Loss effect analysis of irreversible Stirling cycle refrigerator

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Abstract - This paper demonstrates the share of different losses over the respective total losses and the effects of different losses on cooling performance of a Stirling machine using air as a working gas. The modified simple model is applied using the geometrical parameters of the FEMTO-ST engine model. The share of different power and heat losses as well as their effect on cooling performance have been evaluated at different rotational speeds and charging pressure. The share of fluid friction loss and loss due to regenerator ineffectiveness increase with rotational speed and charging pressure.

Keywords: Stirling refrigerator; Power losses; Heat losses; Performance; Share of losses.

Nomenclature

A	cross sectional area, m^2
A_{wg}	wetted area of the metal, m^2
C	average molecular speed, $\mathrm{m.s}^{-1}$
C_p	isobaric specific heat, $\mathrm{J.kg}^{-1}\mathrm{K}^{-1}$
C_v	isochoric specific heat, $\mathrm{J.kg}^{-1}\mathrm{K}^{-1}$
D	piston diameter, m
d	hydraulic diameter, m
f_r	Reynolds friction factor
G	mass flux, $\mathrm{kg.m}^{-2}.\mathrm{s}^{-1}$
J	annular gap between the cylinder and piston displacer, m
K	heat conductivity, $\mathrm{W.m}^{-1}\mathrm{K}^{-1}$
L	length, m
m	mass of working fluid, kg
m_{leak}	mass leakage , kg
n	rotational speed, rpm
NTU	number of transfer unit
N_{st}	Stanton number
Q	heat, m
P	pressure, Pa
Pr	Prandtl number
R	gas constant, $\mathrm{J.kg}^{-1}\mathrm{K}^{-1}$
Re	Reynolds number
s	stroke , m
T	temperature, K
V	volume, m^3
$W_{i,ad}$	ideal adiabatic work, m
Greek symbols	
Δ	difference
ϵ	regenerator effectiveness
γ	ratio of specific heats, (C_p/C_v)
ω	angular speed, $\mathrm{rad.S}^{-1}$
τ	compression ratio
θ	crank angle, rad
Index and exponent	
c	compression space
cr	chiller
e	expansion space
g	gas
h	hot heat exchanger
r	regenerator

1. Introduction

The Stirling engine is one of a variety of external heat engines that can be used to generate power from various thermal energy sources. First, it was invented in 1816 by Robert Stirling as

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a heat engine. However, the benefits of Stirling engines have been revealed with the advancements engineering and the demonstration of new methods to implement this thermodynamic cycle. It could be used in the reverse cycle as the Stirling refrigerator as it was first realized as a cooling machine in 1832 [1]. Due to these advantages, Stirling machines are suitable alternatives for power generation and refrigeration in various applications. The Stirling cycle refrigerating machine designs are categorized as kinematic machines, where the piston and displacer are mechanically linked to the drive shaft, or free-piston machines, where the piston is coupled to the power supply by a linear motor and the displacer is driven by the gas pressure variation in the system. Appropriate design and modeling of Stirling machines will result in better performance as it will minimize irreversibilities associated with the operation and geometrical configuration.

The impact of losses has been investigated on the performance of irreversible solar-powered Gamma type Stirling engine [2]. They highlighted that the two main dissipations are regenerator imperfection loss and leakage loss in the engine. These losses are mostly dependent on geometrical operational parameters. A combination of geometric and operating parameters was optimized for the Beta-type Stirling engine by Ahmed et al. [3]. In this research, it has been discussed that almost all losses increase with operating speed and charging pressure. However, the share of each loss have not been discussed.

Theoretical analysis and optimization of geometrical parameters for Stirling engines have been conducted [4]. In this research, it has been reported that alpha-type Stirling engines are less applicable for low-temperature difference, whereas Beta-type engine produced the highest shaft work.

A comprehensive review of the Stirling cooling machines has been conducted for a range of cooling applications [5]. The review have presented the general working principle, the configuration and drive mechanisms as well as research findings of Stirling refrigerators. Due to its wider application, high theoretical COP, as well as quieter and simpler operation, Stirling cycle cooler is worth researching. Different thermodynamic models for predicting the performance of the Stirling cooler were also proposed and developed in the past decades.

A Stirling refrigerator with a V-type configuration was approximated for a domestic cooling device with a charging pressure of 2 bar and compression ratio ranging between 1.5 and 2 [6]. The refrigerator performance parameters COP and cooling power showed a parabolic shape with a single optimal performance value with respect to operating speed [7]. Eid et al. [8] also investigate the effect of operating speed and they confirmed that cooling power has a single optimum value with respect to speed. It has been presented that, if engine speed increases too much, cycle efficiency will decrease as a result of the rapid increase in flow resistance and regenerator heat transfer losses [9].

The effects of rotational speed and charged pressure on different types of losses have been investigated for a Stirling engine [3, 9, 10]. All types of losses have been increased with an increase in operating speed as well as with charge pressure [3]. Heat conduction, shuttle heat, and flow resistance losses changed slightly as charged pressure increased [9]. Pressure drop loss, mechanical friction loss, and buffer space losses increased with rotational speed [10].

Due to losses associated with real-world conditions compared with ideal situations used for analytical purposes, a practical Stirling refrigerator can't achieve a performance of the Carnot limit. A non-ideal second-order thermal model with losses has been developed for domestic Stirling refrigerator [11]. However, the effect and trend of each loss on the performance have not been investigated so far in detail for the Stirling cycle refrigerator. This research tries to analyze the share of different types of losses with respective categories and their effect on the overall performance of the Stirling cycle refrigerator.

The purpose of this paper is to analyze the percentage share of different losses and to investigate the effects of these losses on engine performance.

2. Theory

The Stirling cycle machine consists of two variable volumes (compression and expansion) spaces physically separated by a regenerator and three constant volumes. For the Stirling cycle refrigerator, heat is absorbed from the low-temperature heat source, part of the heat is stored and released in the regenerator, and rejected to a hot heat sink (see Figure 1).

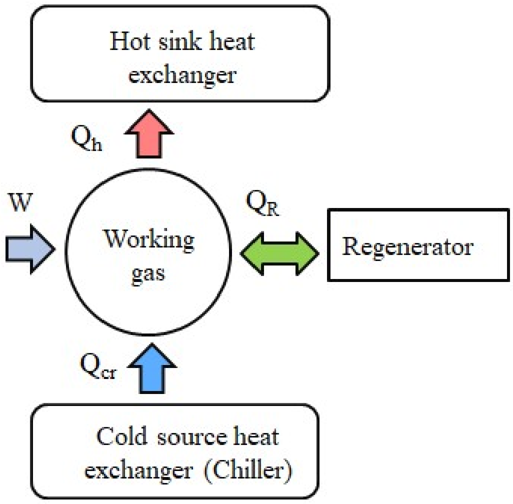

Figure 1 : *Schematic diagram of Stirling refrigerator*

The four thermodynamic transformations for an ideal reversed Stirling cycle includes:

- an isothermal expansion where heat is absorbed from the cold source (chiller),
- isochoric heating where the heat stored in the regenerator is transmitted to the working fluid,
- an isothermal compression where heat is rejected to the heat sink (hot heat exchanger)
- isochoric cooling where the gas transmits heat to the regenerator.

A non-ideal thermodynamic model has been developed in the author's previous article [11]. In such a thermodynamic model, the overall Stirling refrigeration machine is configured into five control volumes (two working spaces and three heat exchangers) serially connected similar to works of Urieli and Berchowitz [12]. The overall approach for driving the equation set that could represent for the analysis of such type of machine is to apply energy conservation equation and equation of state for each control volume and link with the equation of conservation of mass across the whole refrigerating machine. In this paper, such a model is used to evaluate the share of different losses and their effect on the cooling performance of the Stirling refrigerator.

2.1. Numerical model

The performance of Stirling machine is strongly dependent on geometrical and physical parameters such as dimensions, heat transfer coefficients, heat source temperatures and regenerator characteristics. Predicting the performance of Stirling machine is important for the design of the machine. The ideal Stirling cycle has been already studied by several researchers; however the performance of real Stirling machine is far from the ideal Stirling cycle.

The system of ideal adiabatic differential equations was modified as described by author's previous work [11] through including the effects of gas leakage from working space to buffer space (crankcase) and shuttle heat loss by displacer from compression to expansion spaces for the Stirling cycle refrigerating machine. The summary of modified ideal adiabatic equation is presented as shown in Table 1.

Parameters	equations
Pressure	$dP = \frac{-P\gamma(\frac{dV_c}{T_{ch}}+\frac{dV_e}{T_{cre}})+\gamma R\frac{dQ_{shut}}{C_p}(\frac{T_{ch}-T_{cre}}{T_{ch}T_{cre}})-2\gamma Rdm_{leak}}{\frac{V_c}{T_{ch}}+\gamma(\frac{V_h}{T_h}+\frac{V_r}{T_r}+\frac{V_{cr}}{T_{cr}})+\frac{V_e}{T_{cre}}}$
Change of mass	$dm_i = m_i\frac{dP}{P} = \frac{dP}{R}\frac{V_i}{T_i}(where i = h, r, cr)$
	$dm_c = \frac{PdV_c+\frac{V_c dP}{\gamma}}{RT_{ch}} + \frac{dQ_{shut}}{C_pT_{ch}}$
	$dm_e = \frac{PdV_e+\frac{V_e dP}{\gamma}}{RT_{cre}} - \frac{dQ_{shut}}{C_pT_{cre}}$
	$dQ_{shut} = \frac{\pi s^2 K_g D_d}{8JL_d}(T_c - T_e)$
Temperature variation	$dT_e = T_e(\frac{dP}{P} + \frac{dV_e}{V_e} - \frac{dm_e}{m_e})$
	$dT_c = T_c(\frac{dP}{P} + \frac{dV_c}{V_c} - \frac{dm_c}{m_c})$
	$dQ_{h,i} = \frac{V_h dPC_v}{R} - C_p(T_{ch}\dot{m}_{ch} - T_h\dot{m}_{hr})$
	$dQ_{r,i} = \frac{V_r dPC_v}{R} - C_p(T_h\dot{m}_{hr} - T_{cr}\dot{m}_{rcr})$
Energy	$dQ_{cr,i} = \frac{V_{cr} dPC_v}{R} - C_p(T_{cr}\dot{m}_{rcr} - T_{cre}\dot{m}_{cre})$
	$dW_{i,ad} = dW_c + dW_e$ where
	$dW_c = PdV_c$ and $dW_e = PdV_e$

Table 1 : *Summary of modified ideal adiabatic model (adapted from [11]).*

Then, the model was further corrected to modified simple model by including power and heat losses. These losses include regenerator imperfection loss, fluid friction loss, mechanical friction, conduction heat loss in the regenerator wall, pressure drop due to finite speed of piston, conduction heat loss, and gas spring hysteresis loss as shown in Table 2. The details of these losses are also presented in the author's previous work [11].

The numerical model presented above, was evaluated by considering the FEMTO 60 Stirling engine in which the main parameters and dimensions of the device are tabulated in Table 3 operating as a refrigerating machine as a case study.

3. Results and discussion

In this section, the shares of different types of losses associated with the Stirling cycle with the respective category are presented. Furthermore, the trends of these losses with operating

Losses	equations
Loss due to regenerator imperfection	$Q_{rl} = mc_p(1-\epsilon)(T_c - T_e)$
	$\epsilon = \frac{NTU}{NTU+1}$, $NTU = N_{st}\frac{A_{wg}}{A}$
	$N_{st} = 0.023Pr^{-0.6}Re^{-0.2}$
Losses due to pressure drop in heat exchangers	$W_{fr} = \int_0^{2\pi}(\Delta P\frac{dV_e}{d\theta})d\theta$
	where $\Delta P = \Delta P_h + \Delta P_r + \Delta P_{cr}$ and
	$\Delta P_i = \frac{2f_r\mu V_i G_i l_i}{m_i {d_i}^2}$ (Where $i = h, r, cr$)
Mechanical friction losses	$W_{mec.fr} = 2\Delta P_{mec.fr}V_{swc}$
	$\Delta P_{mec.fr} = \frac{(0.94+0.045sn)10^5}{3(1-1/3\tau)}(1-1/\tau)$
Conduction heat loss in the regenerator wall	$Q_{wrl} = k\frac{A_{wg}}{Lf}(T_{wh} - T_{wcr})$
Heat conduction losses	$\dot{Q}_{cond} = k\frac{A}{L}\Delta T$
Losses due to finite speed of piston	$W_{fin-sp} = 2\Delta p_{fin.sp}V_{swc}$
	and, $\Delta p_{fin.sp} = \frac{1}{2}(P\frac{aU_p}{C_c} + P\frac{aU_p}{C_e})$
	$a = \sqrt{3\gamma}$ and $C = \sqrt{3RT}$
Gas spring hysteresis losses	$\dot{W}_{hys} = \sqrt{\frac{1}{32}\omega\gamma^3(\gamma-1)T_wP_{mean}k_g}(\frac{V_d}{2V_t})^2A_{wg}$

Table 2 : *Summary of additional losses included.*

No	Parameters	value
1	Hot heat temperature (K)	305
2	Cooling temperature (K)	270
3	Piston diameter (mm)	60
4	Piston stroke (mm)	40
5	Regenerator length (mm)	50
6	Diameter of regenerator (mm)	82
7	Compression space swept volume (cm^3)	103
8	Expansion space swept volume (cm^3)	113
9	Working gas	Nitrogen
10	Frequency (Hz)	7.5
11	Charging pressure (bar)	17.5

Table 3 : *Main specifications of Stirling cooling machine used in this study (details can be found [13]).*

speed are discussed. These results are based on simulation findings of the second-order thermodynamic model described so far using air as a working fluid.

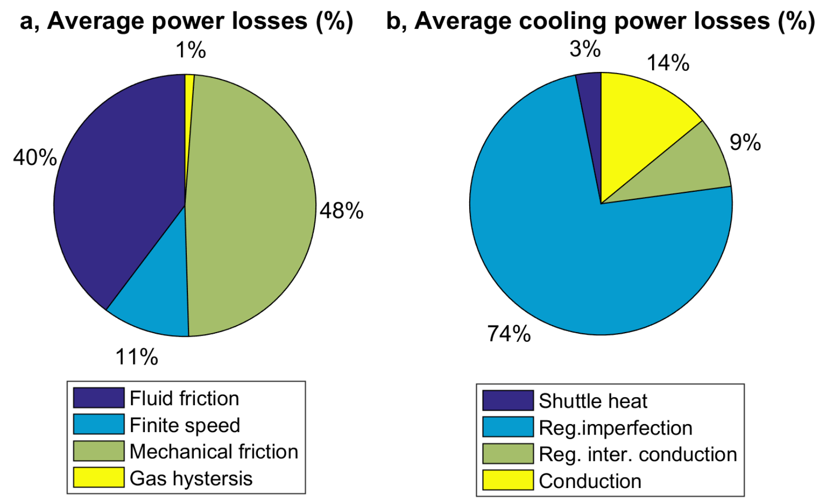

Figure 2 : *Analysis of the percentage of losses at a pressure of* 17.5 *bar*

Figure 2a and 2b demonstrate the percentage of different power and heat losses with respect to total losses at respective categories at a charging pressure of 17.5 bar. In Figure 2a, the average share of power loss is presented. It is clear that from the considered losses, the major share is mechanical friction loss (48%), followed by fluid friction loss due to pressure drop (40%). Others cover only 12% of the total power loss. Similarly in Figure 2b, the average share of heat loss is presented. The major heat loss is due to regenerator ineffectiveness which comprises 74%. The second major heat loss is conduction heat loss (14%) across the displacer due to the temperature difference between the two working spaces. This type of loss is basically more dependent on the temperature difference and hence the share may vary.

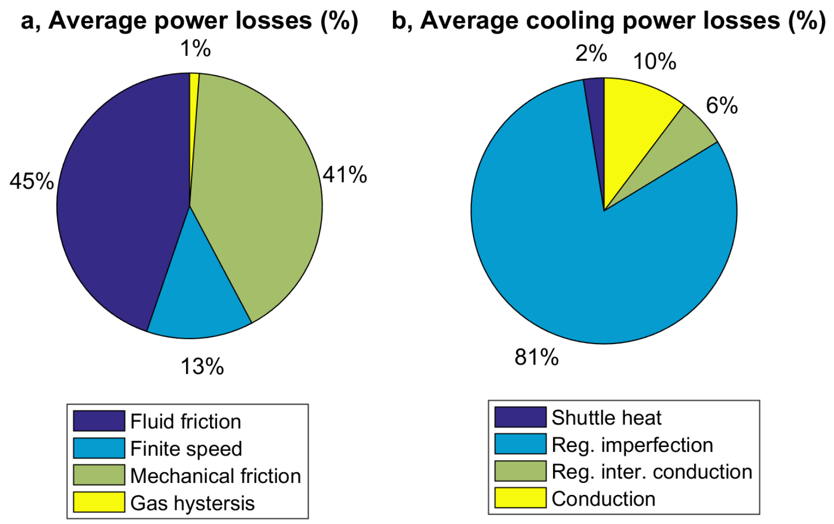

Figure 3 : *Analysis of the percentage of losses at a pressure of* 25 *bar*

Figure 3a and 3b show the percentage of different power and heat losses with respect to total losses at respective categories and at a charging pressure of 25 bar. Figure 3a, demonstrates the percentage share of different power losses and Figure 3b, show the percentage share of various heat losses. According to Figure 2, it is noticed that the percentage share of fluid friction loss and loss due to regenerator ineffectiveness increase with increase charging pressure. Great care should be given to minimization of fruid friction loss and regenerator ineffectiveness for machnines operating at higher pressure.

Figure 4a, 4b, 4c, and 4d illustrate the effect of rotational speed on the major power losses, major heat power losses, cooling power, and COP respectively at an ambient temperature of 300 K, chiller temperature of 270 K, and charging pressure of 17.5 bar. As demonstrated in Figure 4a, except for gas hysteresis power loss all power losses are mainly affected by rotational speed. Fluid friction loss increases strongly at a higher speed. As it is shown in Figure 4b regenerator ineffectiveness loss linearly increases and all other heat power losses relatively remain unchanged with the operating speed. From Figure 4c, it could be seen that cooling power loss increases with an increase in rotational speed. This is mainly due to the increase in the thermodynamic cycle per unit time. Lastly, as it is seen in Figure 4d, COP decreases with an increase in rotational speed. This is expected as demonstrated in Figure 4a and 4b, both power and heat power losses increase which resulted in lower COP at a higher speed.

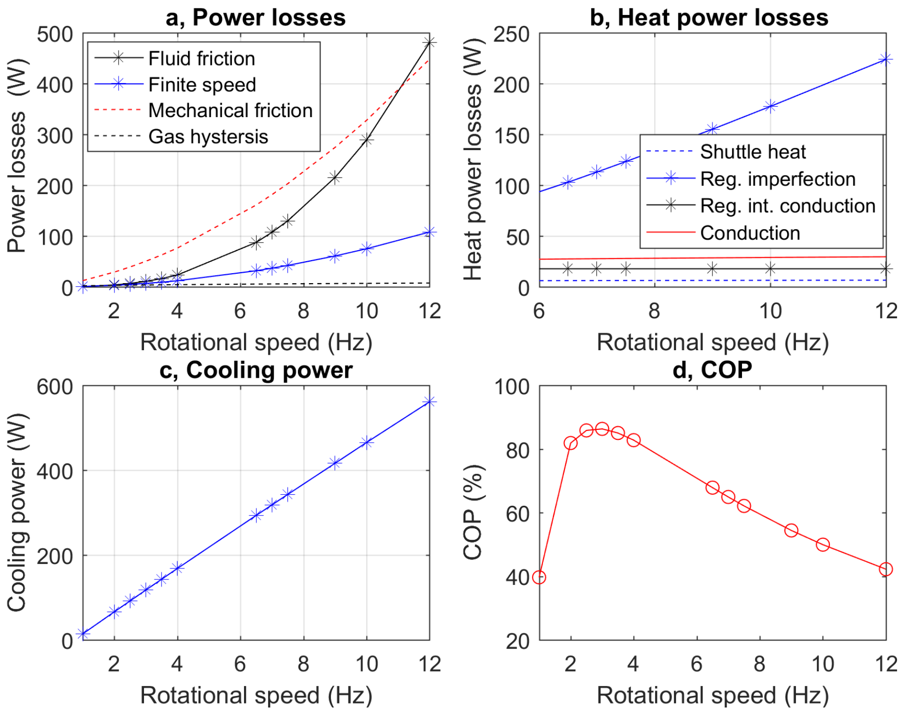

Figure 4 : *Trend of major losses and refrigeration performance with respect to the rotational speed*

4. Conclussion

The design and development of Stirling cycle refrigerators require an understanding of the processes that govern the operation of the machine. The operating condition and processes of the Stirling cycle refrigerator are accompanied by different types of losses. A non-ideal thermodynamic model is used to investigate the share of different losses and the trend of such losses with the operating speed and chargin pressure. The major power and heat losses, as well as their effect on the performance of the cooling machine, have been discussed. Fluid friction power losses and regenerator imperfection losses are found as the two most losses that are mostly affected by charging pressure and operating speed. The shares of these two losses over their respective total losses increase with an increase in operating speed as well as with charging pressure. The coefficient of performance of the refrigerating machine decreases with the increase of rotational speed primarily due to the increase of fluid friction and regenerator imperfection losses. Hence, during the design of the Stirling refrigerator, critical investigation of losses shall be conducted for the intended operating range of the machine to minimize major losses and enhance performance.

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