

Eurotherm Seminar 94Advanced Spring School:
Thermal Measurements & Inverse Techniques 5th edition
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L5-B: Measurements without contact in heat transfer: principles, implementation and pitfalls

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return on innovation

Temperature measurement by sensing the thermal emissive power

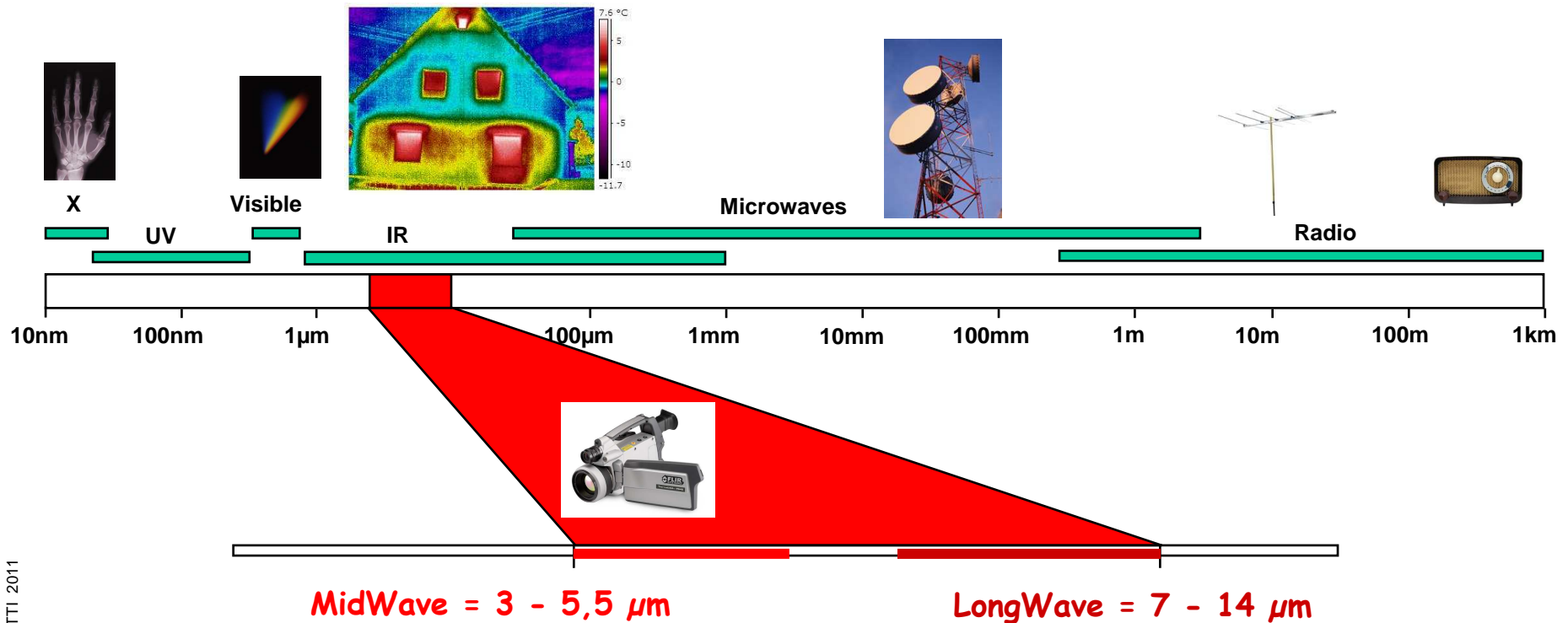
- Basics : Planck's law, Wien's law and s.o.
- Emissivity-Temperature Separation problem (ETS)
 - Pyrometry
 - single-color, bispectral pyrometry
 - multispectral pyrometry
 - ETS in airborne/satellite remote sensing
 - atmosphere compensation
 - Spectral-Smoothness method
 - Multi-temperature method
- Conclusion

Thermal radiation

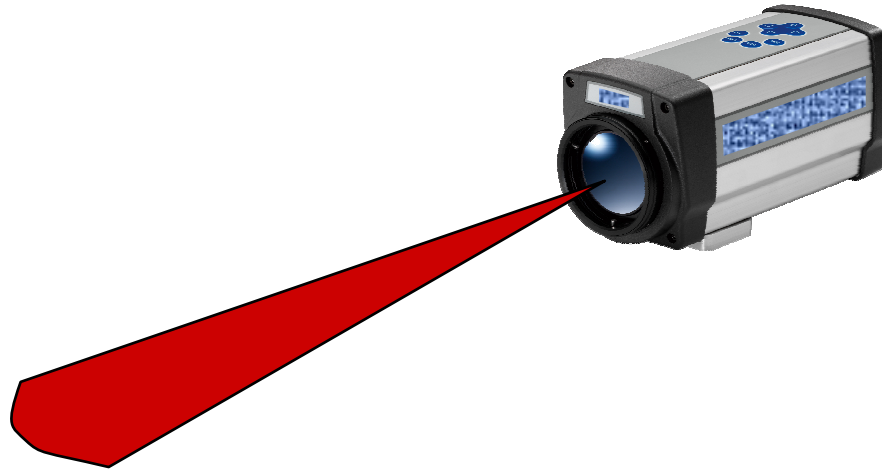
Matter emits EM radiation
Intensity increases with
temperature



Monitoring of emitted
radiation offers a mean for
temperature measurement



Thermal radiation monitoring



Advantages of the radiation method :

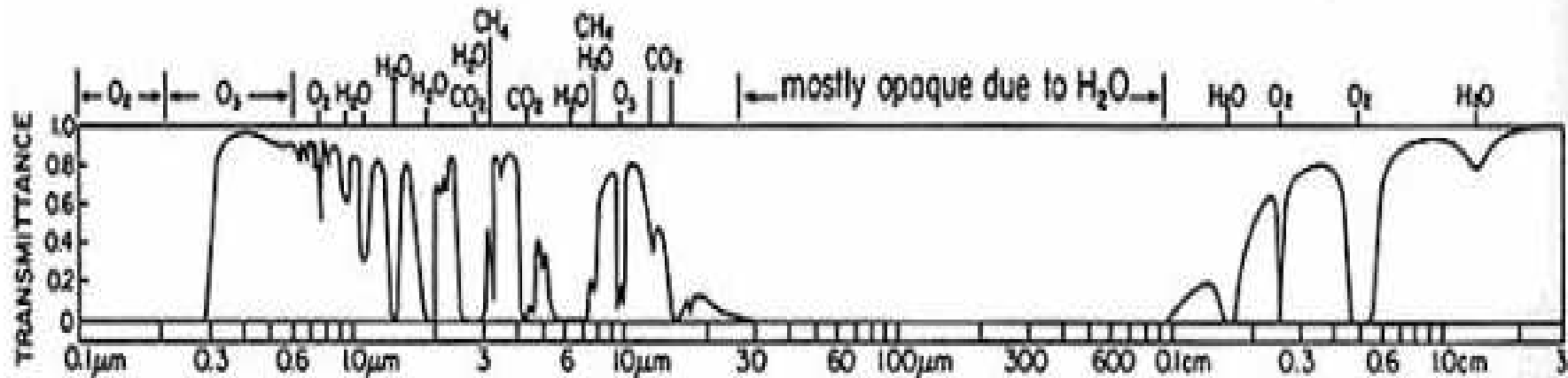
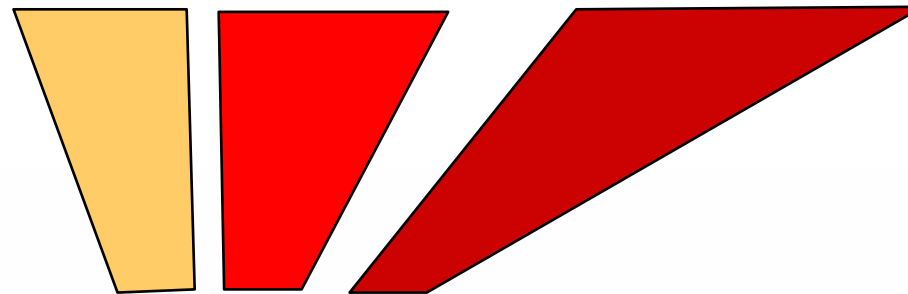
- non-contact
- surface probing (opaque material),
- surface to sub-surface probing (semi-transparent material)
- rapid : detectors with up to GHz bandwidth (and even higher)
- long distance measurement (airborne and satellite remote sensing, astronomy)
- point detectors (local measurement or 2D images by mechanical scanning)
- focal plane arrays (instantaneous 2D images)
- possibility of spectral measurements (multispectral, hyperspectral)

Radiation sensing is dependant on the atmosphere transmission,
(absorption bands of air constituents : H₂O, CO₂, O₃, CH₄, ...)

MidWave: 3 - 5,5 μm

ShortWave: 0.7 - 2,5 μm

LongWave: 7 - 14 μm

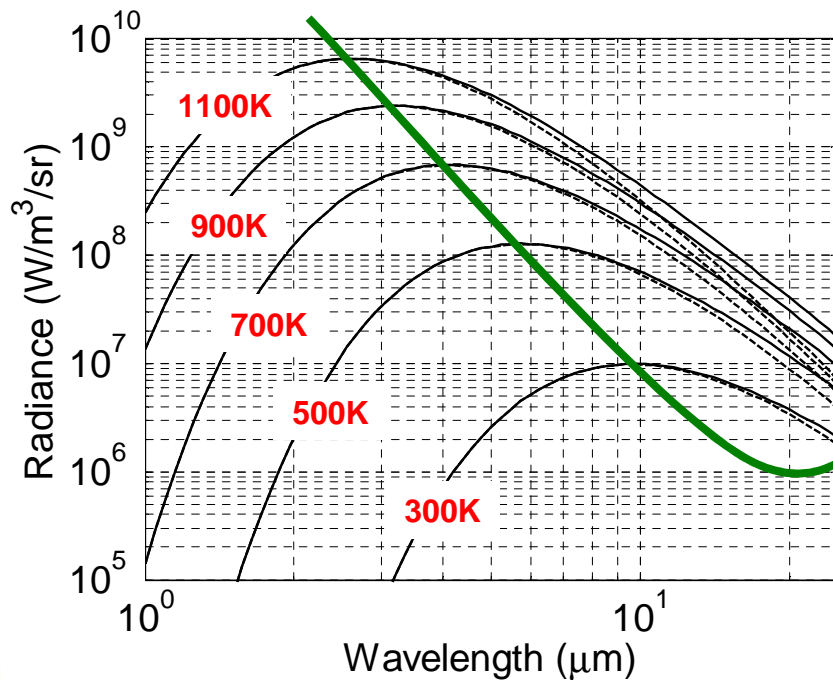


Basics (1/4)

- Blackbody: perfect absorber, perfect emitter (~Holy Grail...)

- Spectral radiance given by Planck's law: $B(\lambda, T) = \frac{C_1}{\lambda^5} \frac{1}{\exp\left(\frac{C_2}{\lambda T}\right) - 1}$

- Wien's approximation: $B_w(\lambda, T) = \frac{C_1}{\lambda^5} \exp\left(-\frac{C_2}{\lambda T}\right)$



—— Planck
 - - - - Wien

Maximum given by Wien's displacement law: $\lambda_{\max} T = 2898 \mu\text{mK}$

Error of Wien's approximation is less than 1% providing that $\lambda T < 3124 \mu\text{mK}$

Basics (2/4)

Wavelength selection for temperature measurement

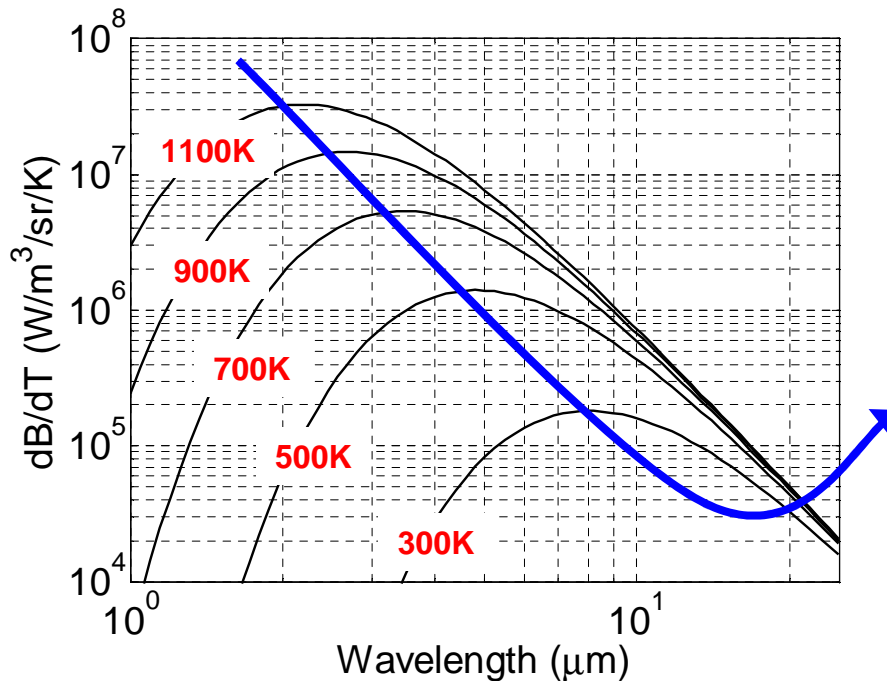
- Maximum of radiance given by Wien's displacement law: $\lambda_{\max} T = 2898 \mu\text{mK}$



$$\lambda = 9.65 \mu\text{m}$$

for $T = 300\text{K}$

- Radiance sensitivity to temperature (*absolute sensitivity*): $\frac{\partial B}{\partial T}$



Maximum corresponding to: $\lambda T = 2410 \mu\text{mK}$



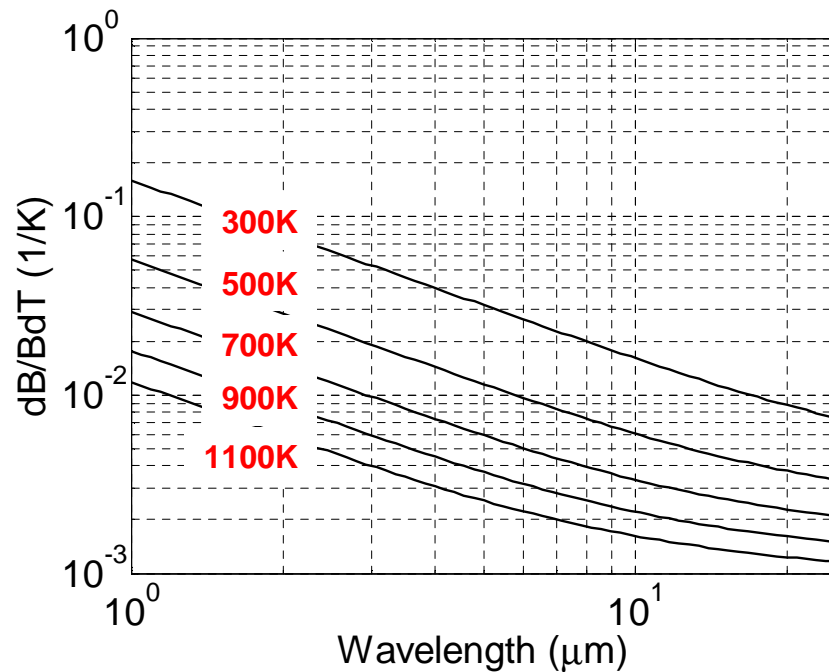
$$\lambda = 8.05 \mu\text{m}$$

for $T = 300\text{K}$

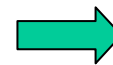
Basics (3/4)

Wavelength selection for temperature measurement

- Radiance sensitivity to temperature (*relative sensitivity*): $\frac{1}{B} \frac{\partial B}{\partial T}$



for $T = 300\text{K}$:
2% radiance increase per K at $8\mu\text{m}$
16% radiance increase per K at $1\mu\text{m}$



Advantage of performing measurements at short wavelengths (sensitivity is nearly in inverse proportion to wavelength)

Interest in visible pyrometry or even UV pyrometry ?

Basics (4/4)

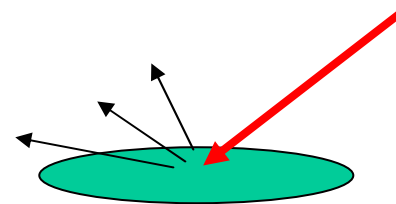
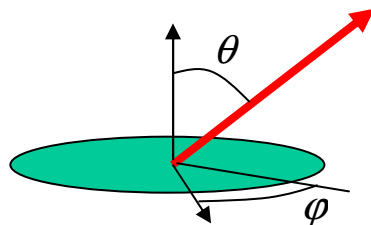
Real materials (non-perfect emitters)

- with respect to blackbody, the emitted radiance $L(\lambda, T, \theta, \varphi)$ is reduced by a factor called emissivity:

$$L(\lambda, T, \theta, \varphi) = \varepsilon(\lambda, T, \theta, \varphi) B(\lambda, T) \quad 0 \leq \varepsilon \leq 1$$

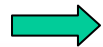
- emissivity depends on wavelength, temperature, and direction
- second Kichhoff's law between emissivity and absorptance:

$$\varepsilon(\lambda, \theta, \varphi) = \alpha(\lambda, \theta, \varphi)$$



- relation between absorptance and directional hemispherical reflectance from the energy conservation law for an opaque material (the energy that is not absorbed by the surface is reflected in all directions):

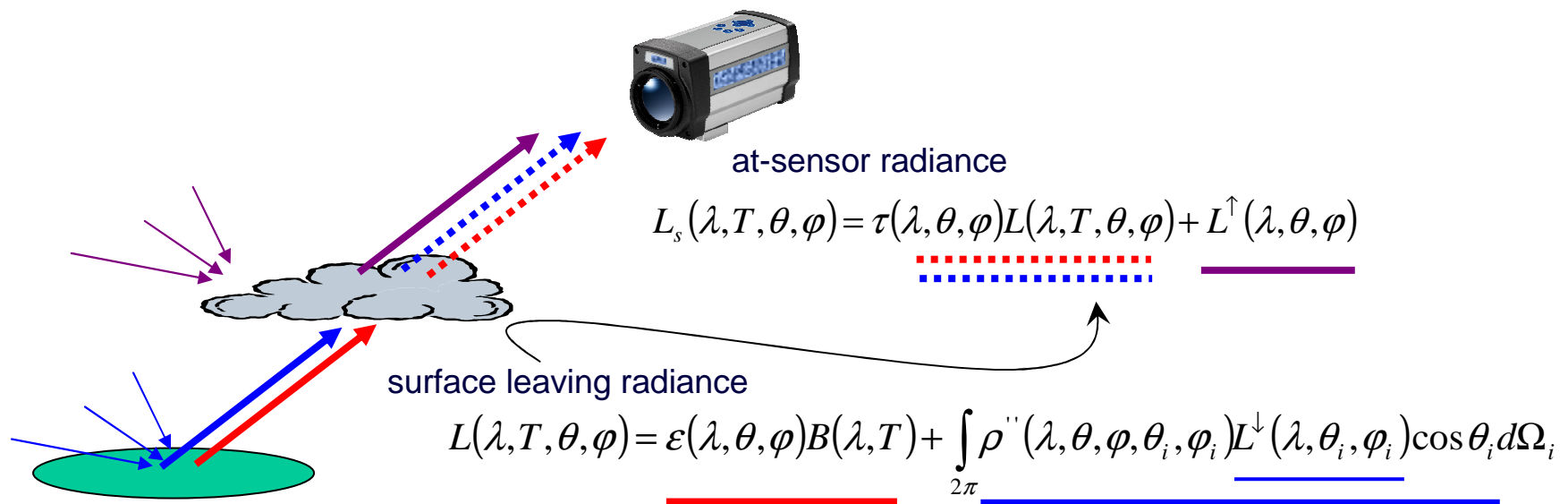
$$\alpha(\lambda, \theta, \varphi) + \rho^{\wedge}(\lambda, \theta, \varphi) = 1$$



Emissivity can be inferred from a reflectance measurement (integrating sphere)
Drawback : need to bring the integrating sphere close to the surface

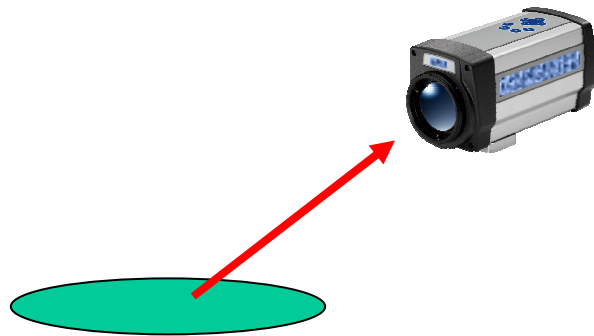
Contributors to the optical signal

- the surface reflects the incoming radiation (non-perfect absorber)
 - downwelling radiance: $L^\downarrow(\lambda, \theta_i, \varphi_i)$
 - bidirectional reflectance: $\rho''(\lambda, \theta, \varphi, \theta_i, \varphi_i)$
- the radiation leaving the surface is attenuated along the optic path (absorption, scattering by atmosphere constituents: gases, aerosols – dust, water/ice particles)
 - transmission coefficient: $\tau(\lambda, \theta, \varphi)$
- atmosphere emits and scatters radiation towards the sensor
 - upwelling radiance $L^\uparrow(\lambda, \theta, \varphi)$



First considered case

- Pyrometry of high temperature surfaces
 - sensor at close range (limited or even negligible atmosphere contributions)
 - environment much colder than the analyzed surface



$$L_s(\lambda, T, \theta, \varphi) = \varepsilon(\lambda, \theta, \varphi) \underline{B(\lambda, T)}$$

Second considered case

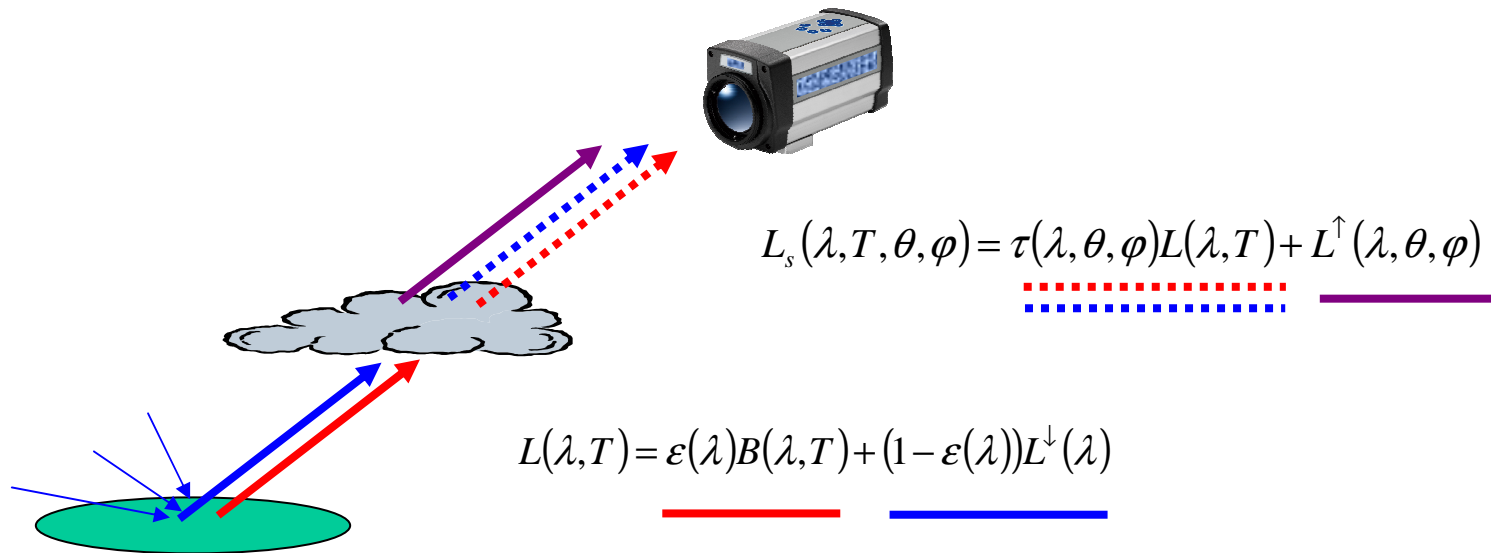
- Airborne/satellite remote sensing

- hypothesis of lambertian surface: isotropic reflectance \longrightarrow isotropic emissivity

- mean downwelling radiance

$$L^\downarrow(\lambda) = \frac{1}{\pi} \int_{2\pi} L_{env}(\lambda, \theta_i, \varphi_i) \cos \theta_i d\Omega_i$$

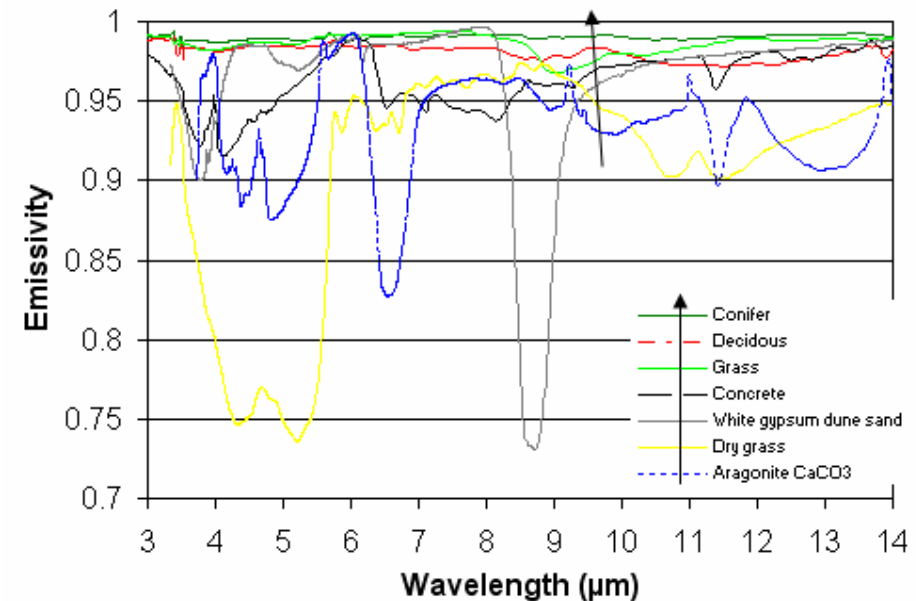
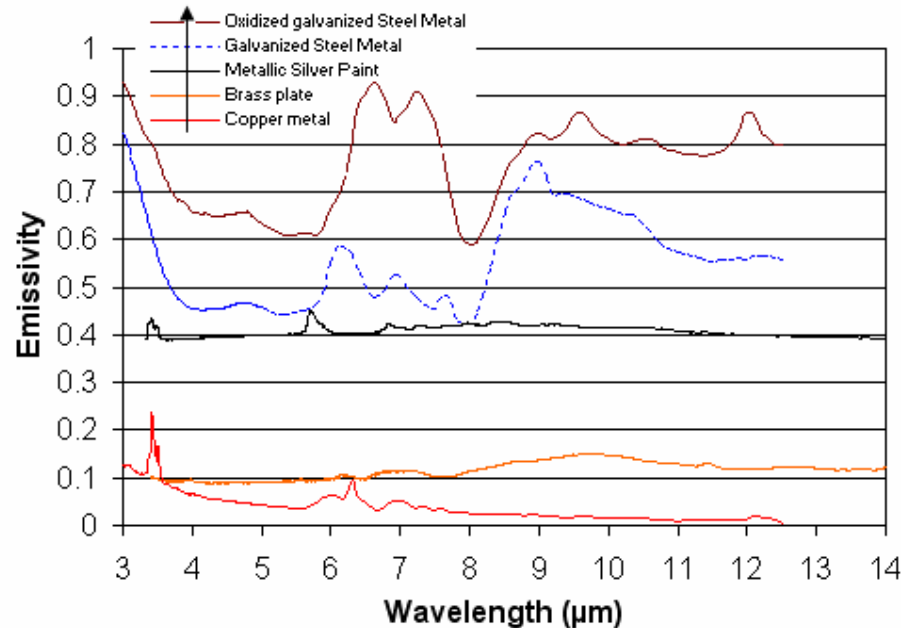
- need for atmosphere compensation step



What about emissivity ?

In all cases we need an information on emissivity to get temperature

- relations for emissivity : **only for ideal materials**, for example Drude law for pure metals (satisfactory only for $\lambda > 2\mu\text{m}$, not valid for corroded or rough surfaces)
- databases for specific materials **in particular state** of roughness, corrosion, coatings, contaminant, moisture content ...



- **Practical solution : simultaneous evaluation of temperature and emissivity**

Single-color pyrometry

- Measurement is performed in a narrow to large spectral band
- In any case, after sensor calibration, the retrieved radiance is of the form

$$L_s(\lambda, T) = \varepsilon(\lambda) B(\lambda, T)$$

One equation, two unknown parameters



One has to estimate the emissivity (a priori knowledge)

- Sensitivity of temperature to an error in emissivity estimation:

$$\frac{dT}{T} = - \left(\frac{T}{B} \frac{dB}{dT} \right)^{-1} \frac{d\varepsilon}{\varepsilon} \approx - \frac{\lambda T}{C_2} \frac{d\varepsilon}{\varepsilon}$$

at 1 μm and T= 1100K : -0.8K/% error
 at 10 μm and T= 300K : -0.6K/% error

- advantage in working at short wavelength (visible or UV pyrometry): sensitivity to emissivity error drops.
- However, the signal drops at short wavelength } → compromise

Two-color pyrometry (1/2)

- Adding a new wavelength adds an equation but also an unknown parameter namely the emissivity at this additional wavelength.

- Two spectral signals:

$$\begin{cases} L(\lambda_1, T) = \varepsilon(\lambda_1)B(\lambda_1, T) \\ L(\lambda_2, T) = \varepsilon(\lambda_2)B(\lambda_2, T) \end{cases}$$

- by ratioing the signals:

$$\ln(L_2 \lambda_2^5) - \ln(L_1 \lambda_1^5) = \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \frac{C_2}{T} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

effective wavelength: $\lambda_{12} = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}$

Effective wavelength can be high : bad news !!

- The problem can be solved if one has a knowledge about the **emissivity ratio**.
- Common hypothesis (but not necessary) : « greybody » assumption $\varepsilon(\lambda_1) = \varepsilon(\lambda_2)$
- Sensitivity of temperature to an error in emissivity estimation:

$$\frac{dT}{T} \approx - \frac{\lambda_{12} T}{C_2} \left(\frac{d\varepsilon_1}{\varepsilon_1} - \frac{d\varepsilon_2}{\varepsilon_2} \right)$$

at 1µm/1.5µm and T= 1100K : -2.5K/% error = **3 times higher**
 at 10µm/12µm and T= 300K : -3.7K/% error = **6 times higher**

Ratio pyrometry vs 1-color pyrometry

	1-color	2-color	3-color
Input	ϵ_1	$\frac{\epsilon_2}{\epsilon_1}$ (slope)	$\frac{\epsilon_2^2}{\epsilon_1 \epsilon_3}$ (curvature)
Error amplification on temperature	$-\frac{\lambda T}{C_2}$	$\times \frac{\lambda}{\Delta\lambda}$	$\times \left(\frac{\lambda}{\Delta\lambda}\right)^2$

Two-color pyrometry (2/2)

- Sensitivity of temperature to an error in emissivity estimation can be reduced by decreasing the effective wavelength (by increasing $1/\lambda_1 - 1/\lambda_2$)

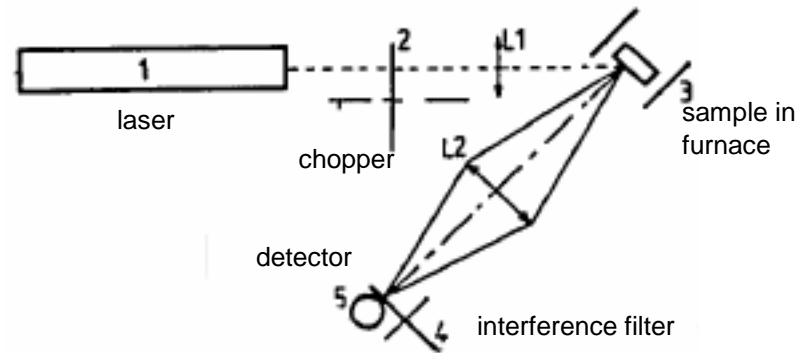
→ dilemma when spreading the wavelengths : will the “greybody” assumption still hold ?

- Advantage of ratio pyrometry over single color pyrometry : **immune to partial occultation, to variations of optical path transmission**

- 2-color photothermal pyrometry:

A laser is used to periodically heat the surface.
A lock-in detection is implemented to capture the modulated radiance **deprived from any reflection.**

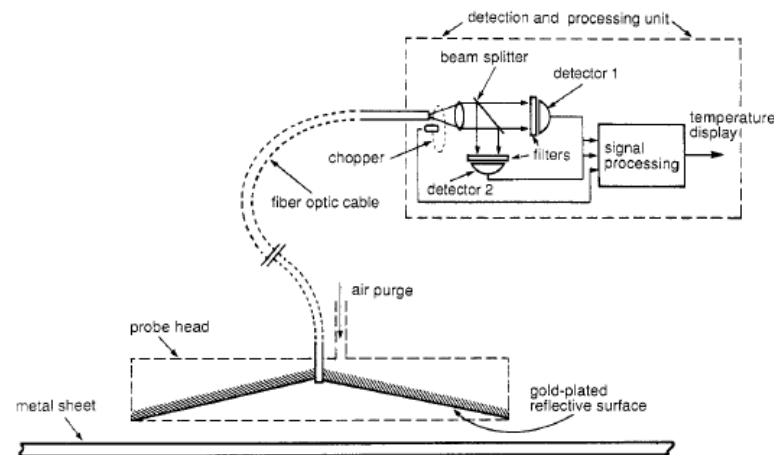
The signal ratio is:
$$\frac{\varepsilon(\lambda_1) \partial B / \partial T(\lambda_1, T)}{\varepsilon(\lambda_2) \partial B / \partial T(\lambda_2, T)}$$



T. Loarer at al., 1990

- Emissivity-enhanced
2-color pyrometry

Additional reflective surface for introducing a cavity effect (increase of both apparent emissivities, reduction of spurious reflections)



J.-C. Krapez at al., 1990

Multiwavelength pyrometry (MWP)

- Emissivity-temperature separation is essentially an **underdetermined inverse problem**:

$$N \text{ observables} \rightarrow L_s(\lambda_i, T) = \underbrace{\varepsilon(\lambda_i)}_{N \text{ unknown parameters}} \underbrace{B(\lambda_i, T)}_{1 \text{ unknown parameter}} \quad i = 1, N$$

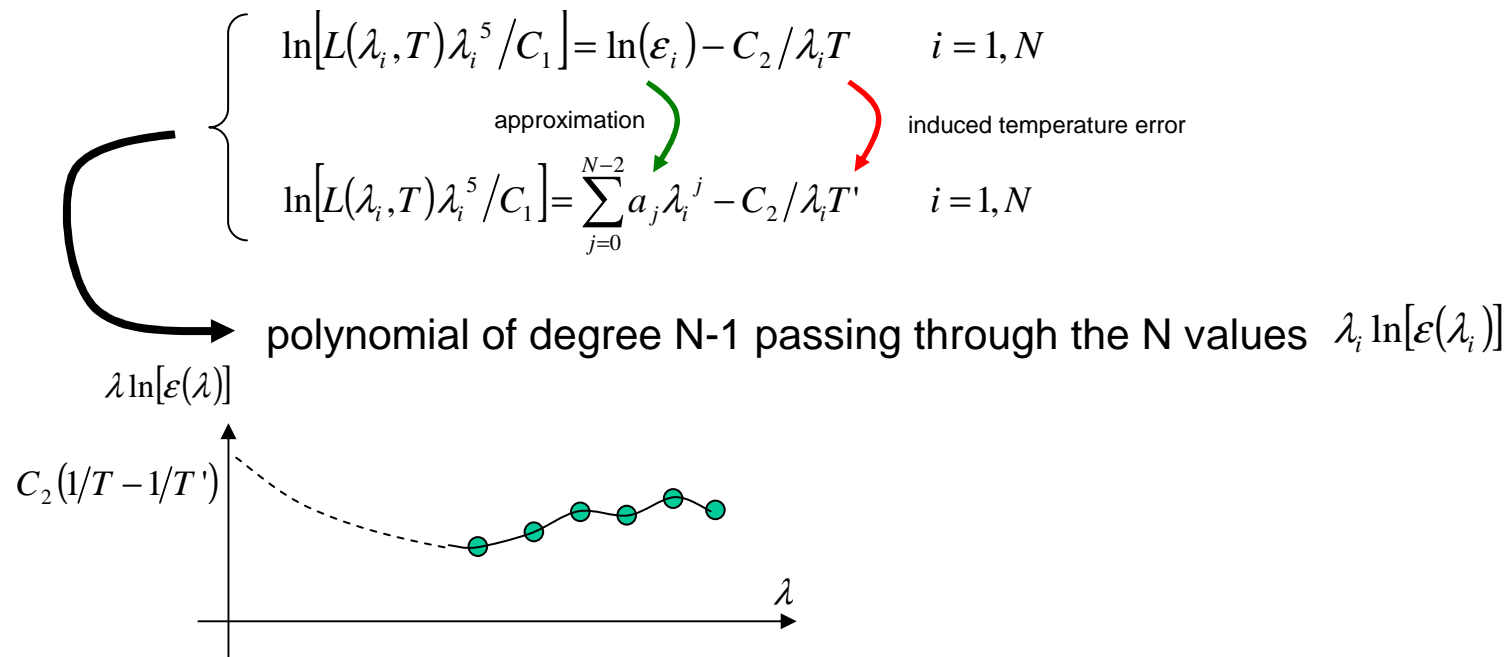
whatever the number of wavelengths/equations, there are always one more unknown parameters

- Two types of solutions:
 - reduce **by one** the degree of freedom of the discretized emissivity spectrum
 - N equations, N unknowns \rightarrow the problem should be solvable (?)
 \rightarrow **interpolation-based method**
 - regularization by using a **low-order emissivity model** (continuous or step function)
 - N equations, much less unknowns
 \rightarrow **least-square based method**
- Long time controversy: does MWP bring a real advantage as compared to single-color or two-color pyrometry ?

Multiwavelength pyrometry. Interpolation-based method (1/2)

“Just as needed” regularization : approximating the emissivity (or its log.) by a N-2 degree polynomial

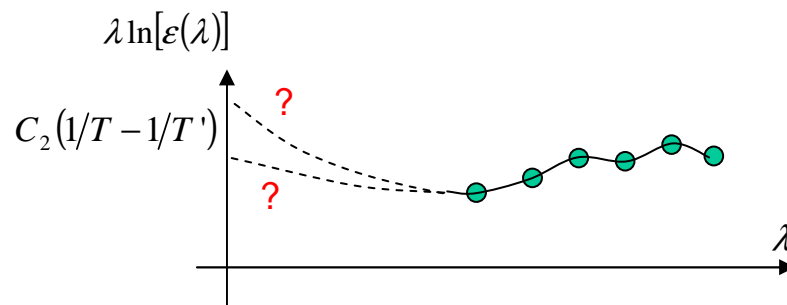
By considering the Wien approximation and taking the logarithm, Coates showed that this may lead to “catastrophic” results:



constant parameter = temperature error $C_2(1/T - 1/T')$
 i.e. **extrapolation result at** $\lambda = 0$

Multiwavelength pyrometry. Interpolation-based method (2/2)

there would be no error if a $N-2$ degree polynomial could be found passing **exactly** through the N values $\ln[\varepsilon(\lambda_i)]$ \longrightarrow highly improbable !



Therefore, in general, one has to count on **extrapolation** properties.

Unfortunately, extrapolation based on polynomial interpolation leads to **increasingly high errors as the polynomial degree rises !**

\longrightarrow **unpredictably high errors when adding new wavelengths**

Previous errors are **systematic**, i.e. **method errors** (errorless signal).

Same bad results are observed for **measurement errors** (they add to the previous ones).



The calculated temperatures are increasingly sensitive to measurement errors as the number of channels increases : **OVERFITTING** problem

Multiwavelength pyrometry. Low-order emissivity models (1/2)

The only solution : reduce the model complexity (low order model !)

Some examples of models :

$$\varepsilon(\lambda_i) = \sum_{j=0}^m a_j \lambda_i^j \quad i = 1, \dots, N \quad m < N - 2$$

$$\ln[\varepsilon(\lambda_i)] = \sum_{j=0}^m a_j \lambda_i^j \quad i = 1, \dots, N \quad m < N - 2$$

$$\varepsilon(\lambda_i) = 1 / (1 + a_0 \lambda_i^2) \quad i = 1, \dots, N$$

- Polynomials of $\lambda^{1/2}$ or $\lambda^{-1/2}$ for $\ln[\varepsilon(\lambda)]$
- Functions involving the brightness temperature $T_R = B^{-1}(\lambda, L_s)$
- Sinusoidal function of wavelength
- Step function (**grey-band model** with N_b bands).
 - 2 or 3 channels per band
 - up to N-1 single-channel bands and one dual channel band

Multiwavelength pyrometry. Low-order emissivity models (2/2)

Observable : $Y_i = \ln[L(\lambda_i, T)\lambda_i^5 / C_1] + e_i$ ↖ measurement error (noise)

Wien approximation

Polynomial approx. of $\ln[\varepsilon(\lambda_i)]$

Minimizing the weighted sum $\sum_{i=1}^N \sigma_i^{-2} \left(Y_i - \left(\sum_{j=0}^m a_j \lambda_i^j - \frac{C_2}{\lambda_i T} \right) \right)^2$ } → Linear least squares problem

Observable : $Y_i = L(\lambda_i, T) + e_i$ ↖ measurement error (noise)

Planck's law

Polynomial approx. of $\varepsilon(\lambda_i)$

Minimizing the weighted sum $\sum_{i=1}^N \sigma_i^{-2} \left(Y_i - B(\lambda_i, T) \sum_{j=0}^m a_j \lambda_i^j \right)^2$ } → Non-linear least squares problem

Multiwavelength pyrometry. Linear least squares problem (1/5)

One is looking for the polynomial coefficients and the temperature such that:

$$\hat{\mathbf{P}} = \left[\hat{a}_0 \quad \dots \quad \hat{a}_m \quad \hat{T} \right]^T = \arg \operatorname{Min}_{a_j, T} \sum_{i=1}^N \left(Y_i - \left(\sum_{j=0}^m a_j \lambda_i^j - \frac{C_2}{\lambda_i T} \right) \right)^2$$

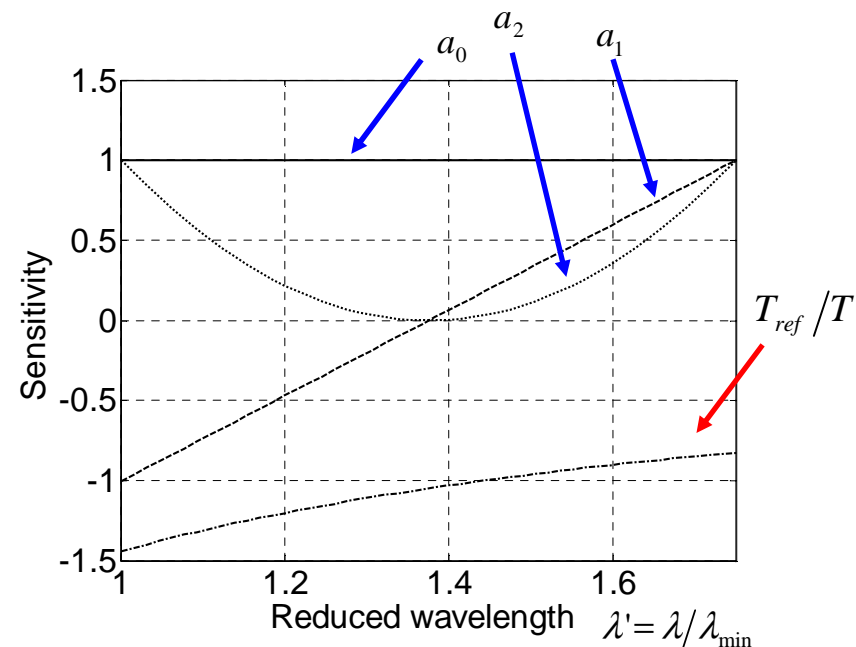
Parameter reduction for numerical purposes:

$$\lambda_i^* = 2 \frac{\lambda_i - \lambda_{\min}}{\lambda_{\max} - \lambda_{\min}} - 1 \quad P_T^* = T_{ref} / T \quad \text{such that} \quad C_2 / \lambda_i T_{ref} \approx 1$$

Sensitivity matrix to the reduced parameters:

$$\mathbf{X} = \begin{bmatrix} 1 & \lambda_1^* & \lambda_1^{*2} & \dots & \frac{-C_2}{\lambda_1 T_{ref}} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \lambda_N^* & \lambda_N^{*2} & \dots & \frac{-C_2}{\lambda_N T_{ref}} \end{bmatrix}_{N, m+2}$$

The sensitivity to the temperature inverse is very smooth, close to linear. We can thus expect a **strong correlation between the parameters (near collinear sensitivity vectors).**



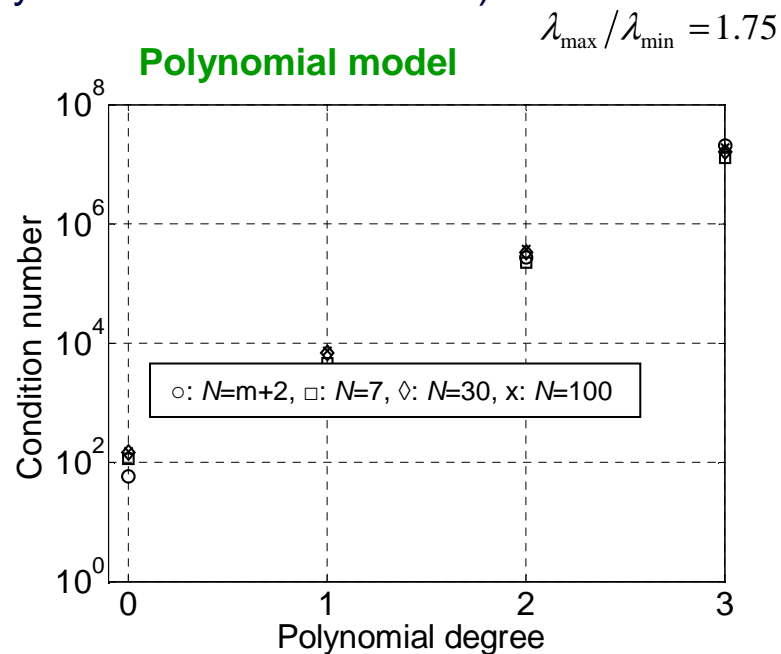
Multiwavelength pyrometry. Linear least squares problem (2/5)

Assuming that the measurement errors are additive, uncorrelated and of uniform variance, an estimation of the parameter vector $\hat{\mathbf{P}}^*$ in the least squares sense is obtained by solving the linear system :

$$(\mathbf{X}^T \mathbf{X}) \hat{\mathbf{P}}^* = \mathbf{X}^T \mathbf{Y}$$

Near collinear sensitivity vectors lead to a **high condition number** of $(\mathbf{X}^T \mathbf{X})$

The **condition number** (ratio of maximum to minimum eigenvalue) indicates the rate at which the identified parameters will change with respect to a change of the observable (sensitivity to measurement errors)



Huge increase of the condition number with the polynomial degree

Problems are expected with models of degree 2 and more

Multiwavelength pyrometry. Linear least squares problem (3/5)

Condition number : only an upper bound of error amplification.

The diagonal of the covariance matrix $(\mathbf{X}^T \mathbf{X})^{-1}$ is of greater value for analyzing the error propagation

$$[\sigma_{p^*}^2] = \text{diag}((\mathbf{X}^T \mathbf{X})^{-1}) \sigma^2$$

← assumed uniform variance of the observable

error around the mean estimator value due to **radiance error propagation to the parameters**
(does not include the bias due to model error, i.e. misfit between true emissivity and emissivity model)

Error amplification factors

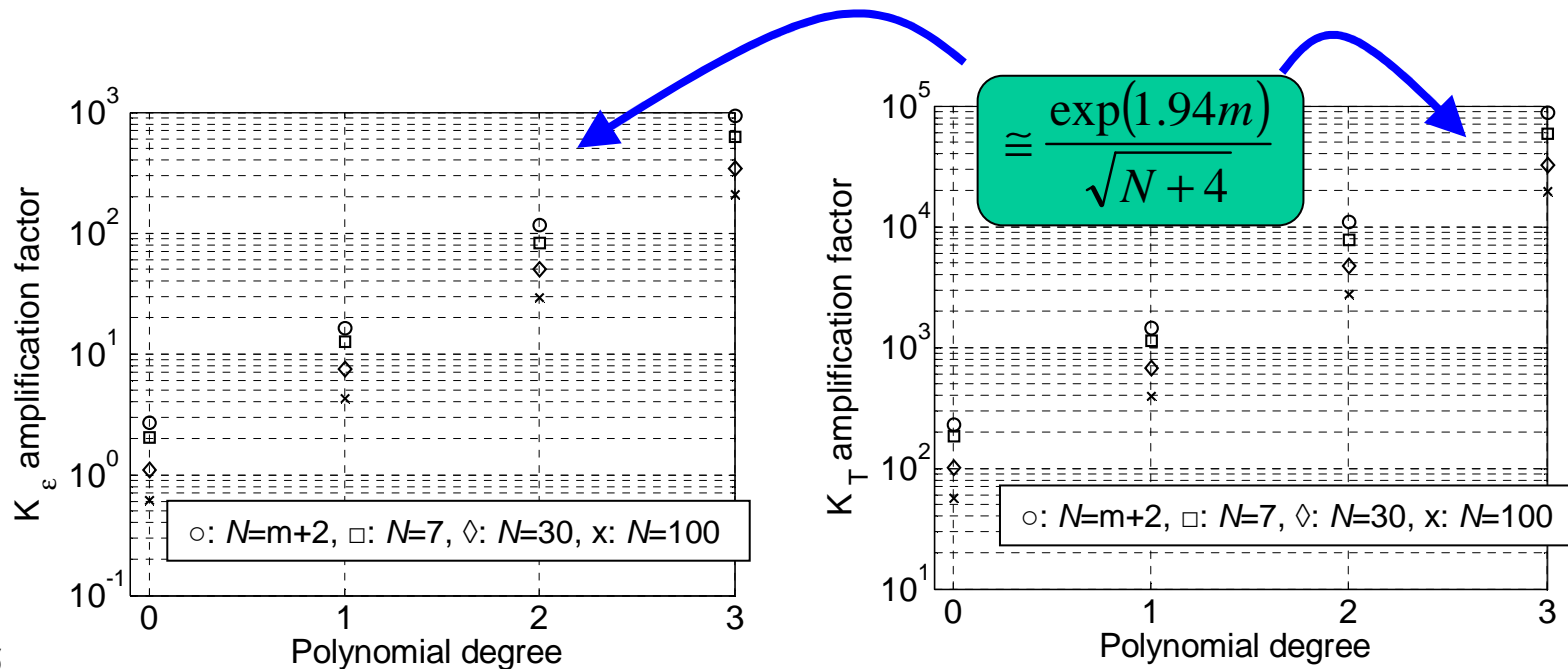
$$\frac{\sigma_{\varepsilon}}{\varepsilon} = K_{\varepsilon} \frac{\sigma_L}{L}$$

$$\frac{\sigma_T}{T} = K_T \lambda_{\min} T \frac{\sigma_L}{L}$$

Multiwavelength pyrometry. Linear least squares problem (4/5)

Polynomial model

Illustration for $\lambda_{\max} / \lambda_{\min} = 1.75$



The errors are rapidly rising with the degree of freedom

Multiwavelength pyrometry. Linear least squares problem (5/5)

Practical application :

- target at 320K,
- 1% radiance noise
- radiometer with **seven wavelengths** between 8 and 14 μ m

Polynomial model

Polynomial degree	σ_T (K)	σ_ε
0	1.5	0.02
1	9.4	0.13
2	64	0.83

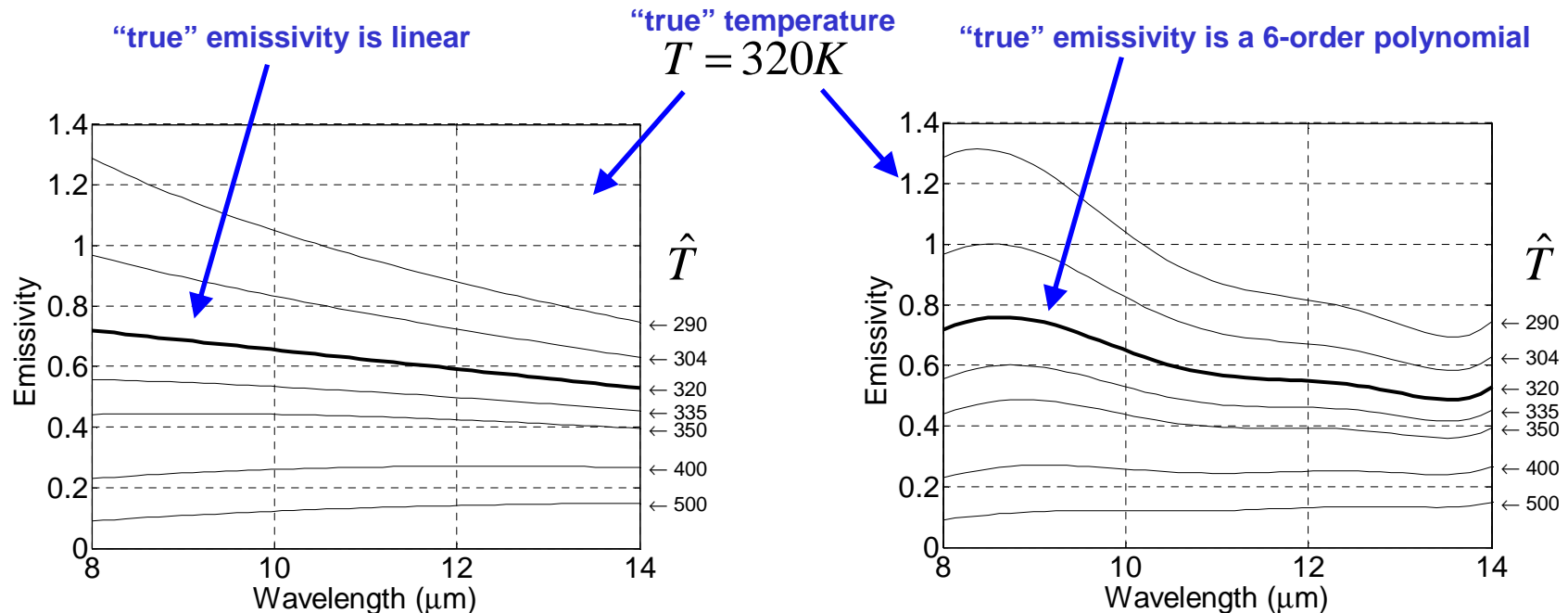
The mentioned standard errors only reflect what happens when noise corrupts the radiance emitted by a surface **which otherwise perfectly follows the chosen model (polynomial model of degree 0, 1 or 2)**

Multiwavelength pyrometry. A look to the ETS solutions (1/2)

To each estimated temperature value \hat{T} one can associate an emissivity profile $\hat{\varepsilon}(\lambda, \hat{T})$ according to:

$$\hat{\varepsilon}(\lambda, \hat{T}) = \frac{L(\lambda, T)}{B(\lambda, \hat{T})} = \underbrace{\varepsilon(\lambda)}_{\text{"true" emissivity}} \underbrace{\frac{B(\lambda, T)}{B(\lambda, \hat{T})}}_{\text{"true" temperature}}$$

They constitute an infinite number of perfect solutions to the underdetermined problem.

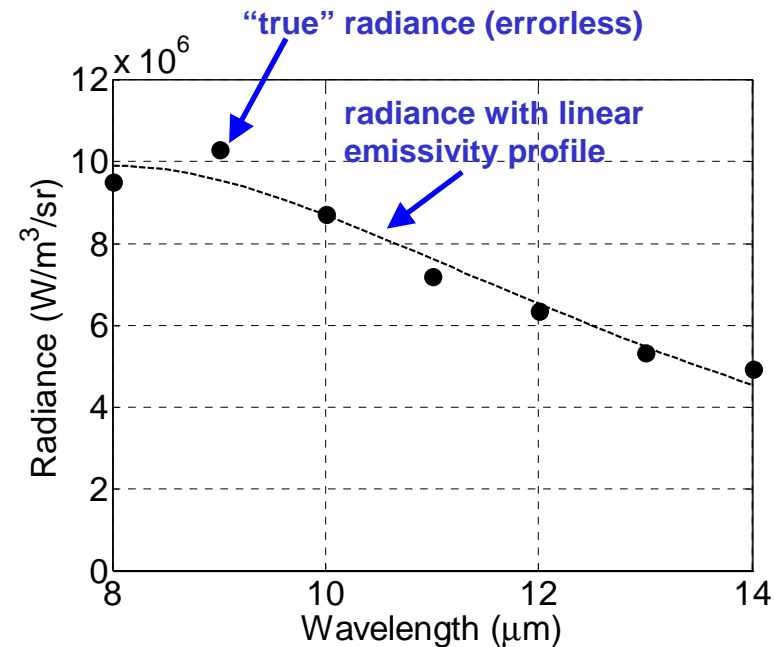
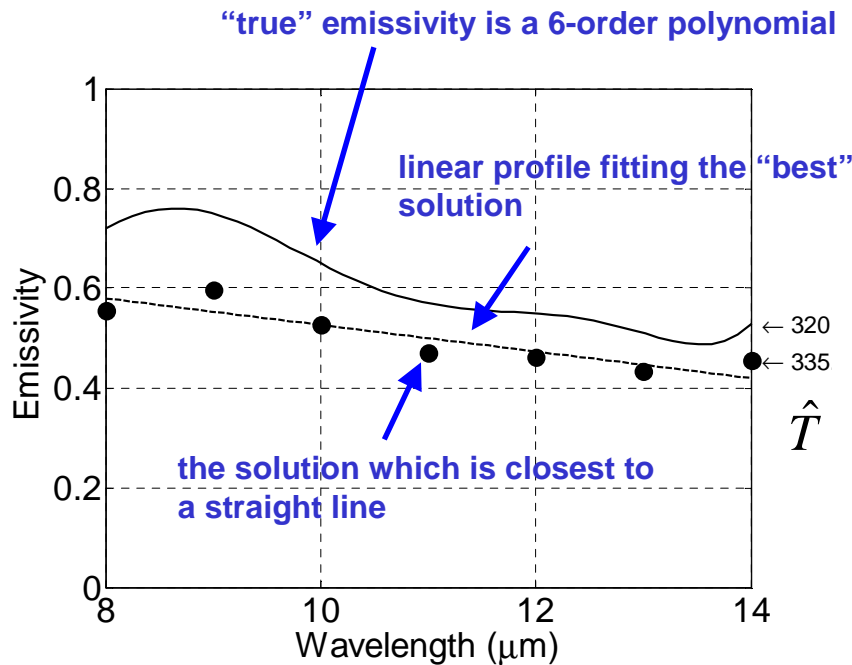


Let us now consider a 1-degree emissivity model. **Which, among all candidate profiles, fits a straight line at best ?**

Multiwavelength pyrometry. A look to the ETS solutions (2/2)

Misleading idea : « the chosen model is used to fit the true emissivity profile »
 Actually, the least squares method selects among all possible solutions, **the one which conforms at best to the model**, taking into account a weighting by $B(\lambda, \hat{T})$

7-channels pyrometer [8-14 μ m]

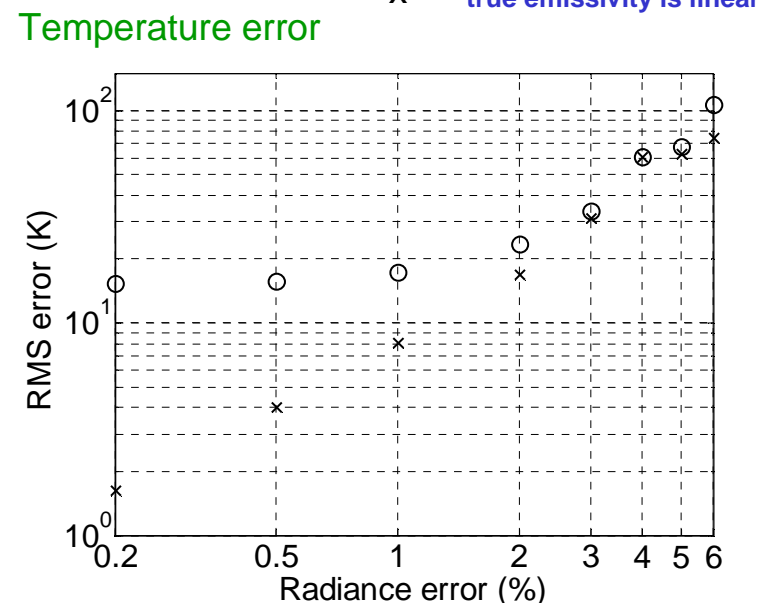
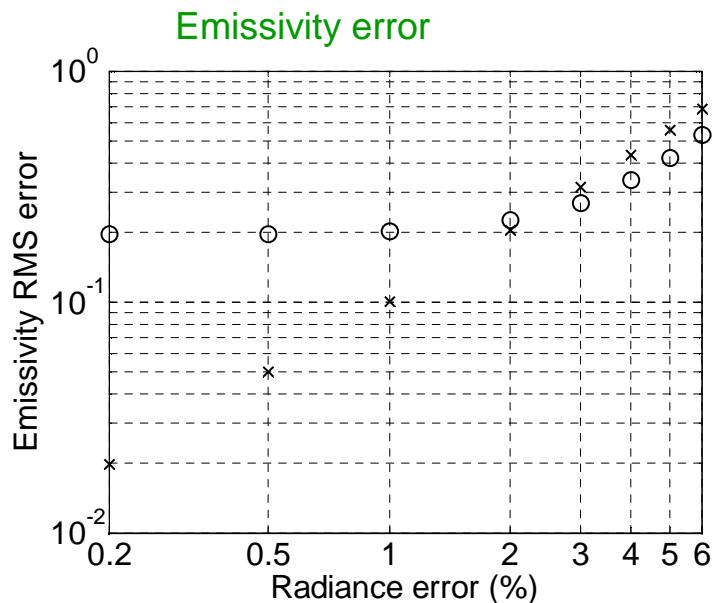


Errorless radiance leads to a 15K bias for temperature and 0.06 to 0.2 emissivity underestimation (**systematic or model error**)

With a 2-degree polynomial model, the results are even worse : $\hat{T} = 230\text{K}$, $\hat{\epsilon} > 2$!

Multiwavelength pyrometry. Non-linear least squares and Monte-Carlo analysis (1/3)

- Measurements are simulated by adding artificial noise to the theoretical emitted radiance (Gaussian distribution with a spectrally uniform standard deviation : 0.2% to 6% of the maximum radiance value)
- Statistical analysis on 200 simulated experiments
- Chosen model : 1-degree polynomial



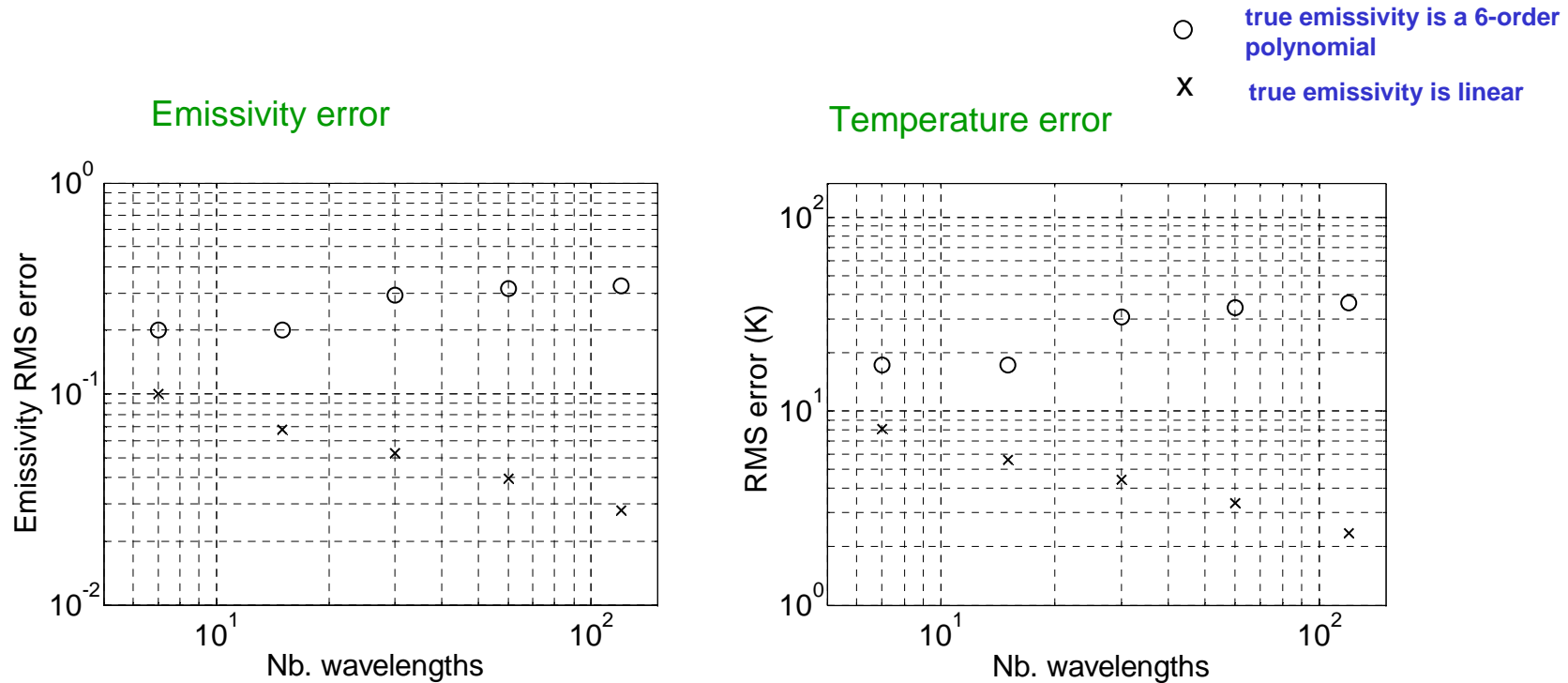
○ true emissivity is a 6-order polynomial
 x true emissivity is linear

High systematic error when the emissivity model (1-degree polynomial) doesn't match with the true profile (**>15K RMS !**).

Otherwise, 0.1 emissivity error and 8K temperature error for 1% radiance error. Same holds when the true profile departs by 1% from a straight line !

Multiwavelength pyrometry. Non-linear least squares and Monte-Carlo analysis (2/3)


- Does it help to increase the number of spectral channels ?



When the emissivity model (1-degree polynomial) matches with the true profile we observe the classical $N^{-1/2}$ uncertainty reduction.

Otherwise, emissivity and temperature RMS error remain high (systematic errors always dominate); they even increase with N for present example !

Conclusion on LSMWP with low-order emissivity models

- Reasonable RMS values can be obtained only when the implemented emissivity model **perfectly matches** the real emissivity spectrum. Otherwise, there are **important systematic errors**
- When can we guaranty that a specific model perfectly matches to reality ?
- LSMWP focuses on **profile shape rather than on magnitude**  Add a penalization based on emissivity level (mean or local) to force the solution to remain close to a predetermined level (*a priori* information)



back to one-color pyrometry !

- When using **only** the **emitted** spectral radiance, there is no valuable reason for implementing MWP instead of the simpler one-color or bispectral pyrometry

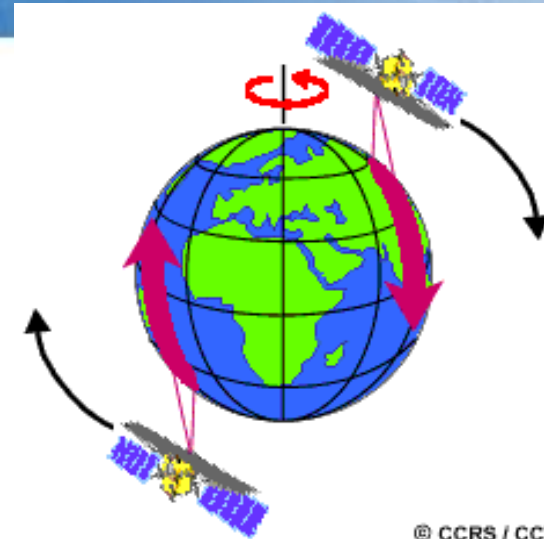
ETS in the field of remote sensing



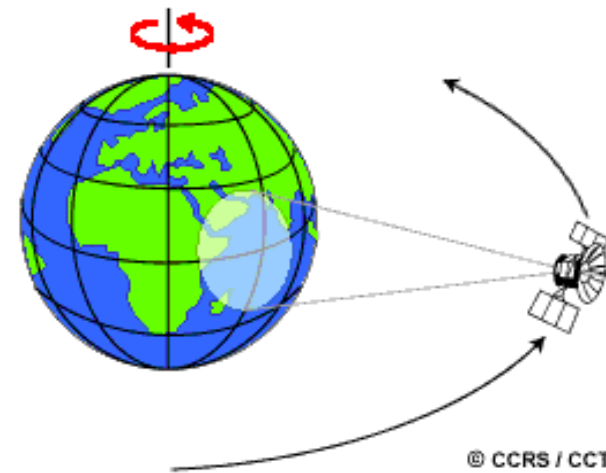
Low-altitude airborne remote sensing



High-altitude airborne remote sensing



Polar-orbiting satellites (low-earth orbit)

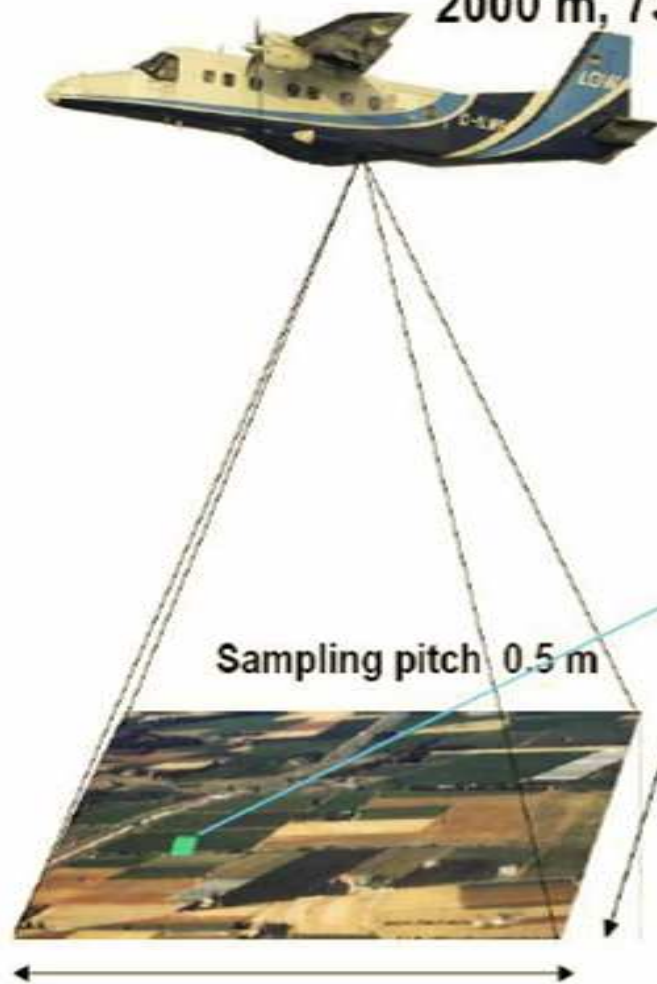


Geostationary satellite

SYSIPHE main characteristics

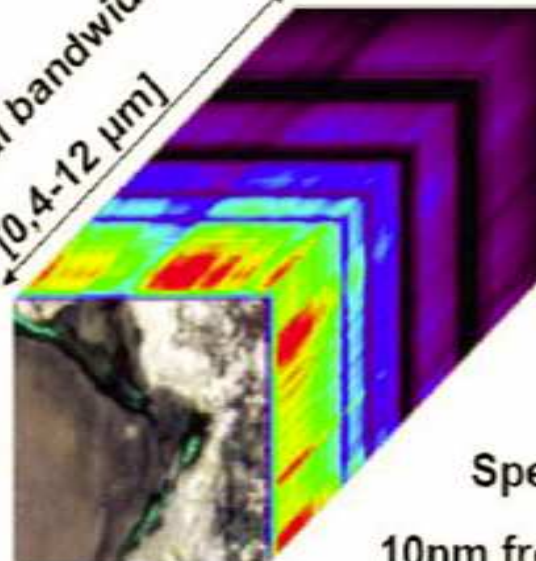
DLR Dornier DO-228

2000 m, 73 m/s



Sampling pitch 0.5 m

Spectral bandwidth
[0.4-12 μm]



« Hypercube »

Spectral sampling

10nm from 0.4 to 2.5 μm

20 cm^{-1} from 3 to 5.3 μm

10 cm^{-1} from 8 to 11.5 μm

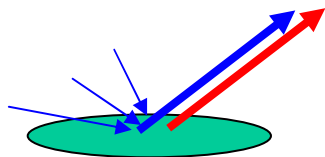
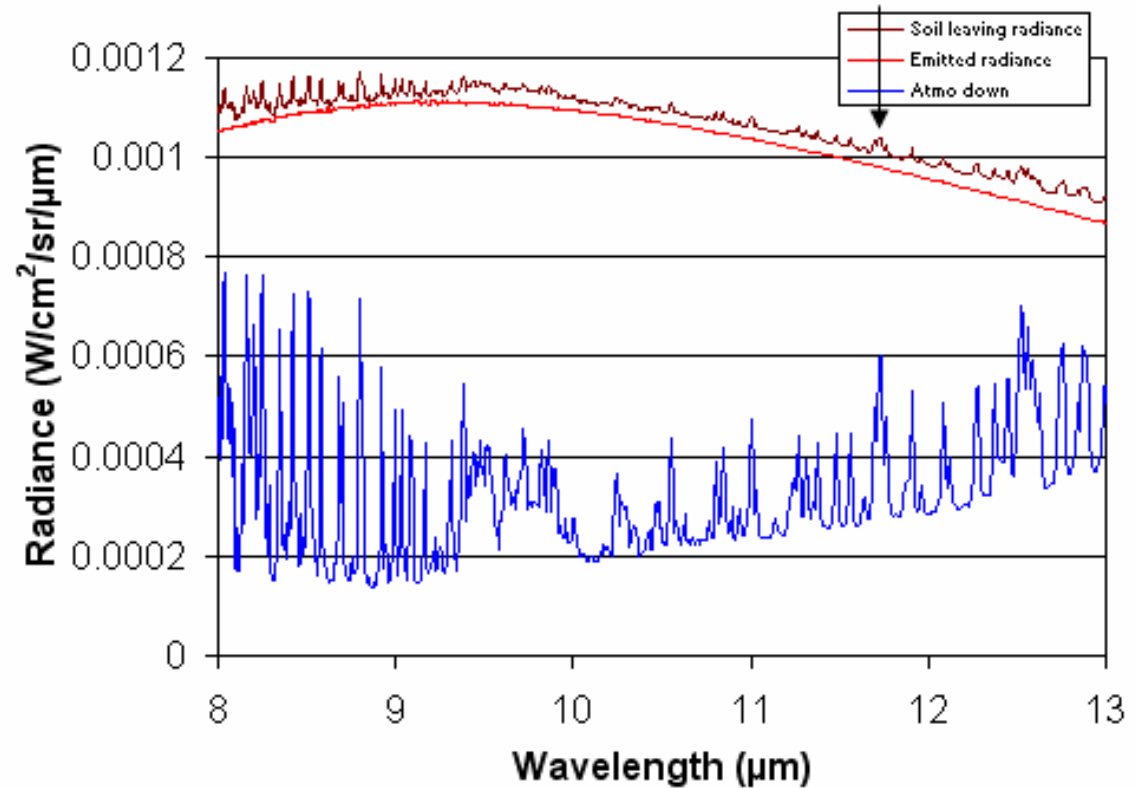
> 3000 m

Specific features of IR remote sensing

- Measurements are highly conditioned by the **radiative properties of the atmosphere** (transmission, emission toward the earth surface and then reflection, emission along the optical path, scattering, ...). Optical path in air from ~100 m to several km.
 - **Atmosphere compensation** is necessary
 - Atmosphere properties considered **uniform** in images of several km²
- Footprint is generally large: from ~10 cm for low altitude airborne sensors to ~2 km for sensors on geostationary satellites → **aggregation** of various materials and temperatures (**desaggregation = inversion problem**)
- In [8-14μm] band, natural surfaces (soil, vegetation, water) have **high emissivity values** (> 0.9). Generally considered as Lambertian.

Evaluation of atmosphere contributions

- Example of a grey surface ($\epsilon=0.9$) at $T=313\text{K}$
- Radiative transfer simulations with MODTRAN; (mid-latitude summer atmospheric model; rural aerosols)

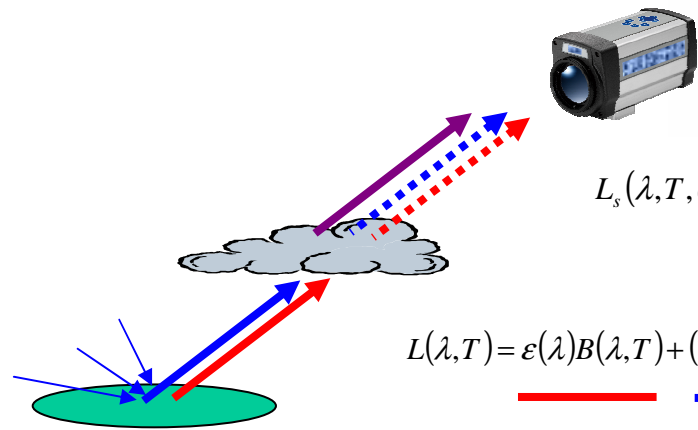
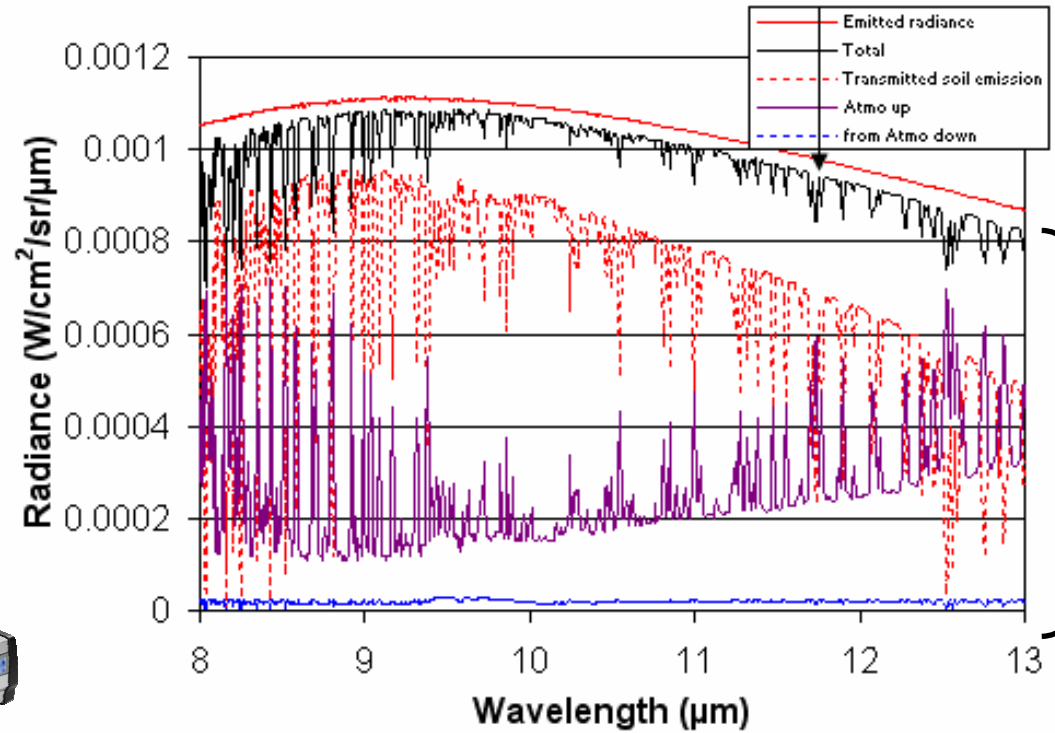


$$L(\lambda, T) = \epsilon(\lambda)B(\lambda, T) + (1 - \epsilon(\lambda))L^\downarrow(\lambda)$$



Evaluation of atmosphere contributions

- Example of a grey surface ($\epsilon=0.9$) at $T=313\text{K}$ sensed by an IR instrument at 1900 m altitude.
- Radiative transfer simulations with MODTRAN; (mid-latitude summer atmospheric model; rural aerosols)



$$L_s(\lambda, T, \theta, \varphi) = \tau(\lambda, \theta, \varphi)L(\lambda, T) + L^\uparrow(\lambda, \theta, \varphi)$$




$$L(\lambda, T) = \epsilon(\lambda)B(\lambda, T) + (1 - \epsilon(\lambda))L^\downarrow(\lambda)$$




at-sensor radiances

Atmosphere separate compensation

Radiative transfer simulation (MODTRAN, MATISSE...) with:

- standard atmospheric models (temperature+humidity profiles)/climate/season/aerosols
- radiosonde data  profiles of pressure, temperature, constituents
- IR sounding near $4.3\mu\text{m}$ for CO_2 and between $4.8\text{-}5.5\mu\text{m}$ for H_2O + neural networks allows retrieving mean atmosphere temperature and columnar water vapor under the sensor. These values are then used to scale a set of standard atmosphere profiles used in MODTRAN and get closer to the true atmosphere profiles. Final MODTRAN computation


$$\left\{ \begin{array}{l} \tau(\lambda, \theta, \varphi) \\ L^\uparrow(\lambda, \theta, \varphi) \\ L^\downarrow(\lambda) \end{array} \right.$$

Emissivity-Temperature separation

- Proper atmosphere compensation provides **ground leaving radiance**:

$$L(\lambda, T) = \frac{L_s(\lambda, T) - \underline{L^\uparrow(\lambda)}}{\underline{\tau(\lambda)}} = \underline{\varepsilon(\lambda)} \underline{B(\lambda, T)} + (1 - \underline{\varepsilon(\lambda)}) \underline{L^\downarrow(\lambda)}$$

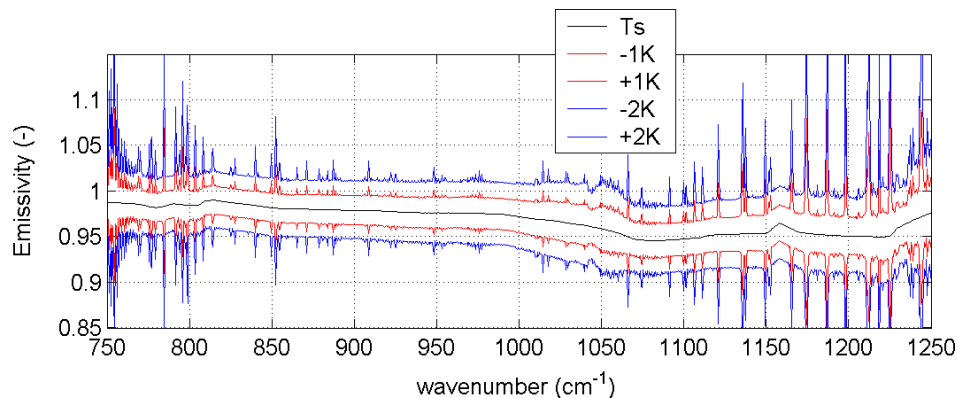
Emissivity estimation $\hat{\varepsilon}(\lambda)$ from a temperature estimation \hat{T} according to

$$\hat{\varepsilon}(\lambda) = \frac{L(\lambda, T) - L^\downarrow(\lambda)}{B(\lambda, \hat{T}) - L^\downarrow(\lambda)}$$

Spectral Smoothness method (SpSm) (1/2)

- When temperature estimation \hat{T} **is in error**, the profile will contain **detailed spectral features** originating from $L(\lambda, T)$ and $L^\downarrow(\lambda)$ (gas absorption bands) $\rightarrow \hat{\epsilon}(\lambda) = \frac{L(\lambda, T) - L^\downarrow(\lambda)}{B(\lambda, \hat{T}) - L^\downarrow(\lambda)}$

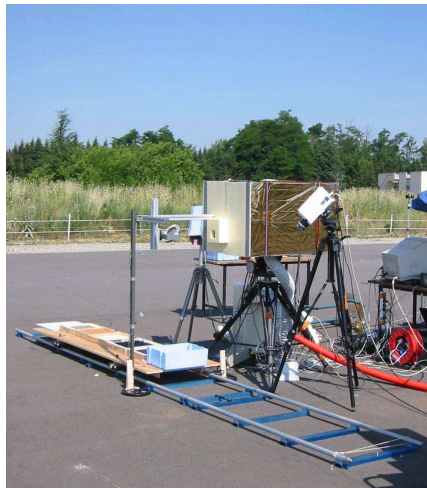
- Adjust \hat{T} until $\hat{\epsilon}(\lambda)$ is **deprived of these artifacts** \rightarrow “smooth” emissivity spectrum



Knuteson, 2006

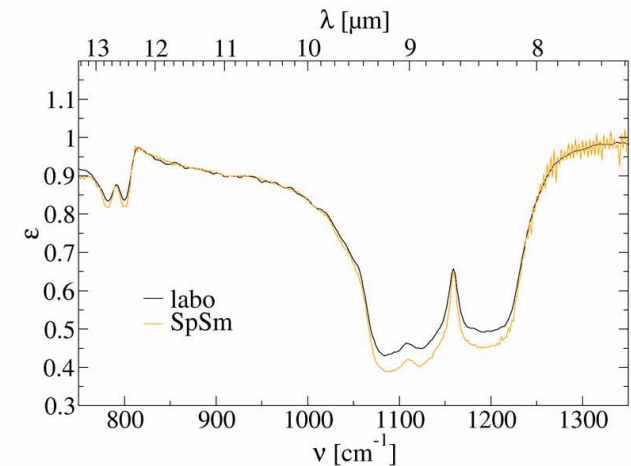
Spectral Smoothness method (SpSm) (2/2)

- Field tests at ONERA (K. Kanani thesis)



Spectroradiometer :
BOMEM MR254

- SpSm requires the **atmospheric compensation to be very precise**
- SpSm requires **high spectral resolution** ($< 10 \text{ cm}^{-1}$) in order to capture sufficient details of the atmosphere spectral features. Restricted to **hyperspectral data**. Spectral calibration errors are highly detrimental
- Radiance error of 0.5% \rightarrow 1.6K RMS and 0.8K bias for temperature and 0.023 RMS and 0.027 bias for emissivity



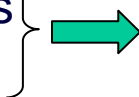
Multi-temperature method : a pitfall ? (1/3)

one more unknown
N more data

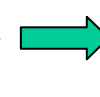


is the ill-conditioning
solved ???

- N_T temperature levels
- N channels



- $N+N_T$ unknowns
- $N \times N_T$ equations



Solvable (in principle)
if $N \geq 2$

However, using Wien's approximation, it can be shown that, when there is no reflection contribution, the problem **remains ill-conditioned !**

- With errorless radiance, there is an infinite number of solutions defined by:

$$\begin{cases} \frac{1}{\hat{T}_t} = \frac{1}{T_t} + cst \\ \hat{\varepsilon}(\lambda) = \varepsilon(\lambda) \exp\left(\frac{C_2}{\lambda} cst\right) \end{cases}$$

- For two temperatures, the sensitivity matrix is

Sensitivities are correlated as $\det(\mathbf{X}^T \mathbf{X}) = 0$

$$\mathbf{X} = \begin{bmatrix} \mathbf{I}_N & \frac{-C_2}{\lambda_1 T_{ref}} & 0 \\ & \dots & \dots \\ & \frac{-C_2}{\lambda_N T_{ref}} & 0 \\ & 0 & \frac{-C_2}{\lambda_1 T_{ref}} \\ \mathbf{I}_N & \dots & \dots \\ & 0 & \frac{-C_2}{\lambda_N T_{ref}} \end{bmatrix}_{2N, N+2}$$

Multi-temperature method (2/3)

- The problem remains badly conditioned when using Planck's law
- Degeneracy is alleviated thanks to the presence of reflections
- Inversion robustness depends on the spectral richness of the reflections

Case of two temperatures.

Nonlinear least-squares approach for identifying the N emissivities and the two temperatures

$$[\varepsilon_i, T_1, T_2]^T = \arg \text{Min}_{\varepsilon_i, T_1, T_2} \sum_{i=1}^N \left(L(\lambda_i, T_1) - (\varepsilon_i B(\lambda_i, T_1) + (1 - \varepsilon_i) L^\downarrow(\lambda_i)) \right)^2 + \left(L(\lambda_i, T_2) - (\varepsilon_i B(\lambda_i, T_2) + (1 - \varepsilon_i) L^\downarrow(\lambda_i)) \right)^2$$

Illustration for the case of a greybody ($\varepsilon=0.9$) at $T_1 = 320\text{K}$.

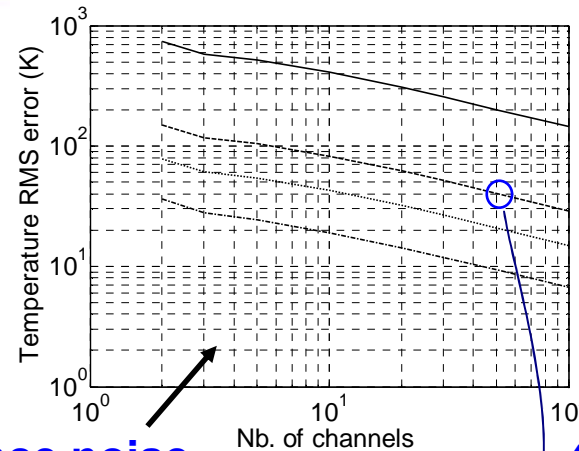
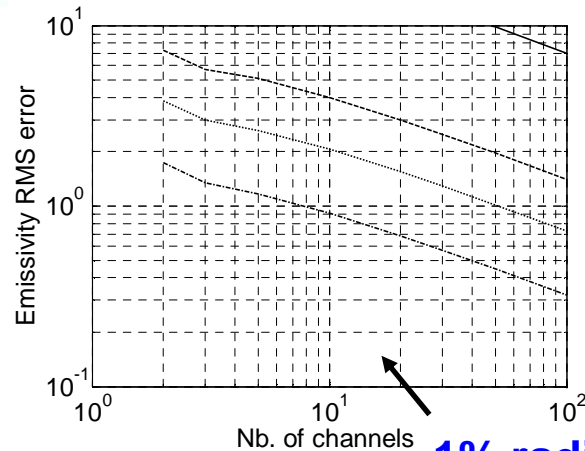
Second temperature is 1K, 5K, 10K or 30K higher.

Downwelling radiance is either:

- blackbody radiance at 300K
- same by weighting with a uniform random distribution (simulation of the presence of detailed spectral features)

Standard errors of identified parameters obtained from covariance matrix (local linearization)

Multi-temperature method (3/3)

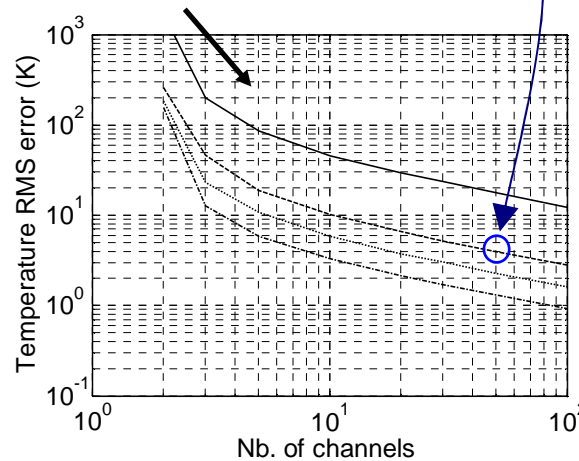
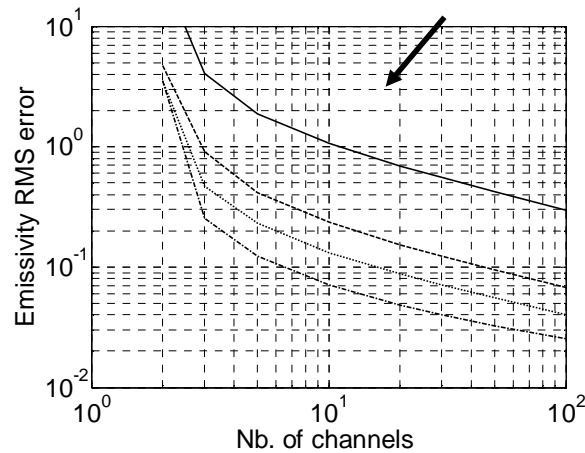


ΔT
 1K
 5K
 10K
 30K

**smooth
 downwelling
 radiance**

1% radiance noise

40K to 4K



ΔT
 1K
 5K
 10K
 30K

**spectrally rich
 downwelling
 radiance**

Better results are obtained by increasing the number of channels and the temperature difference

High constraints to get a temperature RMS error lower than 1K !

Constraints on images co-registration, emissivity stability.

Conclusion

- Radiative temperature measurement
 - advantage : non-contact
 - disadvantage : underdetermined inverse problem due to emissivity
- « Mirage » of multiwavelength pyrometry
 - only very low order emissivity models could be considered (ex: 1 degree polyn.)
 - no significant benefit vs. single or two color pyrometry
- IR remote sensing takes profit from high emissivity of natural surfaces and from their spectral smoothness with respect to downwelling radiance
 - SpSm method : implementation phase for Sysiphe hyperspectral camera
 - Multi-temperature method
 - ineffective without reflections from spectrally rich and well characterized environment
 - additional constraints