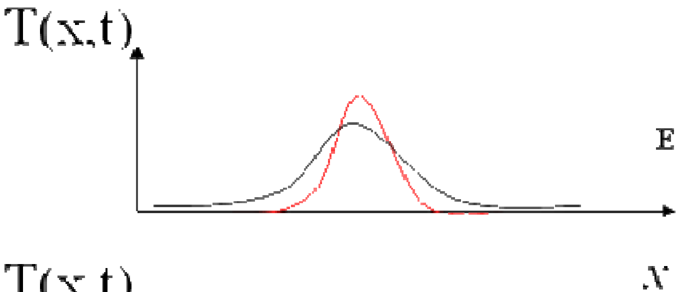
- **Tree categories of errors can be globally considered:**
  - *the random noise* : with zero mean value is an unwanted perturbating noise but able to be processed with simples asumptions (related to the uniform covariance matrix).
  - *the systematic errors*: (NUC, time derive, parasitic effects, sensor positions ...) which must be fought, detected or bypassed by the experimenter.
  - *the space and time convolutions and correlations* of the signal acting on the real time and space resolution limit.
- **The signal is then not-only noisy but also filtered, and truncated in space and in time.**
- **A great amount of data does not significate that all the possible data are available.**
- **Nevertheless a multidimensional signal gives more processing possibilities than a monodimensional one.**

- **It will be assumed for the next steps that the systematic errors are mastered!**

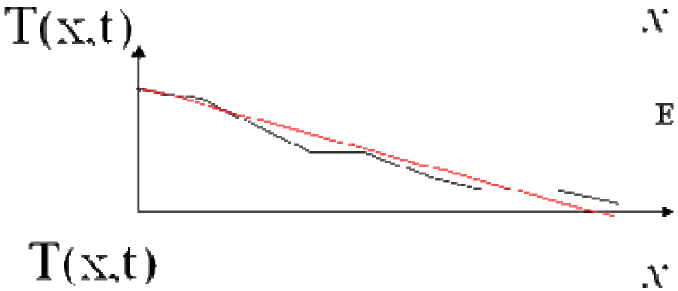
2. Thermal » processing of a T(x,y,t) field



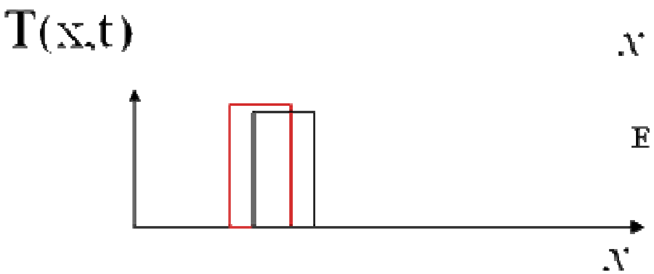
Estimation of a transverse response time $\frac{dT}{dt}+\frac{1}{\tau}T=0$



Estimation of a diffusivity $\frac{\partial T}{\partial t}=a\Delta T$



Estimation of conductivity field $\lambda\frac{\partial^2 T}{\partial x^2}+\frac{\partial \lambda}{\partial x}\frac{\partial T}{\partial x}-HT=0$

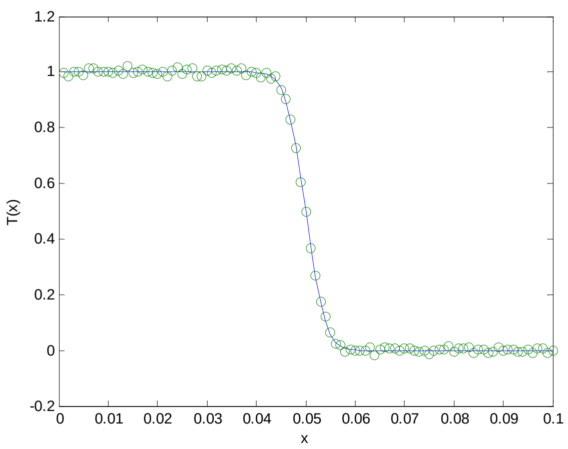


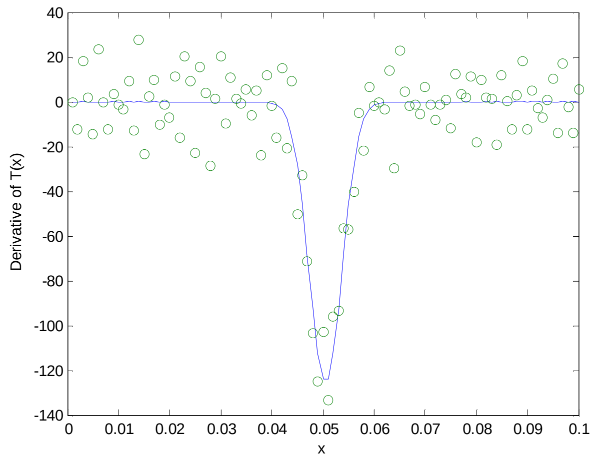
Estimation of a velocity or a shift $\frac{\partial T}{\partial t}+V\frac{\partial T}{\partial x}-a\Delta T-HT+Q$

In a lot of cases, the heat transfer models will consist in derivating the space and time temperature field.

2.1 Strategies for the estimation of the time and space derivative of the signal

$$T(x,t) = b/L + 2/L \sum_{n=1}^{N} \frac{\sin(\alpha_n b)}{\alpha_n} \exp(-a\alpha_n{}^2 t)\cos(\alpha_n x) \qquad \alpha_n = n\pi / L$$





Temperature field from the previous analytical expression at a given time

at time *t=0.5s; a*=10^{-5} *m^2 s^{-1}*; *b=L/2; L=0.1m;* (continuous line: real signal, 'o': discrete noisy signal);

Finite differences space-derivative

2.1.1 Finite differences

$$\hat{T}'_i = \frac{\hat{T}_{i+1} - \hat{T}_i}{\Delta x}$$

$$\hat{T}_i = T_i + e_{T_i}$$

Random variable : “measurement noise”: e_{T_i}

$$\hat{T}'_i = \frac{T(x_{i+1}) - T(x_i)}{\Delta x} + \varepsilon(x_{i+1}) + \frac{e_{Ti+1} - e_{Ti}}{\Delta x}$$

$$\lim_{x \to x_i} \varepsilon(x) = 0$$

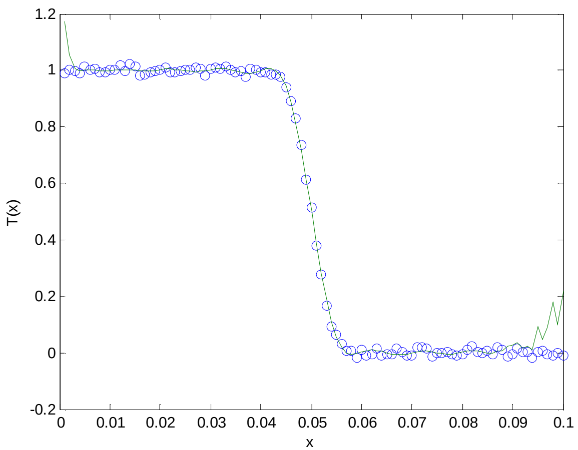
Approximation error: $\varepsilon(x_i)$

Unfortunately, when the space step *Δx* is tending to zero, the approximation error is effectively tending to zero, **but the random error is tending to infinity!**

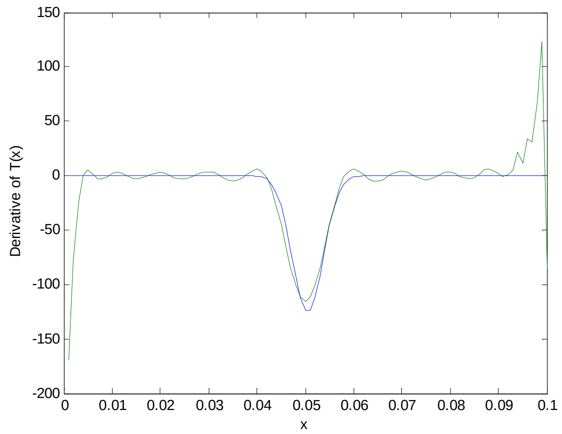
2.1.2 Polynomial fitting

$$T(x) = \sum_{n=0}^{N} \beta_n x^n \qquad \mathbf{B} = [\beta_1, \beta_2, \beta_3 .... \beta_n]^T \qquad \mathbf{X} = \begin{bmatrix} 1 & x_1 & {x_1}^2 & \dots \\ 1 & x_2 & {x_2}^2 & \dots \\ . & & & \\ 1 & x_m & {x_m}^2 & \dots \end{bmatrix}$$

$$\mathbf{B} = \left(\mathbf{X}^{\mathbf{t}}\mathbf{X}\right)^{-1} \mathbf{X}^{\mathbf{t}}\hat{\mathbf{T}}$$



Polynomial fitting



Derivation of the polynomial expression

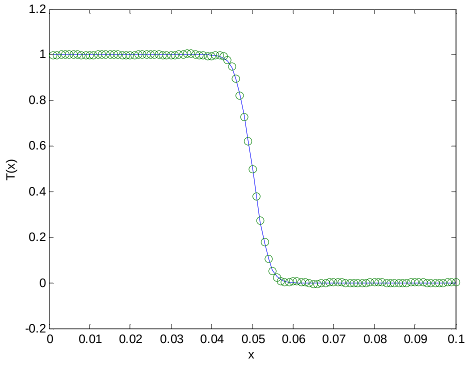
The projection of the signal on a reduced polynomial basis is giving quite good results (excepted with the boundaries) when the rank of the polynom is adapted.

2.1.3 Fourier cosinus basis

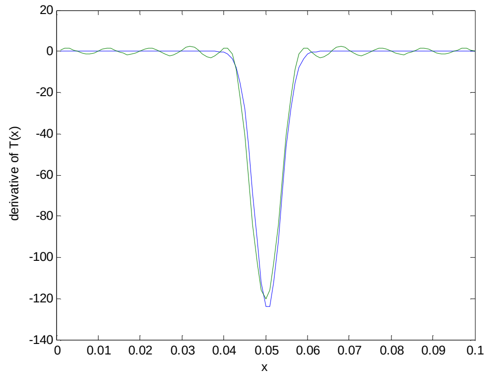
$$T(x) = \sum_{n=0}^{M} \beta_n \cos(\alpha_n x) \qquad \alpha_n = n\pi / L$$

$$\mathbf{B} = [\beta_1, \beta_2, \beta_3 .... \beta_M]^T$$

$\mathbf{X}^{\mathbf{t}}\mathbf{X}$ matrix is orthogonal,



Fourier fitting



Derivation of the serie

The derivation of the “Fourier estimated expression” is giving good results when the rank of the serie is adapted.

2.1.4 Filtering with a convolution kernel

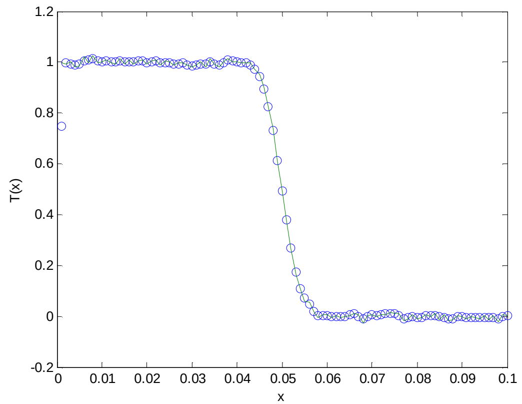
$$\tilde{T}(x) = \int_0^L p(\chi)T(x-\chi)d\chi \qquad \int_0^L p(\chi)d\chi = 1$$

Filtering of the signal

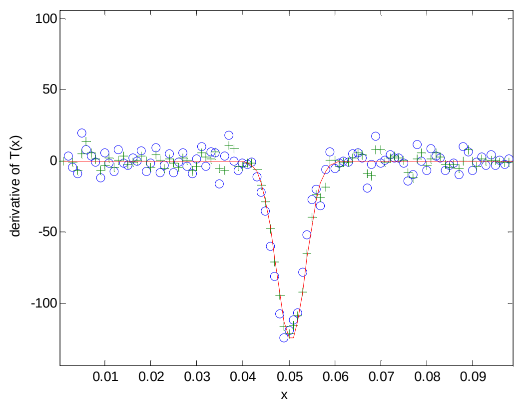
$$\frac{d\tilde{T}}{dx}(x) = \int_0^L p(\chi)\frac{dT(x-\chi)}{dx}d\chi = \int_0^L T(\chi)\frac{dp(x-\chi)}{dx}d\chi$$

Derivation by "derivated kernel"

The discrete approximation of the derivative is then conveniently considered by a convolution with a « derived » kernel.



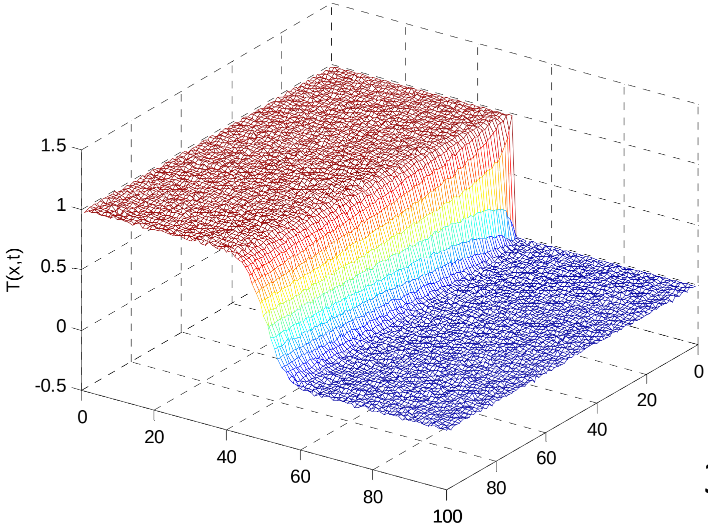
Filtering of the signal

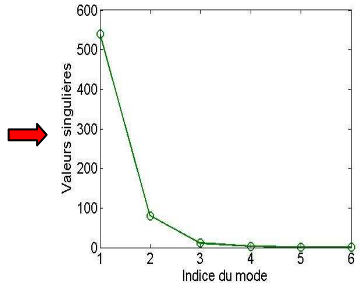


Derivation by a convolution with a "derivated kernel"

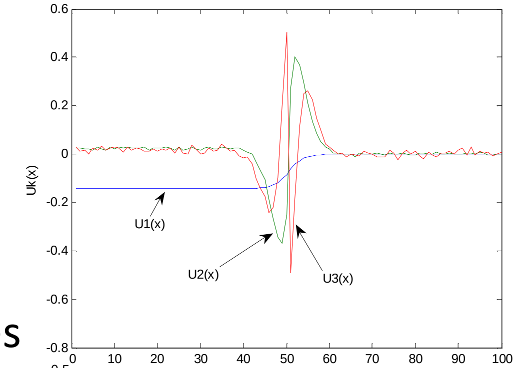
1.2.5 Singular value decomposition of the whole space and time signal

$$\hat{\mathbf{T}} = \mathbf{U}_{nxn} \Sigma_{nxn} \mathbf{V}_{nxm}{}^{T}$$

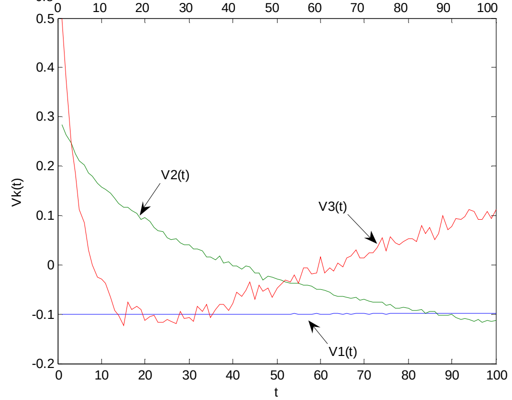




3 dominant Singular values

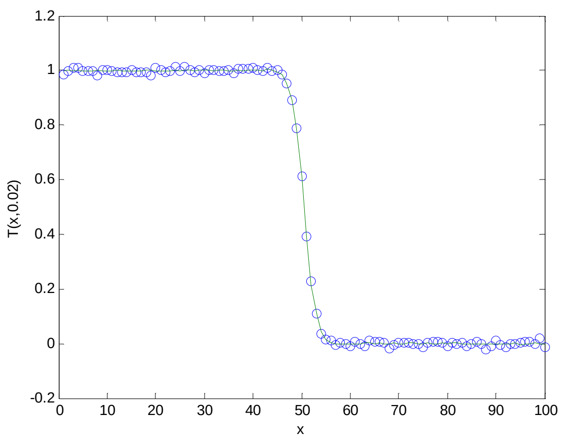


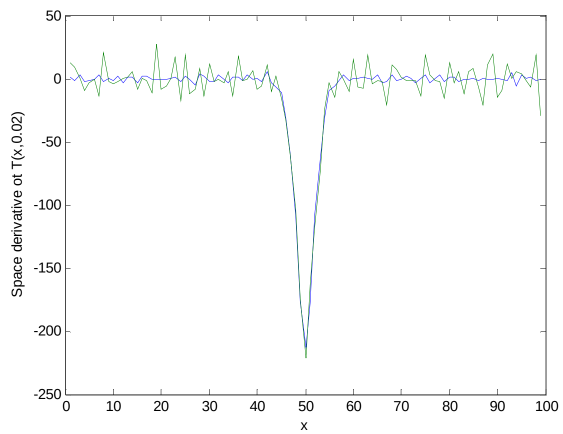
Uk(x)



Vk(t)

Consider not only the Space but also the time signal!



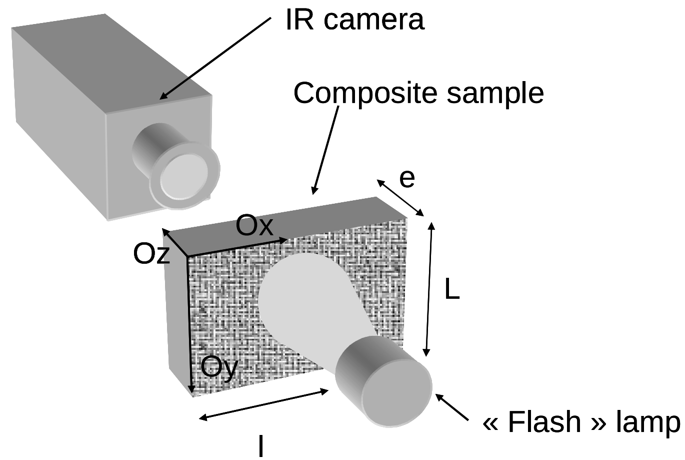


Reconstructed signal with a reduced rank of SVD decomposition

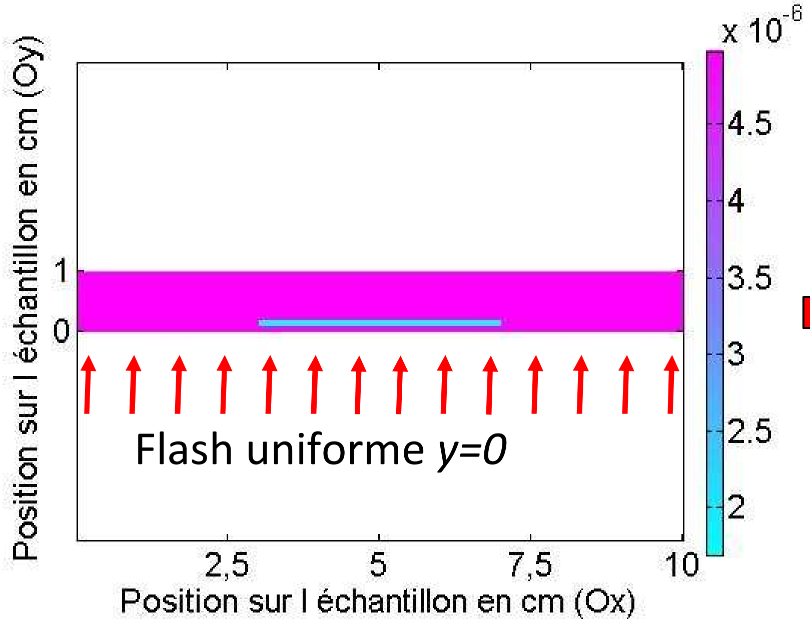
Space derivative of the compressed signal

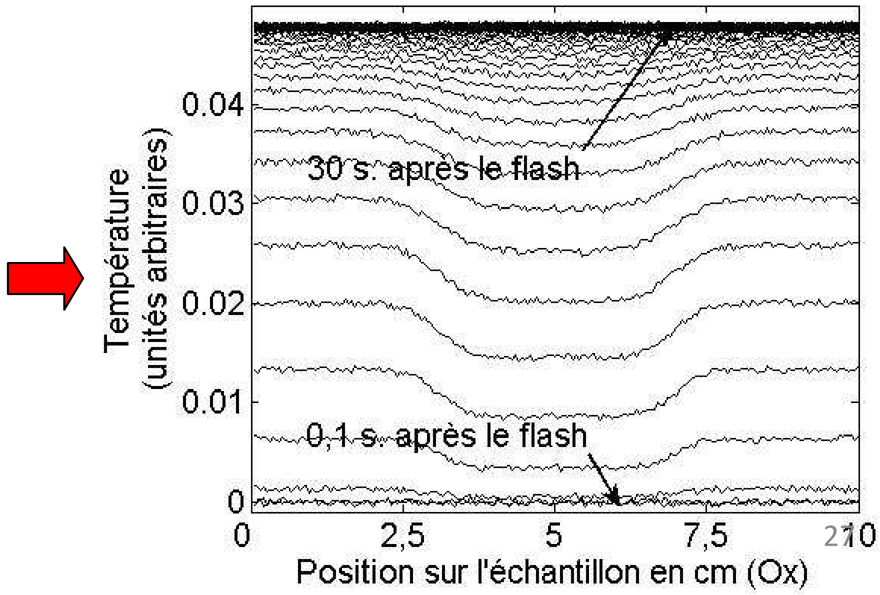
2.2 Estimation of a transverse diffusivity field from flash experiments (comparison of classical Non Destructive Evaluation methods):

Experimental situation and temperature evolution



$$T\left(x, y=e, t\right)$$



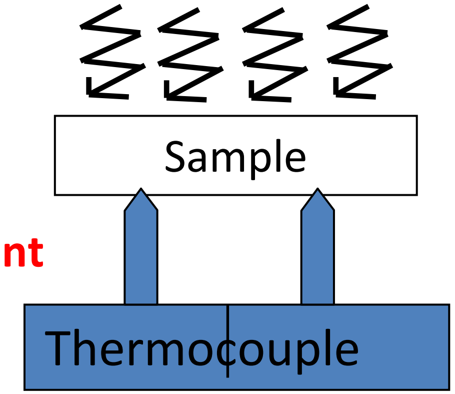


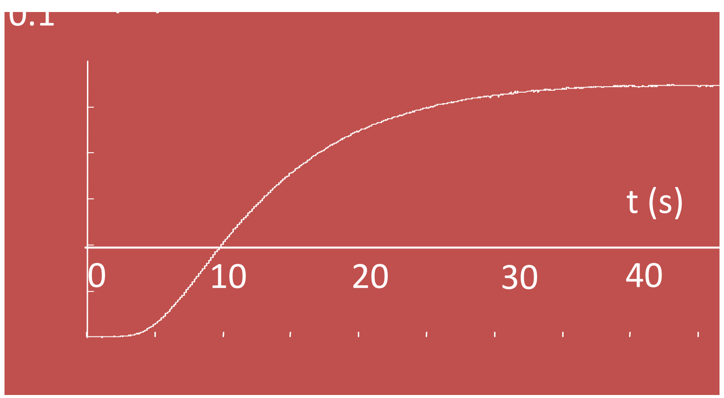
Transverse flash method and IR thermography

Metrology

Laboratory method

1 T(t) measurement



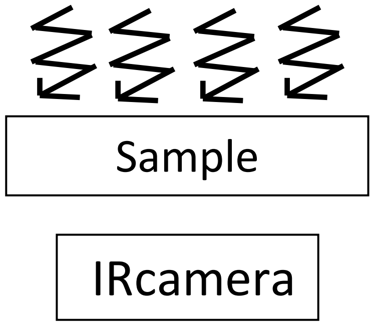


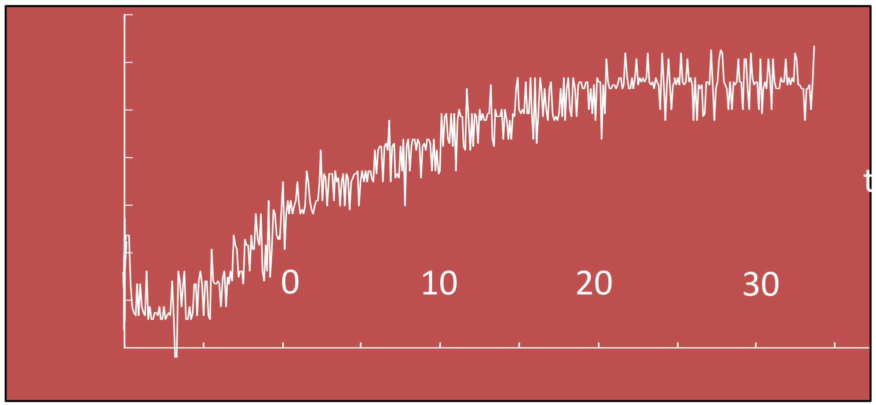
- 1 T(t) measurement
- measurement with contact
- very accurate measurement

Imaging

Industrial method

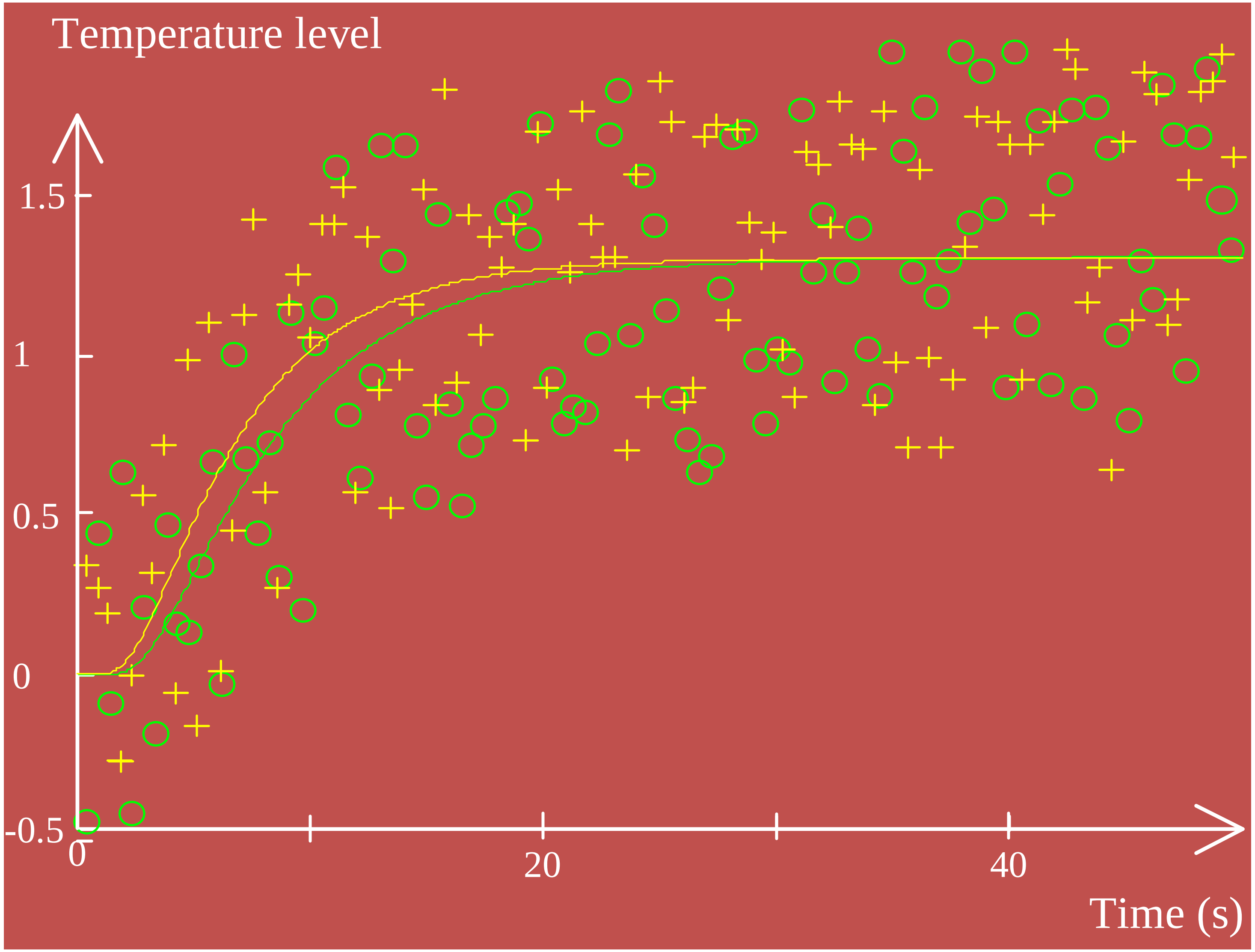
>10 000 pixels T(t) measurement





- >10000 T(t) measurements
- measurements without contact
- very noisy measurements

Can we discern two very noisy thermograms ?



2.2.1 Estimation with physical asymptotic expansions:

1D température response:

$$T(z=0,t)=\frac{Q}{\rho c L}\cdot\left(1+2\cdot\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\cdot\pi\cdot^2\cdot a\cdot t}{L^2}\right)\right)=\frac{Q}{\rho c L}\,f\left(at/L^2\right)$$

Asymptotic expansion by considering a small thermal conductivity variation

$$T(x,y,z,t)=q(x,y)\left(f(z,t,\beta_0)+\Delta\lambda(x,y)\left.\frac{\partial f}{\partial\lambda}\right|_{(z,t,\lambda_0)}\right)$$

With q(x,y) : spatial distribution of energy, and $\Delta\lambda$ (x,y): spatial thermal conductivity variation

Other possibility : consider the conductivity sensitivity function as a logarithmic time derivative

$$T(0,t_i)\approx\frac{Q}{\rho c L}\,f\left(\lambda_0 t_i/\rho c L^2\right)+\frac{Q}{\rho c L}\frac{\Delta\lambda}{\lambda_0}\,t\left.\frac{\partial f}{\partial t}\right|_{(\lambda_0 t_i/\rho c L^2)}$$

Other possibilities with other thermophysical properties :

$$T(0,t_i)\approx\frac{Q}{\rho c L_0}\,f\left(\lambda t_i/\rho c {L_0}^2\right)-\frac{Q}{\rho c L_0}\frac{\Delta L}{L_0}\left(f\left(\lambda t_i/\rho c {L_0}^2\right)+2t\left.\frac{\partial f}{\partial t}\right|_{(\lambda t_i/\rho c {L_0}^2)}\right)$$

$$T(0,t_i)\approx\frac{Q}{\rho c_0 L}\,f\left(\lambda t_i/\rho c_0 L^2\right)-\frac{Q}{\rho c_0 L}\frac{\Delta\rho c}{\rho c_0}\left(f\left(\lambda t_i/\rho c_0 L^2\right)+t\left.\frac{\partial f}{\partial t}\right|_{(\lambda t_i/\rho c_0 L^2)}\right)$$

Linear Estimation method, if **X** is well known

$$T(0,t_i) \approx \beta_1 X_{\beta_1}(t_i) + \beta_2 X_{\beta_2}(t_i)$$

$$\mathbf{T} = \left[T(0,t_1) \quad .. \quad T(0,t_N) \right]^t \qquad \mathbf{X} = \begin{bmatrix} X_{\beta_1}(t_1) & \dots & X_{\beta_1}(t_N) \\ X_{\beta_2}(t_1) & \dots & X_{\beta_2}(t_N) \end{bmatrix}^t$$

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} = \left(\mathbf{X^t X} \right)^{-1} \mathbf{X^t \hat{T}}$$

Is suitable only if **X** is perfectly known. If *f is only a reference curve obtained from experiment, the time logarithmic derivative will noisy and the estimation bad!*
Shepard proposed to decompose the signal with a polynomial fitting, such as:

$$Ln(T(x,y,z=0,t)) = \beta_0(x,y) + \beta_1(x,y) Ln(t) + \beta_2(x,y) Ln^2(t) + \dots$$

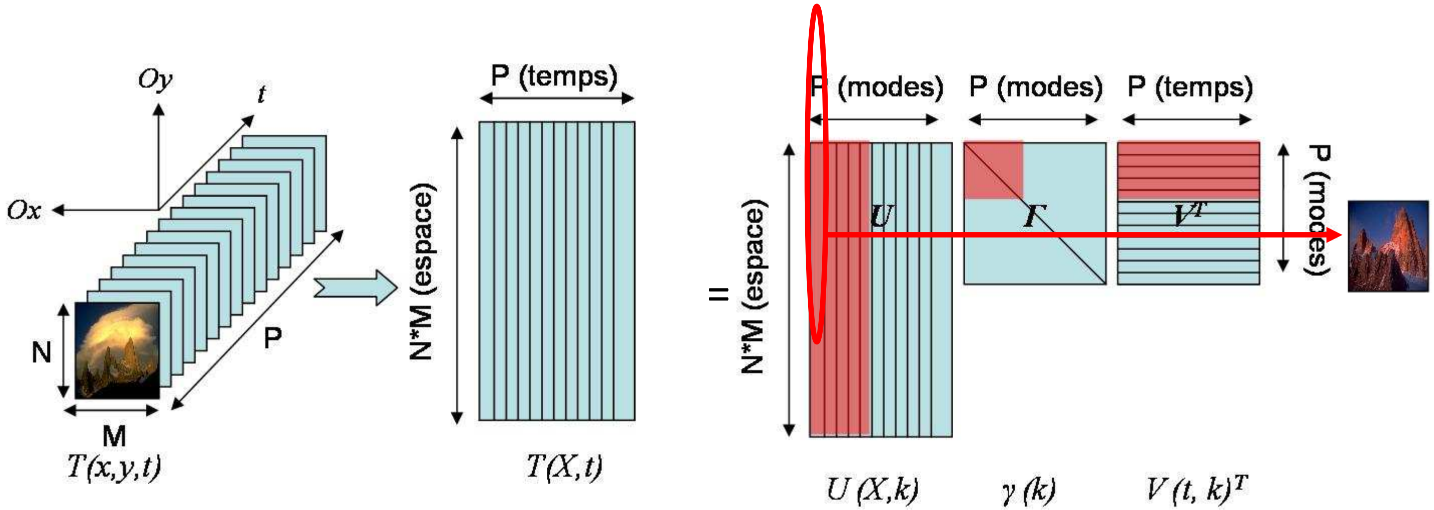
2.2.3 The SVD decomposition

1. Arrangement of the information cube in a space time matrix
2. SVD decomposition of the resulting matrix
3. Arrangement of the spatial *U vectors in spatial matrices*

$$T(x,y,t) \xrightarrow{réarrangement} T(X,t)$$

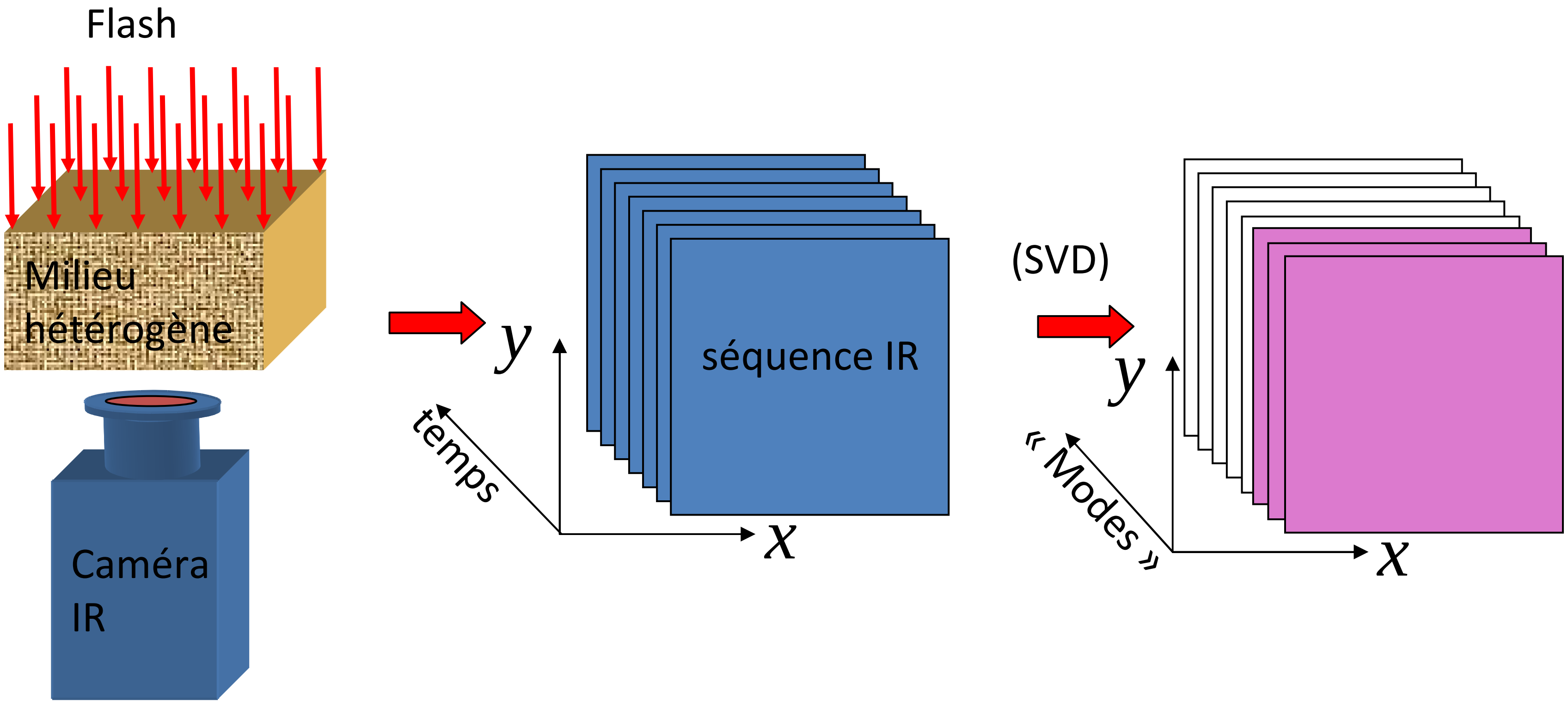
$$T(X,t) = \sum_{k=1}^{P} \lambda_k \cdot U_k(X) \cdot V_k(t)^T$$

$$U_k(X) \xrightarrow{réarrangement} U_k(x,y)$$



SVD and signal compression

From a sequence (about 100 or 1000 images), the SVD will give sometime 3 or 4 images related to the structure of the sample.



SVD and variable separation

$$T(X,t) \overset{SVD}{=} U\Sigma V^T$$

⬇

$$\left[M_{i,j}\right] = \int_t \left[T(X_i,t)\cdot T(X_j,t)\right]\cdot dt \overset{Diagonalisation}{\Rightarrow} U\Sigma^2 U^T$$

M temporal covariance matrix only depending on X

⬇

U Only depending on X

$$\left[N_{i,j}\right] = \int_X \left[T(X,t_i)\cdot T(X,t_j)\right]\cdot dX \overset{Diagonalisation}{\Rightarrow} V\Sigma^2 V^T$$

N Space covariance matrix only depending on t

⬇

V Only depending on t

Comparison between SVD and asymptotic expansions

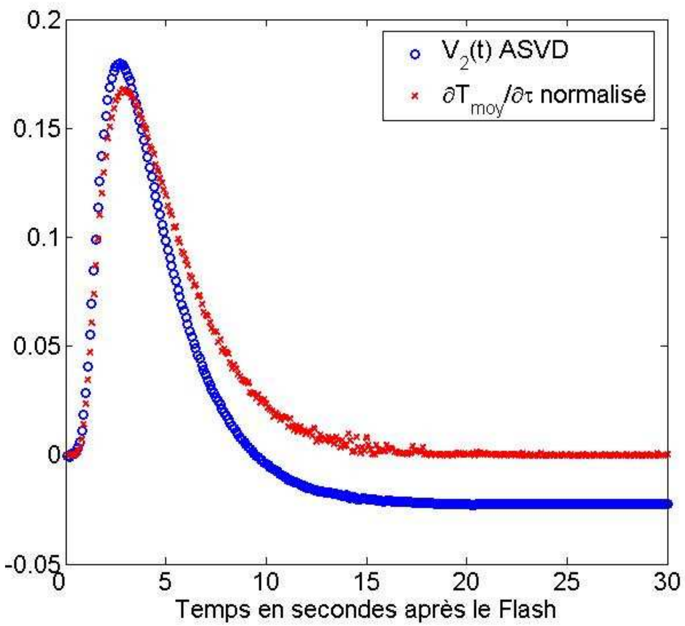
Asymptotic *expansion*:

$$T(x,t) \approx \frac{T_{\max}(x)}{\{T_{\max}\}_x}\left[\{T\}_x(t) + \left(\tau(x) - \tau_{moy}\right)\cdot \frac{\partial \{T\}_x}{\partial \tau_{moy}}(t)\right]$$

U1 (x) and V1(t) are very near from the time and space average signal.

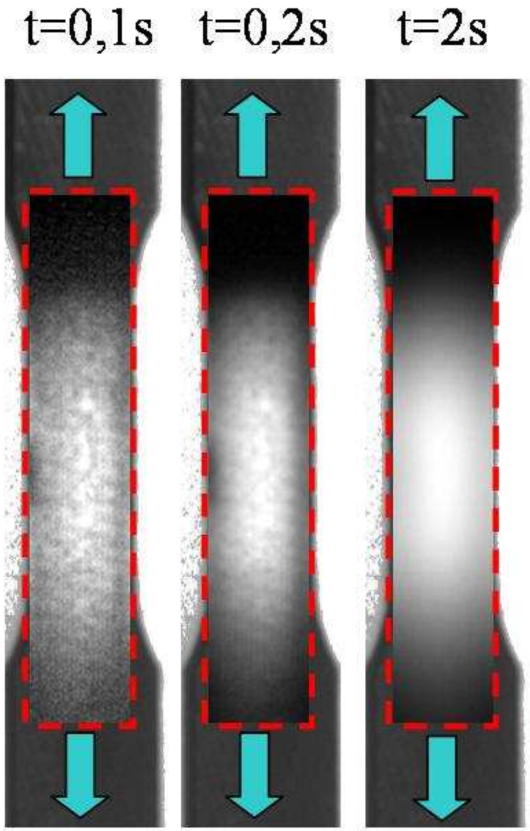
U2(x) and V2(t) are the space and time deviation from the space and time average signal.

$$V_2(t) \approx \frac{\partial \{T\}_x}{\partial \tau_{moy}}(t)$$

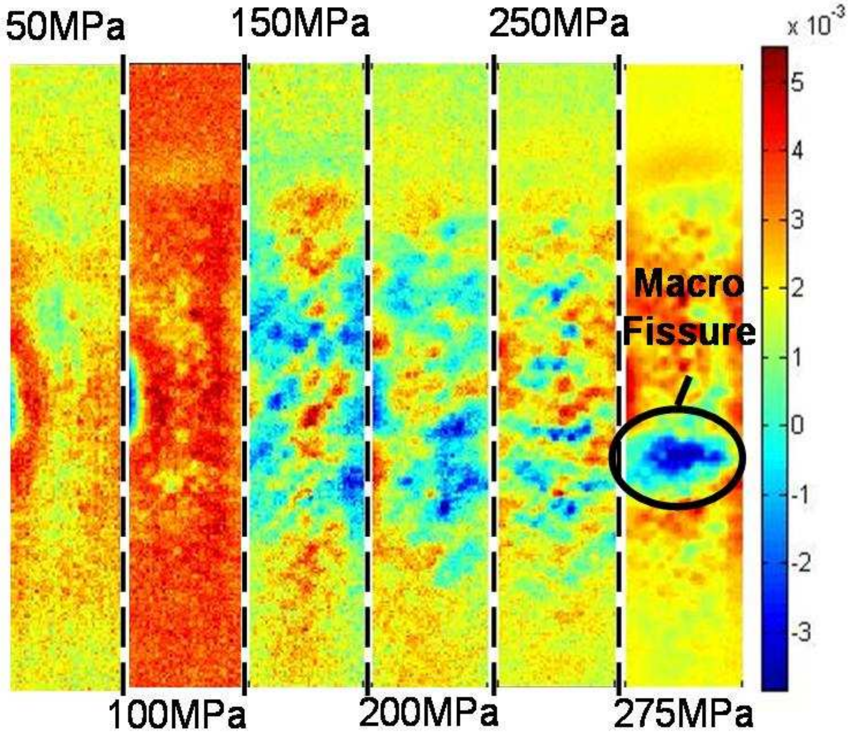


$$U_2(x) \approx \left(\tau(x) - \tau_{moy}\right) / \left\|\left(\tau(x) - \tau_{moy}\right)\right\|$$

Practical example: NDE and tensile test on a composite medium



Relative evolution of $U_2(x)$ versus the stress = transvers diffusivity variations.

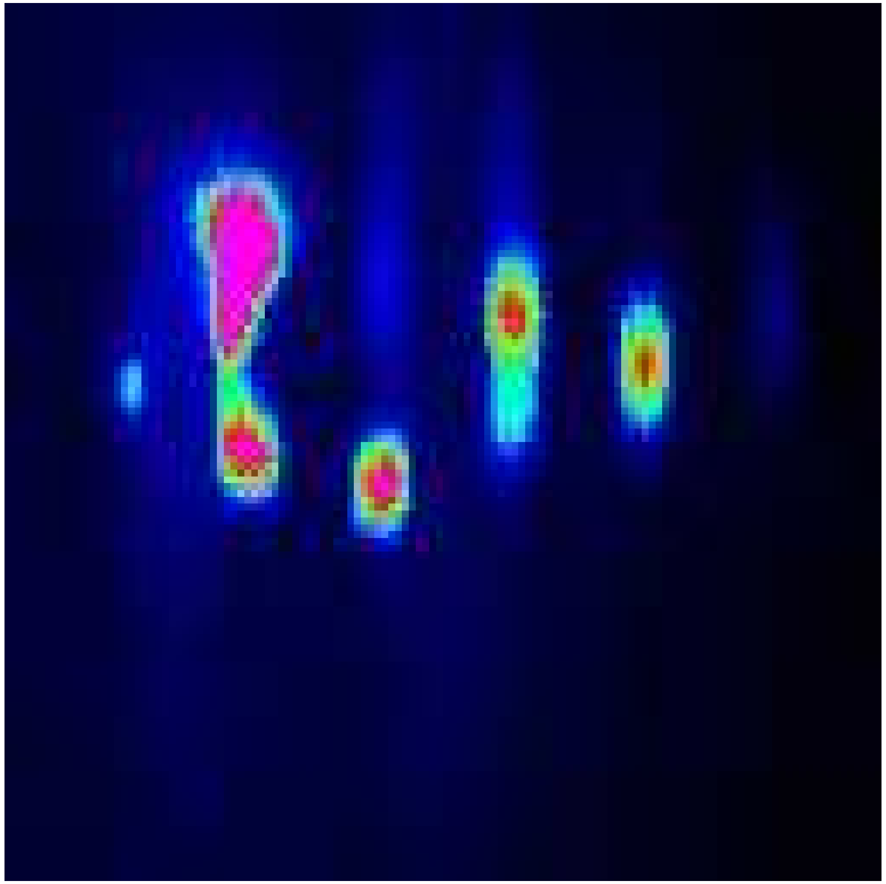


Remarks about the previous methods

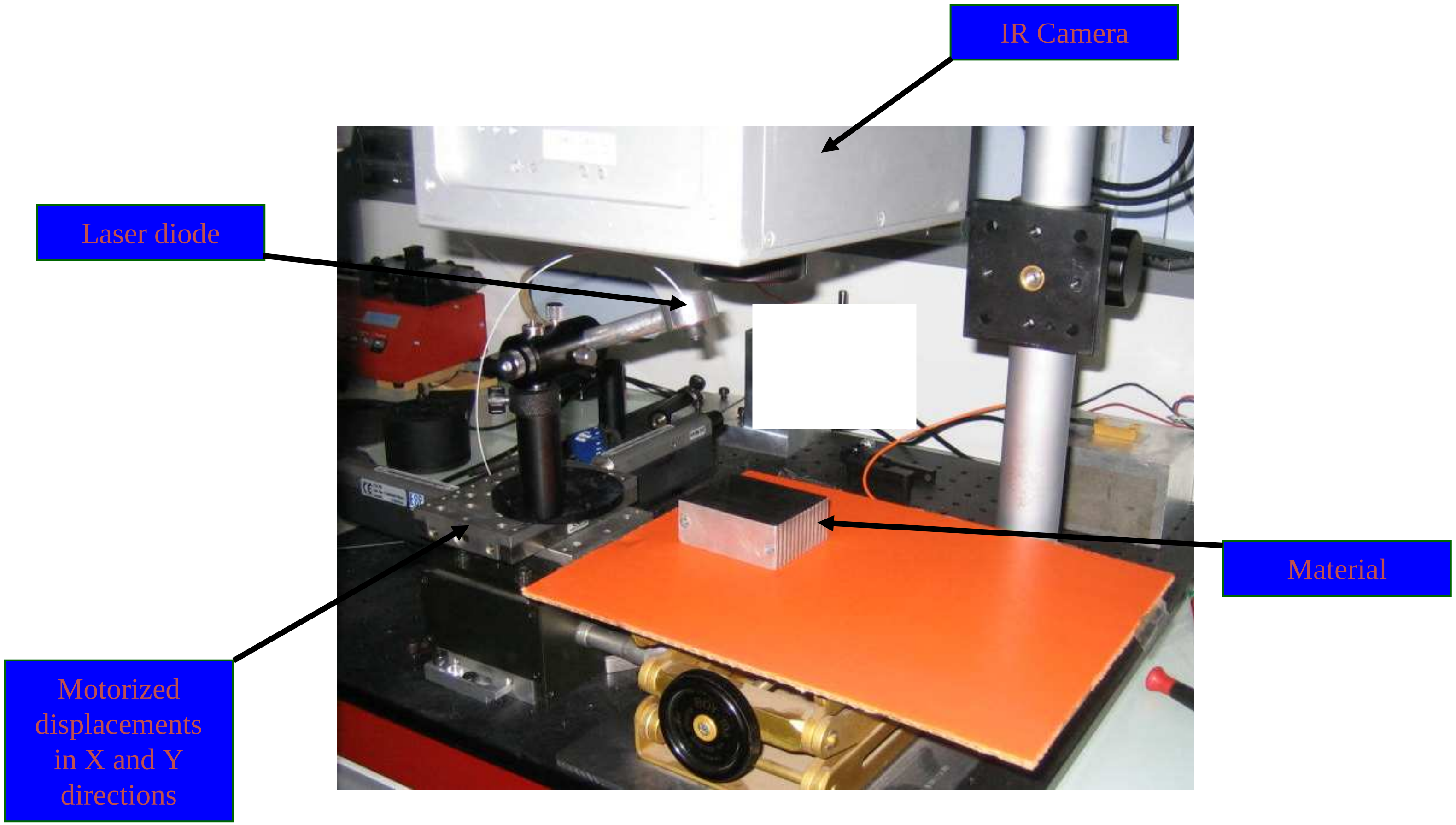
- From the great amount of data, the most previous methods consisted in trying to compress the data, by a projection on a suitable basis (very often non related to the physics)…
- Is it possible to look directly to the phenomena of physical interest?

2.3 Estimation of in-plane diffusivity field-Time-space correlation and elimination of the non useful data

Randomly flying spot



Experiment


IR Camera
Laser diode
Material
Motorized displacements in X and Y directions

« Nodal » method in the case of a source point

- Only a few pixels are available on an image at a given time
- The in-plane diffusion is approximated with a finite difference scheme:

$$Fo_{i,j}\Delta T_{i,j}^{k} + \Phi_{i,j}^{k} = \delta T_{i,j}^{k}$$

with

$$\Delta T_{i,j}^{k} = \left(T_{i+1,j}^{k} + T_{i-1,j}^{k} + T_{i,j+1}^{k} + T_{i,j-1}^{k} - 4T_{i,j}^{k}\right) \; ; \; \delta T_{i,j}^{k} = T_{i,j}^{k+1} - T_{i,j}^{k} \; ; \; Fo_{i,j} = \frac{a_{i,j}\Delta t}{\Delta x^{2}}$$

If the system is in pure relaxation it gives

$$Fo_{i,j}\Delta T_{i,j}^{k} = \delta T_{i,j}^{k}$$

The diffusivity is to be estimated only if the correlation coefficient is near from 1:

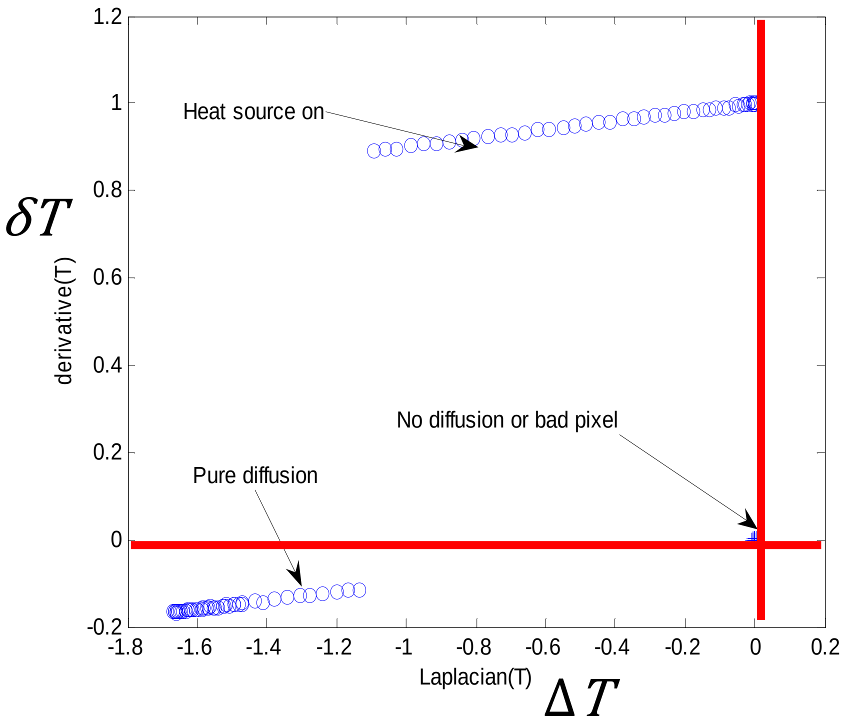
$$\boldsymbol{\rho}_{i,j}^{F_t} = \frac{\sum_{F_t} \Delta T_{i,j}^{k} \delta T_{i,j}^{k}}{\sqrt{\sum_{F_t} {\Delta T_{i,j}^{k}}^{2}} \sqrt{\sum_{F_t} {\delta T_{i,j}^{k}}^{2}}}$$

Correlation indicator method

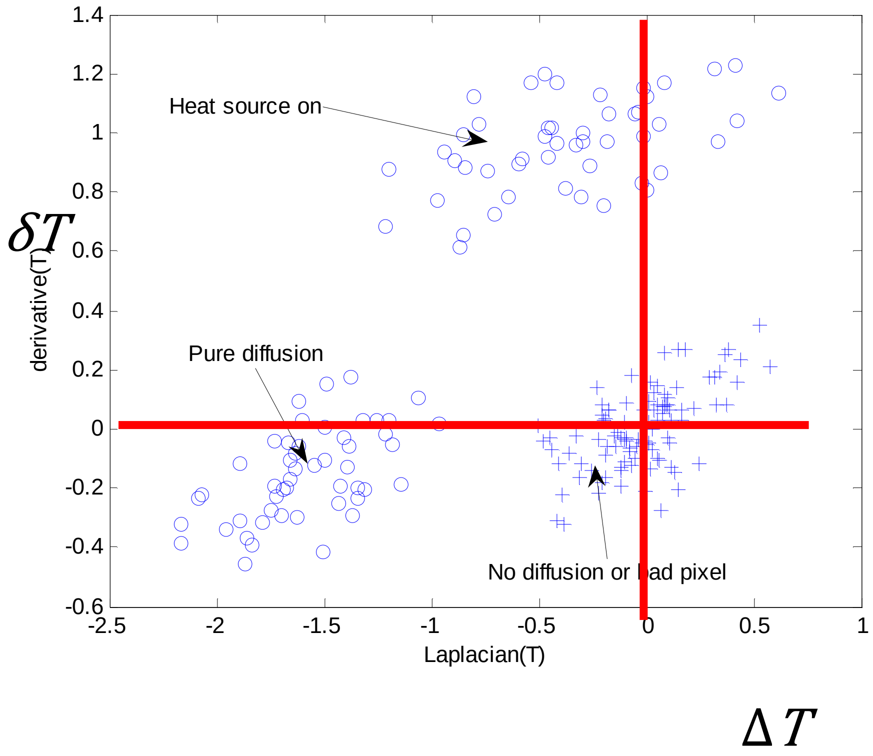
$$Fo_{i,j}\Delta T^{k}_{i,j} = \delta T^{k}_{i,j}$$

$$\rho^{F_t}_{i,j} = \frac{\sum_{F_t} \Delta T^{k}_{i,j}\,\delta T^{k}_{i,j}}{\sqrt{\sum_{F_t} {\Delta T^{k}_{i,j}}^{2}}\sqrt{\sum_{F_t} {\delta T^{k}_{i,j}}^{2}}} \to 1$$

$$\frac{1}{Fo^{F_t}_{i,j}} = \frac{\sum_{F_t} \Delta T^{k}_{i,j}\,\delta T^{k}_{i,j}}{\sum_{F_t} {\delta T^{k}_{i,j}}^{2}} = \frac{\rho^{F_t}_{i,j}\sqrt{\sum_{F_t} {\Delta T^{k}_{i,j}}^{2}}}{\sqrt{\sum_{F_t} {\delta T^{k}_{i,j}}^{2}}}$$

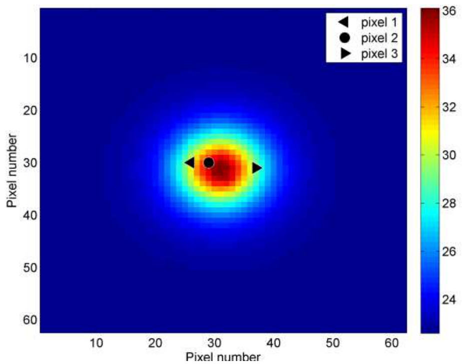


Perfect data

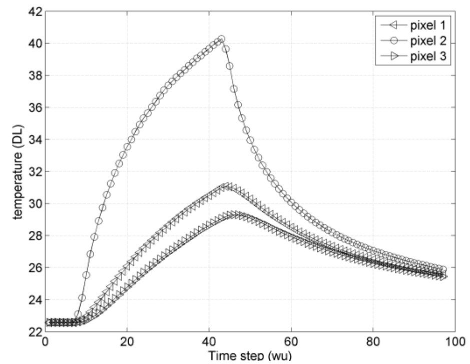


Noisy data

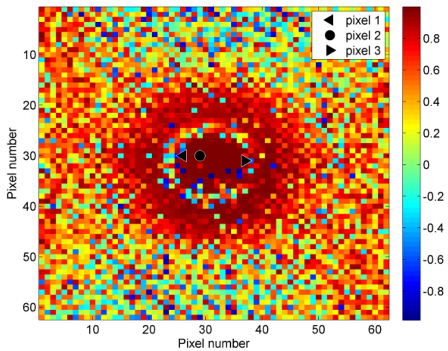
Example with an in-plane source point



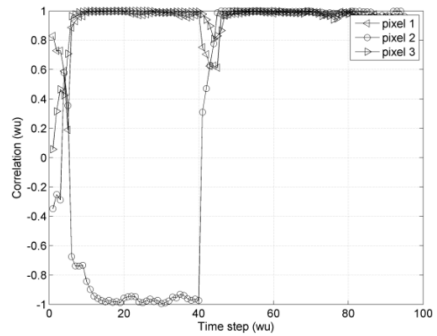
Temperature field



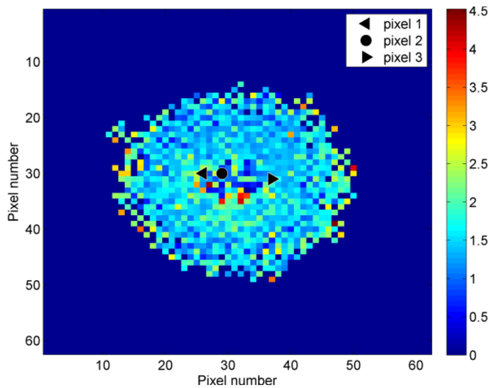
Evolution of a central pixel



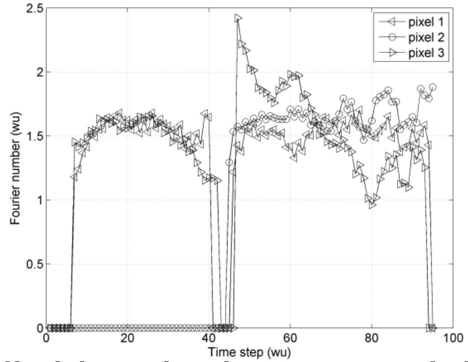
Correlation field



Correlation at a central pixel

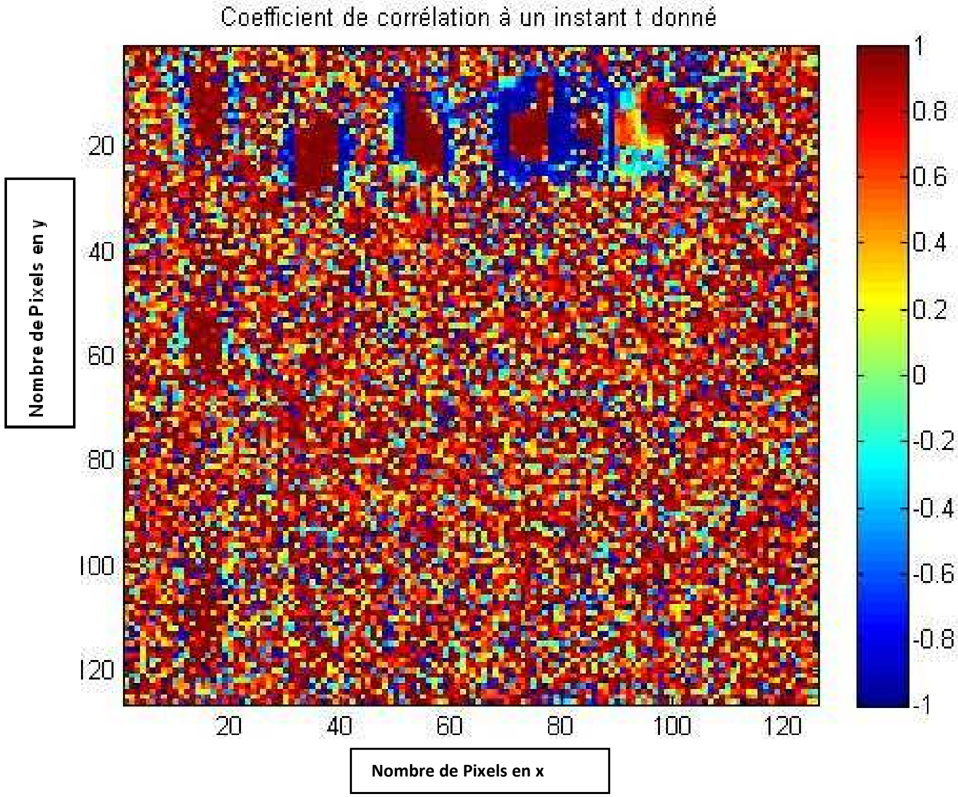


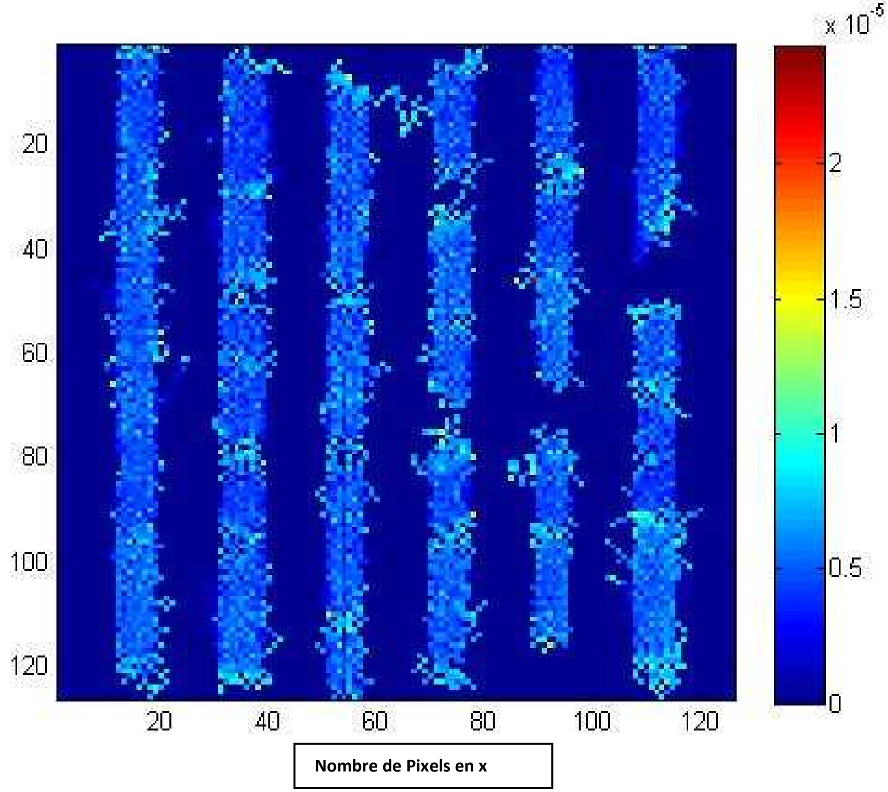
Diffusivity field



Diffusivity estimation at a central pixel

Results for a heterogeneous plate





$$\rho_{i,j}^{F_t} = \frac{\sum_{F_t} \Delta T_{i,j}^{k} \delta T_{i,j}^{k}}{\sqrt{\sum_{F_t} \Delta T_{i,j}^{k\;2}} \sqrt{\sum_{F_t} \delta T_{i,j}^{k\;2}}} \rightarrow 1$$

$$Fo_{i,j} \Delta T_{i,j}^{k} = \delta T_{i,j}^{k}$$

General Conclusion

- Space/ time signal=great amount of noisy and « non-perfect » data.
- Several strategies:
  - Analysis of the different kinds of noise and bias of the signal.
  - Compression (projection, filtering, averaging…) and estimation with a model by the implementation of a « suitable » basis.
  - Direct use of the physical model

    (example: Correlation analysis)