L5-B: Measurements without contact in heat transfer: principles, implementation and pitfalls

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Outline

Temperature measurement by sensing the thermal emissive power

• Basics: Planck’s law, Wien’s law and s.o.

• Emissivity-Temperature Separation problem (ETS)
  • Pyrometry
    • single-color, bispectral pyrometry
    • multispectral pyrometry
  • ETS in airborne/satellite remote sensing
    • atmosphere compensation
    • Spectral-Smoothness method
    • Multi-temperature method

• Conclusion
Thermal radiation

Matter emits EM radiation
Intensity increases with temperature

Monitoring of emitted radiation offers a mean for temperature measurement

MidWave = 3 - 5.5 µm
LongWave = 7 - 14 µm
Advantages of the radiation method:
- non-contact
- surface probing (opaque material),
- surface to sub-surface probing (semi-transparent material)
- rapid: detectors with up to GHz bandwidth (and even higher)
- long distance measurement (airborne and satellite remote sensing, astronomy)
- point detectors (local measurement or 2D images by mechanical scanning)
- focal plane arrays (instantaneous 2D images)
- possibility of spectral measurements (multispectral, hyperspectral)
Radiation sensing is dependant on the atmosphere transmission, (absorption bands of air constituents: H$_2$O, CO$_2$, O$_3$, CH$_4$, …)

**MidWave**: 3 - 5.5 µm

**ShortWave**: 0.7 - 2.5 µm

**LongWave**: 7 - 14 µm
• Blackbody: perfect absorber, perfect emitter (~Holy Grail…)
• Spectral radiance given by Planck’s law:
  \[ B(\lambda, T) = \frac{C_1}{\lambda^5} \exp\left(\frac{C_2}{\lambda T}\right) - 1 \]
• Wien’s approximation:
  \[ B_w(\lambda, T) = \frac{C_1}{\lambda^5} \exp\left(-\frac{C_2}{\lambda T}\right) \]

Maximum given by Wien’s displacement law:
\[ \lambda_{max} T = 2898 \mu mK \]
Error of Wien’s approximation is less than 1% providing that \[ \lambda T < 3124 \mu mK \]
Basics (2/4)

Wavelength selection for temperature measurement

• Maximum of radiance given by Wien’s displacement law: $\lambda_{\text{max}} T = 2898 \mu mK$

• Radiance sensitivity to temperature (absolute sensitivity): $\frac{\partial B}{\partial T}$

  Maximum corresponding to: $\lambda T = 2410 \mu mK$

  $\lambda = 8.05 \mu m$
  for $T = 300K$

  $\lambda = 9.65 \mu m$
  for $T = 300K$
Wavelength selection for temperature measurement

- Radiance sensitivity to temperature (relative sensitivity): \( \frac{1}{B} \frac{\partial B}{\partial T} \)

For \( T = 300K \):
- 2% radiance increase per K at 8\( \mu \)m
- 16% radiance increase per K at 1\( \mu \)m

Advantage of performing measurements at short wavelengths (sensitivity is nearly in inverse proportion to wavelength)

Interest in visible pyrometry or even UV pyrometry?
Real materials (non-perfect emitters)

- with respect to blackbody, the emitted radiance $L(\lambda, T, \theta, \varphi)$ is reduced by a factor called emissivity:
  \[ L(\lambda, T, \theta, \varphi) = \varepsilon(\lambda, T, \theta, \varphi)B(\lambda, T) \quad 0 \leq \varepsilon \leq 1 \]

- emissivity depends on wavelength, temperature, and direction

- second Kichhoff’s law between emissivity and absorptance:
  \[ \varepsilon(\lambda, \theta, \varphi) = \alpha(\lambda, \theta, \varphi) \]

- relation between absorptance and directional hemispherical reflectance from the energy conservation law for an opaque material (the energy that is not absorbed by the surface is reflected in all directions):
  \[ \alpha(\lambda, \theta, \varphi) + \rho^\cap(\lambda, \theta, \varphi) = 1 \]

Emissivity can be inferred from a reflectance measurement (integrating sphere)
Drawback: need to bring the integrating sphere close to the surface
Contributors to the optical signal

- the surface reflects the incoming radiation (non-perfect absorber)
  - downwelling radiance: $L^\downarrow(\lambda, \theta, \varphi_i)$
  - bidirectional reflectance: $\rho^\circ(\lambda, \theta, \theta_i, \varphi_i)$
- the radiation leaving the surface is attenuated along the optic path (absorption, scattering by atmosphere constituents: gases, aerosols – dust, water/ice particles)
  - transmission coefficient: $\tau(\lambda, \theta, \varphi)$
- atmosphere emits and scatters radiation towards the sensor
  - upwelling radiance $L^\uparrow(\lambda, \theta, \varphi)$

\[
L_s(\lambda, T, \theta, \varphi) = \tau(\lambda, \theta, \varphi)L(\lambda, T, \theta, \varphi) + L^\uparrow(\lambda, \theta, \varphi)
\]

\[
L(\lambda, T, \theta, \varphi) = \varepsilon(\lambda, \theta, \varphi)B(\lambda, T) + \int_0^{2\pi} \rho^\circ(\lambda, \theta, \theta_i, \varphi_i)L^\downarrow(\lambda, \theta, \varphi_i)\cos \theta_i d\Omega_i
\]
First considered case

- Pyrometry of high temperature surfaces
  - sensor at close range (limited or even negligible atmosphere contributions)
  - environment much colder than the analyzed surface

\[ L_s(\lambda, T, \theta, \varphi) = \epsilon(\lambda, \theta, \varphi)B(\lambda, T) \]
Second considered case

- **Airborne/satellite remote sensing**
  - hypothesis of lambertian surface: isotropic reflectance
  - mean downwelling radiance
  - need for atmosphere compensation step

\[
L^\downarrow(\lambda) = \frac{1}{\pi} \int L_{\text{env}}(\lambda, \theta_i, \varphi_i) \cos \theta_i d\Omega_i
\]

\[
L_s(\lambda, T, \theta, \varphi) = \tau(\lambda, \theta, \varphi)L(\lambda, T) + L^\uparrow(\lambda, \theta, \varphi)
\]

\[
L(\lambda, T) = \varepsilon(\lambda)B(\lambda, T) + (1 - \varepsilon(\lambda))L^\downarrow(\lambda)
\]
What about emissivity?

In all cases we need an information on emissivity to get temperature

- relations for emissivity: **only for ideal materials**, for example Drude law for pure metals (satisfactory only for $\lambda > 2\mu m$, not valid for corroded or rough surfaces)
- databases for specific materials **in particular state** of roughness, corrosion, coatings, contaminant, moisture content …

• **Practical solution**: simultaneous evaluation of temperature and emissivity
Single-color pyrometry

- Measurement is performed in a narrow to large spectral band
- In any case, after sensor calibration, the retrieved radiance is of the form

\[ L_s(\lambda, T) = \varepsilon(\lambda)B(\lambda, T) \]

One equation, two unknown parameters

One has to estimate the emissivity (a priori knowledge)

- Sensitivity of temperature to an error in emissivity estimation:

\[ \frac{dT}{T} = -\left( \frac{T}{B} \frac{dB}{dT} \right)^{-1} \frac{d\varepsilon}{\varepsilon} \approx -\frac{\lambda T}{C_2} \frac{d\varepsilon}{\varepsilon} \]

at 1µm and T= 1100K : -0.8K/% error
at 10 µm and T= 300K : -0.6K/% error

- advantage in working at short wavelength (visible or UV pyrometry): sensitivity to emissivity error drops.
- However, the signal drops at short wavelength compromise

\[ \varepsilon \]
Two-color pyrometry (1/2)

- Adding a new wavelength adds an equation but also an unknown parameter namely the emissivity at this additional wavelength.
- Two spectral signals:
  \[
  \begin{align*}
  L(\lambda_1, T) &= \varepsilon(\lambda_1)B(\lambda_1, T) \\
  L(\lambda_2, T) &= \varepsilon(\lambda_2)B(\lambda_2, T)
  \end{align*}
  \]

- by ratioing the signals:
  \[
  \ln(L_2\lambda_2^s) - \ln(L_1\lambda_1^s) = \ln\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - \frac{C_2}{T} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)
  \]

  effective wavelength:
  \[
  \lambda_{12} = \frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}
  \]

- The problem can be solved if one has a knowledge about the emissivity ratio.
- Common hypothesis (but not necessary) : « greybody » assumption \( \varepsilon(\lambda_1) = \varepsilon(\lambda_2) \)
- Sensitivity of temperature to an error in emissivity estimation:
  \[
  \frac{dT}{T} \approx \frac{\lambda_{12}T}{C_2} \left(\frac{d\varepsilon_1}{\varepsilon_1} - \frac{d\varepsilon_2}{\varepsilon_2}\right)
  \]

  at 1µm/1.5µm and T= 1100K : -2.5K/% error = 3 times higher
  at 10µm/12µm and T= 300K : -3.7K/% error = 6 times higher
**Ratio pyrometry vs 1-color pyrometry**

<table>
<thead>
<tr>
<th></th>
<th>1-color</th>
<th>2-color</th>
<th>3-color</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
<td>$\varepsilon_1$</td>
<td>$\frac{\varepsilon_2}{\varepsilon_1}$</td>
<td>$\frac{\varepsilon_2^2}{\varepsilon_1 \varepsilon_3}$</td>
</tr>
<tr>
<td><strong>Error amplification on temperature</strong></td>
<td>$- \frac{\lambda T}{C_2}$</td>
<td>$\times \frac{\lambda}{\Delta \lambda}$</td>
<td>$\times \left( \frac{\lambda}{\Delta \lambda} \right)^2$</td>
</tr>
</tbody>
</table>

Error amplification on temperature:

$$
- \frac{\lambda T}{C_2} \times \frac{\lambda}{\Delta \lambda} \times \left( \frac{\lambda}{\Delta \lambda} \right)^2
$$
Two-color pyrometry (2/2)

- Sensitivity of temperature to an error in emissivity estimation can be reduced by decreasing the effective wavelength (by increasing $1/\lambda_1 - 1/\lambda_2$).
  - Dilemma when spreading the wavelengths: will the “greybody” assumption still hold?

- Advantage of ratio pyrometry over single color pyrometry: **immune to partial occultation, to variations of optical path transmission**

- 2-color photothermal pyrometry:
  - A laser is used to periodically heat the surface.
  - A lock-in detection is implemented to capture the modulated radiance deprived from any reflection.
  - The signal ratio is: $\frac{\epsilon(\lambda_1)\partial B/\partial T(\lambda_1, T)}{\epsilon(\lambda_2)\partial B/\partial T(\lambda_2, T)}$

- Emissivity-enhanced 2-color pyrometry
  - Additional reflective surface for introducing a cavity effect (increase of both apparent emissivities, reduction of spurious reflections)
Multiwavelength pyrometry (MWP)

- Emissivity-temperature separation is essentially an underdetermined inverse problem:
  \[ L_s(\lambda_i, T) = \varepsilon(\lambda_i)B(\lambda_i, T) \quad i = 1, N \]
  
  whatever the number of wavelengths/equations, there are always one more unknown parameters

- Two types of solutions:
  - reduce by one the degree of freedom of the discretized emissivity spectrum
    - N equations, N unknowns  the problem should be solvable (?)
      interpolation-based method
  - regularization by using a low-order emissivity model (continuous or step function)
    - N equations, much less unknowns
      least-square based method

- Long time controversy: does MWP bring a real advantage as compared to single-color or two-color pyrometry?
“Just as needed” regularization: approximating the emissivity (or its log.) by a N-2 degree polynomial.

By considering the Wien approximation and taking the logarithm, Coates showed that this may lead to “catastrophic” results:

\[
\begin{align*}
\ln \left[ L(\lambda_i, T) \lambda_i^5 / C_1 \right] &= \ln(\varepsilon_i) - C_2 / \lambda_i T & i = 1, N \\
\ln \left[ L(\lambda_i, T) \lambda_i^5 / C_1 \right] &= \sum_{j=0}^{N-2} a_j \lambda_i^j - C_2 / \lambda_i T' & i = 1, N
\end{align*}
\]

polynomial of degree N-1 passing through the N values \( \lambda_i \ln[\varepsilon(\lambda_i)] \)

constant parameter = temperature error \( C_2 \left(1/T - 1/T' \right) \)
i.e. extrapolation result at \( \lambda = 0 \)
there would be no error if a N-2 degree polynomial could be found passing exactly through the N values \( \ln[\varepsilon(\lambda_i)] \) highly improbable!

Therefore, in general, one has to count on extrapolation properties. Unfortunately, extrapolation based on polynomial interpolation leads to increasingly high errors as the polynomial degree rises!

Previous errors are systematic, i.e. method errors (errorless signal).

Same bad results are observed for measurement errors (they add to the previous ones).

The calculated temperatures are increasingly sensitive to measurement errors as the number of channels increases: OVERFITTING problem
Multiwavelength pyrometry. Low-order emissivity models (1/2)

The only solution: reduce the model complexity (low order model!)

Some examples of models:

\[ \varepsilon(\lambda_i) = \sum_{j=0}^{m} a_j \lambda_i^j \quad i = 1, \ldots, N \quad m < N - 2 \]

\[ \ln[\varepsilon(\lambda_i)] = \sum_{j=0}^{m} a_j \lambda_i^j \quad i = 1, \ldots, N \quad m < N - 2 \]

\[ \varepsilon(\lambda_i) = \frac{1}{1 + a_0 \lambda_i^2} \quad i = 1, \ldots, N \]

- Polynomials of \( \lambda^{1/2} \) or \( \lambda^{-1/2} \) for \( \ln[\varepsilon(\lambda)] \)
- Functions involving the brightness temperature \( T_R = B^{-1}(\lambda, L_s) \)
- Sinusoidal function of wavelength
- Step function (grey-band model with \( N_b \) bands).
  - 2 or 3 channels per band
  - up to \( N-1 \) single-channel bands and one dual channel band
Multiwavelength pyrometry. Low-order emissivity models (2/2)

Observable: \( Y_i = \ln[L(\lambda_i, T)\lambda_i^5 / C_i] + e_i \)

Wien approximation
Polynomial approx. of \( \ln[\varepsilon(\lambda_i)] \)
Minimizing the weighted sum
\[ \sum_{i=1}^{N} \sigma_i^{-2} \left( Y_i - \left( \sum_{j=0}^{m} a_j \lambda_i^j - \frac{C_2}{\lambda_i T} \right) \right)^2 \]

Linear least squares problem

Observable: \( Y_i = L(\lambda_i, T) + e_i \)
Planck’s law
Polynomial approx. of \( \varepsilon(\lambda_i) \)
Minimizing the weighted sum
\[ \sum_{i=1}^{N} \sigma_i^{-2} \left( Y_i - B(\lambda_i, T) \sum_{j=0}^{m} a_j \lambda_i^j \right)^2 \]

Non-linear least squares problem
Multiwavelength pyrometry. Linear least squares problem (1/5)

One is looking for the polynomial coefficients and the temperature such that:

\[
\hat{P} = \left[ \hat{a}_0 \quad \ldots \quad \hat{a}_m \quad \hat{T} \right]^T = \arg \min_{a_j,T} \sum_{i=1}^N \left( Y_i - \left( \sum_{j=0}^m a_j \lambda_i^j - \frac{C_2}{\lambda_i T} \right) \right)^2
\]

Parameter reduction for numerical purposes:

\[
\lambda_i^* = 2 \frac{\lambda_i - \lambda_{\text{min}}}{\lambda_{\text{max}} - \lambda_{\text{min}}} - 1 \quad \quad P_T^* = \frac{T_{\text{ref}}}{T} \quad \text{such that} \quad C_2 / \lambda_i T_{\text{ref}} = 1
\]

Sensitivity matrix to the reduced parameters:

\[
X = \begin{bmatrix}
1 & \lambda_1^* & \lambda_1^*^2 & \ldots & -\frac{C_2}{\lambda_1 T_{\text{ref}}} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & \lambda_N^* & \lambda_N^*^2 & \ldots & -\frac{C_2}{\lambda_N T_{\text{ref}}} \\
\end{bmatrix}_{N \times m+2}
\]

The sensitivity to the temperature inverse is very smooth, close to linear. We can thus expect a strong correlation between the parameters (near collinear sensitivity vectors).
Multiwavelength pyrometry. Linear least squares problem (2/5)

Assuming that the measurement errors are additive, uncorrelated and of uniform variance, an estimation of the parameter vector $\hat{\mathbf{P}}^*$ in the least squares sense is obtained by solving the linear system:

$$(\mathbf{X}^T \mathbf{X}) \hat{\mathbf{P}}^* = \mathbf{X}^T \mathbf{Y}$$

Near collinear sensitivity vectors lead to a **high condition number** of $(\mathbf{X}^T \mathbf{X})$

The **condition number** (ratio of maximum to minimum eigenvalue) indicates the rate at which the identified parameters will change with respect to a change of the observable (sensitivity to measurement errors)

$$\frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} = 1.75$$

Huge increase of the condition number with the polynomial degree

Problems are expected with models of degree 2 and more
Multiwavelength pyrometry. Linear least squares problem (3/5)

Condition number: only an upper bound of error amplification. The diagonal of the covariance matrix \( (X^T X)^{-1} \) is of greater value for analyzing the error propagation.

\[
\begin{bmatrix}
\sigma_{pe}^2
\end{bmatrix} = \text{diag}\left((X^T X)^{-1}\right) \sigma^2
\]

Assumed uniform variance of the observable error around the mean estimator value due to **radiance error propagation to the parameters** (does not include the bias due to model error, i.e. misfit between true emissivity and emissivity model).

Error amplification factors

\[
\frac{\sigma_\varepsilon}{\varepsilon} = K_\varepsilon \frac{\sigma_L}{L}
\]

\[
\frac{\sigma_T}{T} = K_T \lambda_{\text{min}} T \frac{\sigma_L}{L}
\]
Multiwavelength pyrometry. Linear least squares problem (4/5)

The errors are rapidly rising with the degree of freedom
Multiwavelength pyrometry. Linear least squares problem (5/5)

Practical application:
- target at 320K,
- 1% radiance noise
- radiometer with **seven wavelengths** between 8 and 14µm

<table>
<thead>
<tr>
<th>Polynomial degree</th>
<th>$\sigma_T$ (K)</th>
<th>$\sigma_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5</td>
<td>0.02</td>
</tr>
<tr>
<td>1</td>
<td>9.4</td>
<td>0.13</td>
</tr>
<tr>
<td>2</td>
<td>64</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The mentioned standard errors only reflect what happens when noise corrupts the radiance emitted by a surface *which otherwise perfectly follows the chosen model (polynomial model of degree 0, 1 or 2)*
Multiwavelength pyrometry. A look to the ETS solutions (1/2)

To each estimated temperature value \( \hat{T} \) one can associate an emissivity profile \( \hat{\varepsilon}(\lambda, \hat{T}) \) according to:

\[
\hat{\varepsilon}(\lambda, \hat{T}) = \frac{L(\lambda, T)}{B(\lambda, \hat{T})} = \frac{\varepsilon(\lambda) B(\lambda, \hat{T})}{B(\lambda, \hat{T})}
\]

They constitute an infinite number of perfect solutions to the underdetermined problem.

Let us now consider a 1-degree emissivity model. Which, among all candidate profiles, fits a straight line at best?
Multiwavelength pyrometry. A look to the ETS solutions (2/2)

Misleading idea: « the chosen model is used to fit the true emissivity profile »
Actually, the least squares method selects among all possible solutions, the one which conforms at best to the model, taking into account a weighting by \( B(\lambda, \hat{T}) \)

7-channels pyrometer [8-14\( \mu \)m]

- Errorless radiance leads to a 15K bias for temperature and 0.06 to 0.2 emissivity underestimation (systematic or model error)
- With a 2-degree polynomial model, the results are even worse: \( \hat{T} = 230K, \hat{\varepsilon} > 2! \)
Multiwavelength pyrometry. Non-linear least squares and Monte-Carlo analysis (1/3)

- Measurements are simulated by adding artificial noise to the theoretical emitted radiance (Gaussian distribution with a spectrally uniform standard deviation: 0.2% to 6% of the maximum radiance value)
- Statistical analysis on 200 simulated experiments
- Chosen model: 1-degree polynomial

High systematic error when the emissivity model (1-degree polynomial) doesn’t match with the true profile (>15K RMS !).
Otherwise, 0.1 emissivity error and 8K temperature error for 1% radiance error. Same holds when the true profile departs by 1% from a straight line!
• Does it help to increase the number of spectral channels?

When the emissivity model (1-degree polynomial) matches with the true profile we observe the classical $N^{-1/2}$ uncertainty reduction.

Otherwise, emissivity and temperature RMS error remain high (systematic errors always dominate); they even increase with $N$ for present example!
Conclusion on LSMWP with low-order emissivity models

- Reasonable RMS values can be obtained only when the implemented emissivity model perfectly matches the real emissivity spectrum. Otherwise, there are important systematic errors.
- When can we guarantee that a specific model perfectly matches to reality?
- LSMWP focuses on profile shape rather than on magnitude. Add a penalization based on emissivity level (mean or local) to force the solution to remain close to a predetermined level (a priori information).
- When using only the emitted spectral radiance, there is no valuable reason for implementing MWP instead of the simpler one-color or bispectral pyrometry.
ETS in the field of remote sensing

- Low-altitude airborne remote sensing
- High-altitude airborne remote sensing
- Polar-orbiting satellites (low-earth orbit)
- Geostationary satellite
SYSIPHE main characteristics

DLR Dornier DO-228
2000 m, 73 m/s

Sampling pitch 0.5 m

Spectral bandwidth [0.4-12 μm]

Spectral sampling
10 nm from 0.4 to 2.5 μm
20 cm⁻¹ from 3 to 5.3 μm
10 cm⁻¹ from 8 to 11.5 μm
Specific features of IR remote sensing

- Measurements are highly conditioned by the **radiative properties of the atmosphere** (transmission, emission toward the earth surface and then reflection, emission along the optical path, scattering, …). Optical path in air from ~100 m to several km.
  - **Atmosphere compensation** is necessary
  - Atmosphere properties considered **uniform** in images of several km²
- Footprint is generally large: from ~10 cm for low altitude airborne sensors to ~2 km for sensors on geostationary satellites → **aggregation** of various materials and temperatures (**desaggregation** = inversion problem)
- In [8-14µm] band, natural surfaces (soil, vegetation, water) have **high emissivity values** (> 0.9). Generally considered as Lambertian.
Evaluation of atmosphere contributions

- Example of a grey surface (ε=0.9) at T=313K
- Radiative transfer simulations with MODTRAN; (mid-latitude summer atmospheric model; rural aerosols)

\[ L(\lambda, T) = \varepsilon(\lambda)B(\lambda, T) + (1 - \varepsilon(\lambda))L^\uparrow(\lambda) \]
Evaluation of atmosphere contributions

- Example of a grey surface ($\varepsilon=0.9$) at $T=313$K sensed by an IR instrument at 1900 m altitude.
- Radiative transfer simulations with MODTRAN; (mid-latitude summer atmospheric model; rural aerosols)

\[
L_s(\lambda, T, \theta, \varphi) = \varepsilon(\lambda, \theta, \varphi)L(\lambda, T) + L^\uparrow(\lambda, \theta, \varphi)
\]

\[
L(\lambda, T) = \varepsilon(\lambda)B(\lambda, T) + (1-\varepsilon(\lambda))L^\uparrow(\lambda)
\]

at-sensor radiances
Atmosphere separate compensation

Radiative transfer simulation (MODTRAN, MATISSE…) with:

- standard atmospheric models (temperature+humidity profiles)/climate/season/aerosols

- radiosonde data profiles of pressure, temperature, constituents

- IR sounding near 4.3µm for CO₂ and between 4.8-5.5µm for H₂O +
  neural networks allows retrieving mean atmosphere temperature and columnar water vapor under the sensor. These values are then used to scale a set of standard atmosphere profiles used in MODTRAN and get closer to the true atmosphere profiles. Final MODTRAN computation

\[
\begin{aligned}
\tau(\lambda, \theta, \varphi) \\
L^i(\lambda, \theta, \varphi) \\
L^s(\lambda)
\end{aligned}
\]
Emissivity-Temperature separation

• Proper atmosphere compensation provides **ground leaving radiance**:

\[
L(\lambda, T) = \frac{L_s(\lambda, T) - L^{\dagger}(\lambda)}{\tau(\lambda)} = \varepsilon(\lambda)B(\lambda, T) + (1 - \varepsilon(\lambda))L^{\dagger}(\lambda)
\]

Emissivity estimation \( \hat{\varepsilon}(\lambda) \) from a temperature estimation \( \hat{T} \) according to

\[
\hat{\varepsilon}(\lambda) = \frac{L(\lambda, T) - L^{\dagger}(\lambda)}{B(\lambda, \hat{T}) - L^{\dagger}(\lambda)}
\]
**Spectral Smoothness method (SpSm) (1/2)**

- When temperature estimation $\hat{T}$ is in error, the profile will contain **detailed spectral features** originating from $L(\lambda,T)$ and $L^\dagger(\lambda)$ (gas absorption bands)

\[ \hat{\epsilon}(\lambda) = \frac{L(\lambda,T) - L^\dagger(\lambda)}{B(\lambda,\hat{T}) - L^\dagger(\lambda)} \]

- Adjust $\hat{T}$ until $\hat{\epsilon}(\lambda)$ is deprived of these artifacts

  "smooth" emissivity spectrum

Knuteson, 2006
Spectral Smoothness method (SpSm) (2/2)

- Field tests at ONERA (K. Kanani thesis)

Spectroradiometer :
BOMEM MR254

- SpSm requires the atmospheric compensation to be very precise
- SpSm requires high spectral resolution (< 10 cm\(^{-1}\)) in order to capture sufficient details of the atmosphere spectral features. Restricted to hyperspectral data. Spectral calibration errors are highly detrimental
- Radiance error of 0.5% \(\rightarrow\) 1.6K RMS and 0.8K bias for temperature and 0.023 RMS and 0.027 bias for emissivity
Multi-temperature method : a pitfall ? (1/3)

- $N_T$ temperature levels
- $N$ channels

However, using Wien’s approximation, it can be shown that, when there is no reflection contribution, the problem remains ill-conditionned!

- With errorless radiance, there is an infinite number of solutions defined by:

\[
\frac{1}{T_i'} = \frac{1}{T_i} + \text{cst}
\]

\[
\hat{\varepsilon}(\lambda) = \varepsilon(\lambda) \exp \left( \frac{C_2}{\lambda} \text{cst} \right)
\]

- For two temperatures, the sensitivity matrix is

\[
\begin{bmatrix}
-C_2 \\
\frac{C_2}{\lambda_{T_{\text{ref}}}} \\
\vdots \\
\vdots \\
\frac{C_2}{\lambda_{T_{\text{ref}}}} \\
0 \\
-I_N \\
\vdots \\
\vdots \\
I_N \\
\end{bmatrix}
\]

\[
\text{det}(X^T X) = 0
\]
Multi-temperature method (2/3)

• The problem remains badly conditioned when using Planck’s law
• Degeneracy is alleviated thanks to the presence of reflections
• Inversion robustness depends on the spectral richness of the reflections

Case of two temperatures.
Nonlinear least-squares approach for identifying the $N$ emissivities and the two temperatures

$$[\varepsilon_i, T_1, T_2]^T = \arg \min_{\varepsilon_i, T_1, T_2} \sum_{i=1}^{N} \left( L(\lambda_i, T_1) - \left( \varepsilon_i B(\lambda_i, T_1) + (1 - \varepsilon_i) L^B(\lambda_i) \right) \right)^2 + \sum_{i=1}^{N} \left( L(\lambda_i, T_2) - \left( \varepsilon_i B(\lambda_i, T_2) + (1 - \varepsilon_i) L^B(\lambda_i) \right) \right)^2$$

Illustration for the case of a greybody ($\varepsilon=0.9$) at $T_1 = 320$K.
Second temperature is 1K, 5K, 10K or 30K higher.
Downwelling radiance is either:
• blackbody radiance at 300K
• same by weighting with a uniform random distribution (simulation of the presence of detailed spectral features)

Standard errors of identified parameters obtained from covariance matrix (local linearization)
Better results are obtained by increasing the number of channels and the temperature difference.

High constraints to get a temperature RMS error lower than 1K!

Constraints on images co-registration, emissivity stability.
Conclusion

• Radiative temperature measurement
  • advantage: non-contact
  • disadvantage: underdetermined inverse problem due to emissivity
• « Mirage » of multiwavelength pyrometry
  • only very low order emissivity models could be considered (ex: 1 degree polyn.)
  • no significant benefit vs. single or two color pyrometry
• IR remote sensing takes profit from high emissivity of natural surfaces and from their spectral smoothness with respect to downwelling radiance
  • SpSm method: implementation phase for Sysiphe hyperspectral camera
  • Multi-temperature method
    • ineffective without reflections from spectrally rich and well characterized environment
    • additional constraints