Polarization imaging of multiply-scattered radiation based on Integral-Vector Monte Carlo Method

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Polarization imaging

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Context

Size & structure characterization of complex-shaped particles in semi-transparent media

Non intrusive techniques $\rightarrow$ Electromagnetic waves (light)
Intensity + polarization (Stokes formalism, Mueller matrix)

Existing applications
- Optically thin media $\rightarrow$ single scattering regime
- Optically thick media $\rightarrow$ Diffusion approximation
- **in-between**

multiple scattering effects

To increase information: 2D distribution = imaging
Polarization pattern = signature of the medium
Objectives

2D Optical diagnostic of semi transparent heterogeneous media analyzing polarization state with the Stokes parameter formalism

Characterization of:
- Optical / Radiative properties
- Morphology
- Size dispersion
- Volumetric fraction

Examples of a large scope of applications
- Particle suspensions, soot
- Biological cells
- Circumstellar disks

Polarization pattern from multiple scattering effects → need of comprehensive models

Modeling polarization imaging in a 3D system: Integral-Vector Monte Carlo Method
**Formalism – Polarization**

Plane harmonic wave propagating in homogeneous medium

\[ E_x(z,t) = E_{0x} \cos(kz - \omega t) \hat{x} \]

\[ E_y(z,t) = E_{0y} \cos(kz - \omega t - \delta) \hat{y} \]

- \( \omega \): angular frequency
- \( k \): wave vector
- \( E_{0x}, E_{0y} \): component amplitudes
- \( \delta \): phase difference between components
Formalism – Polarization

If \( \delta = 0 \) \([2\pi]\)

\[
E(z,t) = (E_{0x} \cdot x + E_{0y} \cdot y) \cos(kz - \omega t)
\]

constant orientation of \( E \) (time independent)

linear polarization
Formalism – Polarization

If $\delta = \pi/2 \, [2\pi]$

$$E_{0x} = E_{0y} = E_0$$

$$E_x(z,t) = E_0 \cos(kz - \omega t) \, x$$

$$E_y(z,t) = E_0 \cos(kz - \omega t - \pi/2) \, y$$

$$= E_0 \sin(kz - \omega t) \, y$$

$$E(z,t) = E_0 \left[ \cos(kz - \omega t) \, x + \sin(kz - \omega t) \, y \right]$$

constant module of $E$ ($E_0$)

end of $E$ (in transverse plane) describes a circle

right circular polarization
Polarization imaging

### Formalism – Stokes vector & Mueller matrix

**General case:** Polarization state → elliptic polarization → ellipsometry

**How to describe polarization state of light with measurable quantities?**

→ **Stokes vector**

\[
I = \begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix} = \begin{pmatrix}
E_x E_x + E_y E_y \\
E_x E_x - E_y E_y \\
E_x E_y + E_y E_x \\
i(E_x E_y - E_y E_x)
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>Information about</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear polarization (horizontal / vertical)</td>
<td></td>
</tr>
<tr>
<td>Linear polarization (oblique: +/- 45°)</td>
<td></td>
</tr>
<tr>
<td>Circular polarization (right / left)</td>
<td></td>
</tr>
</tbody>
</table>

**Relation between a Stokes vector source and a detected Stokes vector**

→ **effective Mueller matrix**

\[
\begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix}_{\text{detected}} = \begin{pmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{pmatrix}\begin{pmatrix}
I \\
Q \\
U \\
V
\end{pmatrix}_{\text{source}}
\]
Problem definition

- System = Scattering, absorbing, non emitting cold medium
- Data = 2D distribution of $M_{ij}$ on a defined surface in the space surrounding the system $\rightarrow$ subdivision in pixels
- Detection within a given solid angle
- External radiation source
- Laboratory frame to keep track of the polarization state $\rightarrow$ definition of the meridian plan containing the direction of propagation under consideration

$$I_{sca} = \frac{\sigma}{4\pi} L(\pi - i_2) S L(-i_1) I_{inc} = \frac{\sigma}{4\pi} S_R I_{inc}$$
Integral formulation of the VRTE with a SOSS

The Vector Radiative Transfer Equation (VRTE)

\[
I(s_d, u_d) = I(s_{bd}, u_d) t(s_{bd}, s_d) + \int_{s_{bd}} ds' t(s', s_d) \frac{\sigma}{4\pi} \int du' S_R(s', u', u_d) I(s', u')
\]

\[
I(s_d, u_d) = I(s_{bd}, u_d) t(s_{bd}, s_d) + \int_{s_{bd}} ds' t(s', s_d) \frac{\sigma}{4\pi} \int du' S_R(s', u', u_d) \left\{ I(s', u') t(s'_b, s'_b) + \right. \\
\int ds'' t(s'', s') \frac{\sigma}{4\pi} \int du'' S_R(s'', u'', u') I(s'', u'') \left. \right\}
\]
Polarization imaging

**Integral formulation of the VRTE with a SOSS**

The Scattering Order of Scattering Series (SOSS)

$$I(s_d, u_d) = \sum_{k=0}^{\infty} I_k$$

$I_k$ is the $k^{th}$ scattering order Stokes vector
viz. the ensemble of contributions which are considered with $k$ scattering modifications of propagation direction between a source and the detector

$I_0 \neq 0$ only if a source is aligned with the detector

$$I_0(s_d, u_d) = I(s_s, u_s)t(s_s, s_d)$$

$I_1 \neq 0$ only if a source and the detector directions $(u_s, u_d)$ are in the same plane

General 3D case
at least two modifications of propagation direction are necessary to reach the detector from the source,
viz. $I_0 = I_1 = 0$
Polarization imaging

Integral formulation of the VRTE with a SOSS

First scattering orders in a uniform media

\[ I_1(s_d, u_d) = \frac{\omega}{4\pi \rho_d \sin \varphi_d} \exp \left( -\beta \left( x_d + \rho_d \tan \left( \frac{\varphi_d}{2} \right) \right) \right) S_R(u_d, x) I(s_s, x) \]

\[ I_2(s_2, u_2) = \int_{s_{b2}}^{s_2} \int_{s_{bs}}^{s_s} \left( \frac{\omega}{4\pi} \right)^2 \frac{t(s_s, s_0) t(s_0, s_1) t(s_1, s_2)}{\| s_1 - s_0 \|^2} S_R(u_2, u_1) S_R(u_1, x) I(s_s, x) ds_0 ds_1 \]

Position and direction of the source
Distance from the source on x axis
Distance from x axis
Extinction coefficient
Scattering angle
Albedo
Position and direction of the source
Distance from x axis
Distance from the source on x axis
Extinction coefficient
Scattering angle
Albedo
Integral formulation of the VRTE with a SOSS

Recursive formulation for higher orders

\[ I_k(s_d, u_d) = \frac{\omega}{4\pi} \int_{s_{bd}} \int_{0}^{2\pi} \int_{-1}^{1} t(s_{k-1}, s_d) S_R(u_d, u_{k-1}) I_{k-1}(s_{k-1}, u_{k-1}) d\eta_{k-1} d\psi_{k-1} ds_{k-1} \]

These integrals cannot be solved analytically for complex geometries.
Monte Carlo integration

Principle

\[
\int_D f(x)dx = \int_D pdf_X (x) \frac{f(x)}{pdf_X (x)} dx = \int_D pdf_X (x) w(x)dx = \lim_{N \to \infty} m, \quad m = \frac{1}{N} \sum_{i=1}^{N} w(x_i)
\]


- Judicious choices for **probability density functions (pdf)**
  
  + Computation of the series is a **backward** process: a priori better adapted to **directional detection** than a forward process

  ➔ **Efficient reduction of variance**

MC Integration ➔ sampling integration domains thanks to appropriate chosen pdfs
Monte Carlo integration

- Implementation
  - Variance and accuracy control (sample variance)

\[
\text{var}_m = \frac{1}{N} \left( \frac{1}{N} \sum_{i=1}^{N} \left( w(x_i) \right)^2 - \left( \frac{1}{N} \sum_{i=1}^{N} w(x_i) \right)^2 \right), \quad \text{var}_{sp} = \frac{N_{sp}}{N_{sp} - 1} \left( \frac{1}{N_{sp}} \sum_{i=1}^{N_{sp}} m_i^2 - \left( \frac{1}{N_{sp}} \sum_{i=1}^{N_{sp}} m_i \right)^2 \right)
\]

- MC Formulation

\[
I_k(s_d, u_d) = \frac{\omega}{4\pi} \int_{s_{bd}}^{s_d} \int_{0}^{2\pi} \int_{-1}^{1} t(s_{k-1}, s_d) S_R(u_d, u_{k-1}) I_{k-1}(s_{k-1}, u_{k-1}) d\eta_{k-1} d\psi_{k-1} ds_{k-1}
\]

- pdf choice → cumulative density function (cdf) → variable change

\[
R_{s_{k-1}} = \frac{1 - t(s_{k-1}, s_d)}{1 - t(s_{bd}, s_d)}, \quad R_{\psi_{k-1}} = \frac{\psi_{k-1}}{2\pi}, \quad dR_{\eta_{k-1}} = \frac{S_{R_{11}}(u_d, u_{k-1})}{2} d\eta_{k-1}
\]

\[
I_k(s_d, u_d) = \omega \left( 1 - t(s_{bd}, s_d) \right) \int_{0}^{1} \int_{0}^{1} \int_{0}^{1} \frac{S_R(u_d, u_{k-1})}{S_{R_{11}}(u_d, u_{k-1})} I_{k-1}(s_{k-1}, u_{k-1}) dR_{\eta_{k-1}} dR_{\psi_{k-1}} dR_{s_{k-1}}
\]
Backscattering configuration (validation)

i. against the semi-analytical results of Crosbie and Dougherty (1982) for **scalar backscattered intensities** in the case of a plane parallel layer of **isotropic scattering** media subjected to a Gaussian narrow beam

ii. against Ambirajan and Look (1997) results for the **backscattered Stokes vector** intensities calculated as a function of the distance of observation from a right circularly polarized narrow beam illuminating a plane-parallel medium laden with **spherical particles**
Rakovic et al (1999) 2D effective Mueller matrix contours from a half-space filled with a suspension of monodispersed spheres with a size parameter $x=13.4$ and a refractive index contrast of 1.192
Lateral configuration

Effective Mueller matrix images of a uniform non absorbing particle-laden solution

Polarized monochromatic source collimated at the center 2D Gaussian shape optical FWHM of 1.10^{-3} (narrow) \( \lambda = 0.633 \ \mu m \)

The refractive index ratio \( \frac{n_{\text{particles}}}{n_{\text{surrounding medium}}} = 1.195 \) (polystyrene particles in water at \( \lambda = 0.633 \ \mu m \))

Normal detection direction within a conical aperture of 2°
Sensitivity to morphology

“Aggregation”

- Random generation
- Fractal dimension $D_f = 1.8$
- Prefactor $k_f = 2$
- Monomer diameter : 40 nm
- Constant volume fraction, $f_v = 2 \times 10^{-4}$

<table>
<thead>
<tr>
<th>Agg</th>
<th>Mono</th>
<th>N10 (x=0.57)</th>
<th>N25 (x=0.77)</th>
<th>N50 (x=0.97)</th>
<th>N75 (x=1.1)</th>
<th>N75 bis (x=1.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ (m$^{-1}$)</td>
<td>3.822</td>
<td>11.87</td>
<td>20.93</td>
<td>26.87</td>
<td>27.30</td>
<td>29.02</td>
</tr>
</tbody>
</table>
Sensitivity to morphology

Monomer

N10

N25

N50

N75

N75bis

Scale

0
- 0.1

M_{11}

M_{12}

M_{21}

M_{22}

others

M_{ij}

\pm 0.1
Experimental setup

- Measure of Stokes parameters \( I = (I, Q, U, V)^t \), on each pixel of different pictures, at 90°, in a small solid angle.

Sample

Shaping the polarized collimated laser sheet:
- \( 458 < \lambda < 514 \) nm
- \( 1 < \) Power < 4 W

Polarizers (generators and analyzers)

Extinction ratio < 10^{-5}
For a given source, polarized by G, \( (I,Q,U,V) \) are measured with combinations of R and A:

- \( I = I_{0^\circ} + I_{90^\circ} \)
- \( Q = I_{0^\circ} - I_{90^\circ} \)
- \( U = I_{+45^\circ} - I_{-45^\circ} \)
- \( V = I_{\text{right}} - I_{\text{left}} \)

Then other quantities can be computed: \( Q/I, U/I \)

- Polarization degree
- Total linear circular
- 2D Mueller matrix
### Some experimental results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size parameter</td>
<td>$x = 0.8$</td>
</tr>
<tr>
<td>Volume fraction</td>
<td>$f_v = 0.01 %$</td>
</tr>
<tr>
<td>Scattering coefficient</td>
<td>$\sigma = 20 \text{ m}^{-1}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source polarization</th>
<th>$I/\text{Imax}$</th>
<th>$Q/I$</th>
<th>$U/I$</th>
<th>$V/I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source vertically polarized $(1,1,0,0)^t$</td>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
<td><img src="image4" alt="Image" /></td>
</tr>
<tr>
<td>$x = 2.35$</td>
<td>$f_v = 0.001 %$</td>
<td>$\sigma = 17 \text{ m}^{-1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A new model to generate polarization images in multiple scattering particle-laden media

- From the integral formulation of the VRTE
- With efficient statistical principles for convergence optimization

\[ \text{Integral-Vector Monte Carlo Method} \]

Could potentially be applied to any 3D geometry and kind of particles (provided that the issue of reflections at boundaries is addressed)

Validated in the case of plane-parallel backscattering configurations

2D lateral Mueller matrix elements for a cubic tank filled with a uniform suspension of monodispersed particles

\[ \text{Sensitivity of different Mueller matrix elements to particle size and morphology} \]
Expectation

- Further work
  - Continuation of this analysis from physical and statistical points of view
  - Extension to realistic situations such as systems undergoing an aggregation process
  - Experimental investigations for comparison and application
  - Step by step development of a parameter estimation methodology
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